7 February 2013
Computational Science and Engineering I
Paul E. Hand
hand@math.mit.edu

Problem Set 1

Due: **21 February 2013** in class

1. (10 points) What is the rank of A? Answer by stating a collection of columns that are independent, showing that those columns are independent, and arguing that there can be no additional independent columns.

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

2. (10 points) Find and sketch the null spaces of the following matrices.

   (a) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$

3. (10 points) Find c_1, c_2, c_3, c_4, c_5 such that

$$\begin{pmatrix} 6 \\ 4 \\ 5 \\ 6 \\ 9 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + c_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + c_5 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

   Let Matlab do the computation.

4. (20 points) Consider the following truss. The horizontal bar has length L. The diagonal bars have length $\sqrt{2}L$.
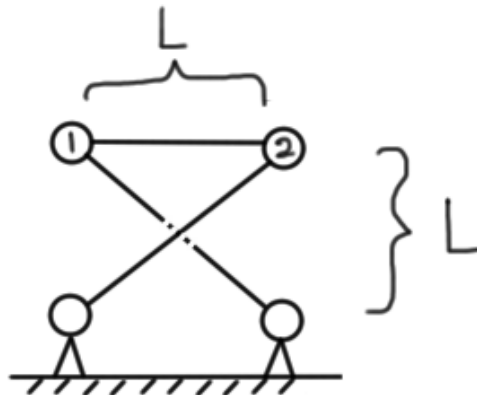
(a) Write down the 3×4 linear system, $Ax = 0$, that enforces the constraint that all bars are of fixed length. How many independent modes of deformation of the truss are there?

(b) Find the mode of deformation by physical reasoning. Check that it works by verifying $Ax = 0$.

(c) Use Matlab to find the mode. Does it agree with (b)?

5. (20 points) Consider the following truss. All vertical and horizontal rods have length L. Both diagonal rods have length $\sqrt{2}L$.
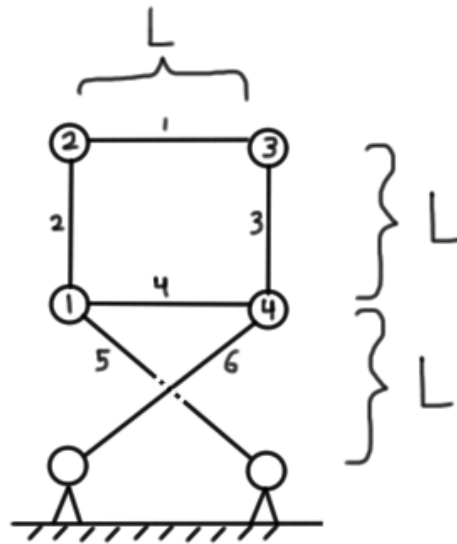
(a) Write out the 6×8 linear system, $Ax = 0$, that enforces the constraint that all bars are of fixed length. How many independent modes of deformation of the truss are there? You may assume A has rank 6.

(b) Sketch the independent modes of deformation based on physical reasoning. Find one mode and check that it works by verifying $Ax = 0$.

(c) Use Matlab to find the null space of A. Sketch the output. Does it include your answers from (b). Reconcile any discrepancies.