7 March 2013
18.085
Computational Science and Engineering I
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Quiz 1

Rules: Open notes, open book, closed electronics. Time limit: 90 minutes. Please show all of your work.

1. Let z be a nonzero column vector in $\mathbb{R}^n$. Let $A = zz^t$.

   (a) (10 points) What is the rank of A?

   (b) (10 points) Show that the null space of A is the set of all vectors perpendicular to z.

2. (20 points) Find an orthonormal basis for the space of points $(x, y, z) \in \mathbb{R}^3$ satisfying

$$x + y + z = 0 \text{ and } -x + y + 2z = 0.$$

3. (20 points) Find the rank and null space of B. Specify the null space entirely; do not just state its dimension. Justify your answer completely.

$$
B = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
$$

4. (20 points) Find the LU decomposition of C.

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 \end{pmatrix}$$

5. (20 points) A way to find the inverse of a matrix.

    (a) (4 points) Suppose B is a 4×4 matrix. For what vector y is $By =$ 2nd column of B + 3rd column of B?

    (b) (8 points) Let A be a 4×4 nonsingular matrix. Find the vector b for which the solution to $Ax = b$ is the first column of A^{-1}.

    (c) (8 points) Inspired by (b), describe a method for finding the inverse of an $n \times n$ matrix. How many floating point operations are needed to implement this method?

        Use the fact that an LU factorization of A involves $\sim \frac{2}{3}n^3$ floating point operations, and that back substitution of a triangular system involves $\sim n^2$ floating point operations.