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Analysis I
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HW 5

Due: 4 Oct 2016

The problems are written in the format ‘chapter.section.problem-number’ from Lang’s book. Practice problems are not to be handed in. The HW problems will be graded thoroughly and may be revised once, by the Tuesday after they were returned. Please submit each problem on a detached sheet of paper with your name on it.

Practice problems:

1. VII.3.3
2. VII.3.7

Homework problems:

P13. Draw and *completely* justify the Venn diagram for the following sets of sequences of functions:

- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ pointwise}, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ uniformly}, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ in } L_1, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$
- $\{\{f_n\}_{n=1}^{\infty} \mid f_n \rightarrow 0 \text{ in } L_2, f_n : \mathbb{R} \rightarrow \mathbb{R}\}$

To show that any particular region in the Venn diagram is nonempty, provide an example of a function in that region. It is possible to complete this exercise using only functions that are bounded, have bounded support, and are piecewise constant.

P14. Same as (P13) but with $f_n : [0, 1] \rightarrow \mathbb{R}$.

P15. Same as (P13) but with $f_n : \mathbb{N} \rightarrow \mathbb{R}$. This problem is the same as comparing convergence of a sequence of sequences in the following senses: pointwise, uniform, ℓ_1 , and ℓ_2 .