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Analysis I
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Week 6 — Summary — Limits of Functions

60. Functions of multiple variables may have a limit in each variable separately but not in all variables together.

61. Pointwise convergence vs. uniform convergence vs L_1 convergence vs L_2 convergence.

62. The space of bounded maps from one complete normed vector space to another is complete with respect to the sup norm.

63. The uniform limit of continuous functions is continuous.

64. Limits do not interchange in general. That is, $\lim_{x\to x_0} \lim_{y\to y_0} f(x,y) \neq \lim_{y\to y_0} \lim_{x\to x_0} f(x,y)$ in general.

65. If $\lim_{x\to x_0} f(x,y)$ exists for all y, and $\lim_{y\to y_0} f(x,y)$ exists uniformly for all x, then

$$\lim_{x\to x_0} \lim_{y\to y_0} f(x,y) = \lim_{y\to y_0} \lim_{x\to x_0} f(x,y) = \lim_{(x,y)\to(x_0,y_0)} f(x,y).$$