CS U200 Discrete Structures **Fall 2003**

Written Homework 5 **Due November 25, 2003 at the start of class**

Reading: Rosen 3.3 Mathematical Induction

Use mathematical induction to prove the statements in the problems below. Follow the steps used in the proofs in section 3.3 of Rosen.

a) DEFINE P(n): Let P(n) be the proposition ...

b) BASIS STEP: P(1) is true since ...
(You may have to start with P(3) or some other starting case.)

c) INDUCTIVE STEP: Assume that P(k) is true for positive integer k and use that to prove that P(k+1) is true.

WRITE OR TYPE YOUR PROOFS NEATLY.

1. If n is a positive integer, then

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2} \right)^2 .$$

2. For every natural number n, $n \mid (n^3 - n)$.

3. If n is an integer and $n \geq 5$, then $2^n \geq n^2$.

4. If n is an integer $n \geq 1$, then $\sum_{k=1}^{n} \frac{1}{k^2} \leq 2 - \frac{1}{n}$.

5. **a)** Find a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + + \frac{1}{n \cdot (n+1)}$$

by examining the values of this expression for small values of n.

b) Use mathematical induction to prove your result.