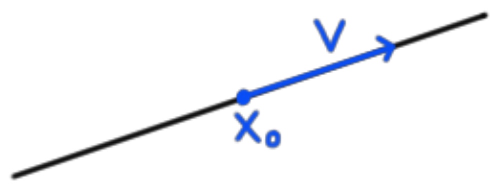
Lecture 3 - Parameterizations, velocity, acceleration, arc length - 7/7/2014 — Interphase 2014 Calc 3

13. Parameterizations

a. A line in any number of dimensions is parametrized by

$$\mathbf{x}(t) = \mathbf{x}_0 + t\mathbf{v}, \text{ for } -\infty < t < \infty,$$

where $\mathbf{v}$ is the tangent vector.



b. A line segment is given by $\mathbf{x}(t) = \mathbf{x}_0 + t\mathbf{v}$ for $a \le t \le b$.

c. A circle of radius r traversed counterclockwise can be parametrized as

$$\langle r\cos\theta, r\sin\theta\rangle \text{ for } 0 \le \theta < 2\pi.$$

It can also be parameterized by time or arc length.

d. A parametrization of a curve is a function which takes a single variable (a parameter) and a range of values of that parameter.

For example, a parametrization in 2d with respect to time is of the form

$$\langle x(t), y(t)\rangle \text{ for } a \le t \le b$$

for some functions $x(t), y(t)$.

e. Convenient parameters include time, angle swept, distance traveled. When in doubt, choose time.

f. A given curve can be parametrized in many ways.

g. The graph $y = f(x)$ can be parameterized by $(t, f(t))$ for $a \le t \le b$.

h. To parametrize a complication motion, express the motion as the vector sum of many separate parts (e.g. translations and rotations) and figure out how each of those parts are related.

14. Velocity, speed, acceleration, arclength

a. The velocity of a parameterization is $\mathbf{v}(t) = \frac{d\mathbf{x}}{dt}$.

b. The speed of a parameterization is $|\mathbf{v}(t)| = \left|\frac{d\mathbf{x}}{dt}\right|$.

c. The acceleration of a parameterization is $\mathbf{a}(t) = \frac{d\mathbf{v}}{dt}$.

d. The velocity vector $\mathbf{v}(t)$ is tangent to the curve $\mathbf{x}(t)$.

e. The arc length of the curve $\mathbf{x}(t)$ traced out between $t = a$ and $t = b$ is given by

$$s = \int_a^b \left| \frac{d\mathbf{x}(t)}{dt} \right| dt.$$