18 April 2013
18.085
Computational Science and Engineering I
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Problem Set 5

Due: **25 April 2013** in class.

Print or write out any Matlab input and output.

1. (10 points) The Heaviside function $H(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$.

   (a) Plot $\int_{-\infty}^{x} \delta(y-2)dy$ as a function of x and express it in terms of Heaviside functions.

   (b) Same, but for $\int_{-\infty}^{x} \left(\delta(y-3) - \frac{1}{2}\delta(y+2)\right) dy$

2. (10 points) Without solving analytically, sketch the solution to

$$-\frac{d^2u}{dx^2} = f(x)$$
$$u(0) = 0$$
$$\frac{du}{dx}(1) = 0$$

   for

   (a) $f(x) = -1$

   (b) $f(x) = \delta(x - 1/2) + \delta(x - 3/4)$

   (c) $f(x) = \begin{cases} 1 & \text{if } x \leq 1/2 \\ -1 & \text{if } x > 1/2 \end{cases}$

3. (10 points) Find the solution to

$$-\frac{d^2u}{dx^2} = \sin\frac{\pi x}{L}$$
$$u(0) = 0$$
$$u(L) = 0$$

4. (10 points) Find the solution to

$$-\frac{d^2u}{dx^2} = \delta(x - L/2)$$
$$u(0) = 0$$
$$\frac{du}{dx}(L) = 0$$

5. (20 points) Consider

$$\frac{dy}{dt} = Ay$$
$$y(0) = y_0$$

where $y(t) \in \mathbb{R}^2$ and A is a 2×2 matrix. Let y_n be the computed value of y at $t = n\Delta t$.

(a) Write out a Matlab expression to compute y_{n+1} in terms of y_n, A, Δt, the identity matrix I, and the Matlab command "\" for

i. Forward Euler

ii. Backward Euler

iii. Trapezoidal rule

(b) Let $A = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$ and $y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Use Matlab to compute $y(1)$ using Forward Euler with $\Delta t = 0.01$.

(c) Same as (b), but with Trapezoidal rule.

(d) Same as (b), but use Matlab's built-in 'ode45' command.