<div align="right">**Name:**</div>

Quiz 2 Makeup

Due: Tuesday, May 1 in class

Rules: You may not collaborate with classmates. You may use any course notes, textbooks, or websites. You may check your work with Matlab, but your solutions must be complete without Matlab. Please show all of your work. Your quiz 2 grade will be replaced by $\frac{1}{3}$(quiz 2 grade) $+ \frac{2}{3}$(makeup quiz 2 grade).

1. (20 points) Let A be a 3×3 real, symmetric, positive-definite matrix:

$$A = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

    (a) (5 points) Is it possible for $d = 0$? Justify why not or produce an example matrix A.

    (b) (5 points) Is it possible for $c = 0$? Justify why not or produce an example matrix A.

    (c) (10 points) Show that $\begin{pmatrix} a & b \\ b & d \end{pmatrix}$ is positive definite.

2. (20 points) Let A be an $m \times n$ matrix with rank n and $m > n$. Let b be a point in m dimensional space.

(a) (5 points) Set up a least squares problem to find the nearest point to b on the range of A.

(b) (10 points) Let $A = \tilde{Q}\tilde{R}$ be the reduced QR factorization of A. Use the normal equations to show that the nearest point to b is given by $\tilde{Q}\tilde{Q}^t b$.

(c) (5 points) Let

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}, \text{ and } b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

Find the reduced QR factorization of A and use it to find the nearest point to b on the range of A.

3. (20 points) Without a computer, find the fft of

$$
\begin{pmatrix}
1 \\
3 \\
1 \\
-1 \\
1 \\
3 \\
1 \\
-1 \\
1 \\
3 \\
1 \\
-1
\end{pmatrix}
$$

4. (20 points)

(a) (10 points) Let $q = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$. Find the singular values and condition number of $A = qq^t$.

(b) (10 points) Let

$$Q = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

Find the singular values and condition number of $A = QQ^t$.

5. (20 points) Suppose you have measurements $(t_i, y_i) = (0, 0.1), (1, 0.9), (2, -0.3), (3, 0), (4, 0.5)$ that belong on a curve $y = a + b\sin\left(\frac{\pi}{2}t\right) + c\cos\left(\frac{\pi}{2}t\right)$ for unknown a, b, c. Set up but do not solve a least squares problem to find a, b, c.