CSU200 Discrete Structures Professor Fell Special Functions

We define here some functions of numbers that you will frequently use in computing practice and theory.

factorial function, n!

If n is a positive integer, the *factorial n!* is defined to be

$$n! = n(n-1)\cdots 2\cdot 1.$$

So, for example

$$3! = 3\cdot 2\cdot 1 = 6$$

$$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720.$$

We also define $0! = 1.$

This fits with the combinatorial formulas we will derive later in the semester.

floor and ceil

The functions *floor* and *ceiling* (or *ceil*) are functions that take real arguments and give integer values.

$$floor(x) = \lfloor x \rfloor = \text{the greatest integer less than or equal to } x.$$

$$ceiling(x) = \lceil x \rceil = \text{the least integer greater than or equal to } x.$$

procedure: `(floor real)`
returns: the integer closest to *real* toward $-\infty$

```
(floor 19) ⇒ 19
(floor 2/3) ⇒ 0
(floor -2/3) ⇒ -1
(floor 17.3) ⇒ 17.0
(floor -17/2) ⇒ -9
```

procedure: `(ceiling real)`
returns: the integer closest to *real* toward $+\infty$

```
(ceiling 19) ⇒ 19
(ceiling 2/3) ⇒ 1
(ceiling -2/3) ⇒ 0
(ceiling 17.3) ⇒ 18.0
(ceiling -17/2) ⇒ -8
```

These and other Scheme examples are from The Scheme Programming Language, *Second Edition* © 1996.

trunc and round

The functions *truncate (or trunc)* and *round* are also functions that take real arguments and give integer values. You probably learned about *round* back in elementary school.

procedure: `(round real)`
returns: the integer closest to *real*

If *real* is exactly between two integers, the closest even integer is returned.

```
(round 19) ⇒ 19
(round 2/3) ⇒ 1
(round -2/3) ⇒ -1
(round 17.3) ⇒ 17.0
(round -17/2) ⇒ -8
(round 2.5) ⇒ 2.0
(round 3.5) ⇒ 4.0
```

procedure: (truncate real)
returns: the integer closest to real toward zero

```
(truncate 19) ⇒ 19
(truncate 2/3) ⇒ 0
(truncate -2/3) ⇒ 0
(truncate 17.3) ⇒ 17.0
(truncate -17/2) ⇒ -8
```

absolute value

The *absolute value* of a real number is defined by

$$abs(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

procedure: `(abs` *`real`*`)`
returns: the absolute value of *real*

```
(abs 1) ⇒ 1
(abs -3/4) ⇒ 3/4
(abs 1.83) ⇒ 1.83
```

References:

"Discrete Mathematics, second edition" by James L. Hein, pages 79, 84, Jones and Bartlett Mathematics, 2003.
floor and ceiling functions
The Scheme Programming Language, *Second Edition* © 1996. Electronically reproduced by permission of Prentice Hall, Upper Saddle River, New Jersey