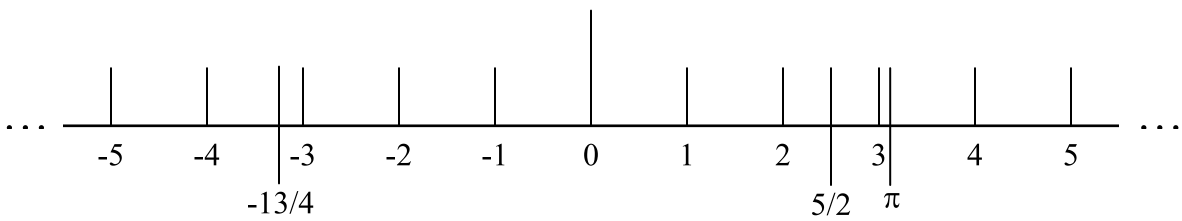
CSU200 Discrete Structures Professor Fell Sets of Numbers

There are several sets of numbers that we will talk about in this course. You have probably met these sets already but we want to make their meaning precise and make sure that we are all calling things by the same names.

Real Numbers

The *real numbers* are all the numbers we plot on the number line.



The symbol " . . ." is used to show that the number line keeps going in both directions. The set of real numbers is usually denoted by $\mathbb{R}$ or R or R.

Subsets of Real Numbers

Most of the numbers we will study this semester are real numbers and we will often need to refer to certain special subsets of the real numbers.

The *positive real numbers*, $\mathbb{R}^+$, is the set of all real numbers that are strictly greater than 0 and the *negative real numbers*, $\mathbb{R}^-$, is the set of all real numbers that are strictly less than 0.

The *natural numbers*, $\mathbb{N}$, are the numbers that are used for counting, including 0.

$$\mathbb{N} = \{0,1,2,3,\cdots\}$$

where {0, 1, 2, …} is read, "the set containing zero, one, two, and so on."

The *integers*, $\mathbb{Z}$, include the natural numbers and their negatives.

$$\mathbb{Z} = \{\cdots -3,-2,-1,0,1,2,3,\cdots\}$$

"Most mathematicians, at least when they're talking to each other, use Z to refer to the set of integers. In German the word "zahlen" means "to count" and "Zahl" means "number." See: http://mathforum.org/dr.math/faq/faq.integers.html.

As with the real numbers, we use $\mathbb{Z}^+$ to denote the positive integers and $\mathbb{Z}^-$ to denote the negative integers. Here are some other subsets of the integers that we will talk about:

The *even integers* are all the integers that are divisible by 2, and the *odd integers* are the integers that are not divisible by 2. Note that 0 is even.

The *digits* are the integers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 .

The *rational numbers*, $\mathbb{Q}$, are all the numbers that can be expressed as ratios of integers, e.g. $\frac{1}{2}, \frac{397}{23}, \frac{-4}{7}, \frac{19}{1}$. The name $\mathbb{Q}$, no doubt, comes from "quotient."

The *irrational numbers*, are all the real numbers that are not rational, i.e. not in $\mathbb{Q}$. Irrational numbers sometimes arise as solutions to algebraic equations. For example, $\sqrt{5}$ is a solution to $x^2 - 5 = 0$ but $\sqrt{5}$ cannot be expressed as a ratio of integers.

There are irrational numbers, π for example, that are not solutions to algebraic equations. These real numbers are called *transcendental numbers*.

Imaginary and Complex Numbers
There are very simple algebraic equations that do not have real number solutions, for example

$$x^2 + 1 = 0.$$

Mathematicians invented *i*, to solve this equation. By definition, $i^2 = -1$. The real multiples of *i* are called the *imaginary numbers*.

The *complex numbers*, $\mathbb{C}$, are all the numbers that can be formed by adding real and imaginary numbers to each other. All complex numbers can be written in the form, $a + bi$, where a and b are real numbers. This may seem like an abstract exercise but, in fact, complex numbers have many application in physics and electrical engineering. We need complex numbers, not just imaginary numbers to solve all quadratic equations. The solutions to

$$x^2 - 2x + 2 = 0$$

are x = 1 + i and x = 1 - i.

This material is based of section 1-1 of "Algebra and Trigonometry: Functions and Applications" by Paul A.Foerster, Addison-Wesley, 1980.