

11 October 2014  
Analysis I  
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## HW 7

Due: Oct 21 in class. Justify all of your work.

1. VI.3.4
2. VI.3.5
3. VI.3.6. Make sure your norms are in fact norms on the space. In particular, make sure they are finite for any element in the vector space.
4. VI.4.1
5. VI.4.2
6. VI.4.4
7. Let  $S$  be a subset of a normed vector space. Prove that the intersection of all closed sets containing  $S$  equals the set of adherent points of  $S$ . This proves the equivalence of the two definitions of the closure of  $S$ . Recall that an adherent point of a set  $S$  is a point  $x$  such that  $\forall \varepsilon > 0, \exists y \in S$  such that  $\|y - x\| < \varepsilon$ .
8. VI.5.2
9. VI.5.8
10. Let  $V = \ell_\infty$ . Let  $S = \{x \mid \|x\|_1 \leq 1\}$ . Is  $S$  open with respect to the  $\ell_\infty$  norm? Is it closed with respect to the  $\ell_\infty$  norm? Prove it.
11. If you were to present Theorem VII.3.2 (Uniform limit of continuous functions is continuous) in class, write up the notes of what you would say.