CS U200 Discrete Structures **Fall 2003**

Written Homework 4 **Due November 7, 2003 at the start of class**

1. A laboratory timer is set to 0 at midnight. The timer keeps a count of the number of hundredths of a second that have passed. For example, if the count is 3330367, the actual time is: 9:15:03 67, i.e. 9 hours, 15 minutes, 3 seconds and 67 hundredths. Use **div** and **mod** to give an algorithm that will convert this count to the actual time of day.

2. Analyze the theorems below, as indicated.

Theorem: If n is an integer greater than 1 and n is not prime, then $2^n - 1$ is not prime.

a) Identify the given (hypothesis)
b) Identify the goal (conclusion)
c) Does 6 satisfy the given conditions? What does the theorem tell you in this case? Check that this conclusion is correct.
d) What can you conclude from the theorem when n = 15? Check that this conclusion is correct.
e.) What can you conclude from the theorem when n = 11?

Theorem: If x is a real number and x > 0, then there is a real number y such that y(y-1) = x.

a) Identify the given (hypothesis)
b) Identify the goal (conclusion)
c) Does 6 satisfy the given conditions? What does the theorem tell you in this case? Check that this conclusion is correct.
d) Does 5 satisfy the given conditions? What does the theorem tell you in this case? Check that this conclusion is correct.
e) Does -6 satisfy the given conditions? What does the theorem tell you in this case?
f) Prove the theorem.

3. The **two's complement** representation of integers is commonly used in computers to simplify arithmetic. To represent an integer x with $-2^{n-1} \le x \le 2^{n-1} - 1$ for a given n, a total of n bits is used. The leftmost bit is used to represent the sign. A 0 bit in this position is used for positive integers and a 1 bit in this position is used for negative integers. For a positive integer, the remaining bits are just the binary representation of the number. For a negative number, the remaining bits are the binary expansion of $2^{n-1} - |x|$.

a. Using bit string of length 8, give the two's complement representation of these integers.

i. 54 ii. 29 iii -13 iv. -6

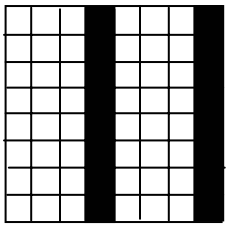
b. What integer, in standard base 10 notation, is represented by each of the following 8-bit two's complements numbers?

i. 01010110 ii. 11010110 ii. 01100101 iv. 10011010

4. Patterns

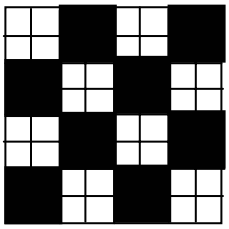
Patterns, like the ones above, are available in most drawing programs for filling regions. A pattern is defined by an 8 by 8 array of bits. So, for example, the pattern on the left is given by

0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1



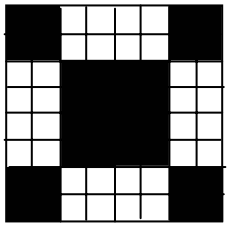
and the second pattern by

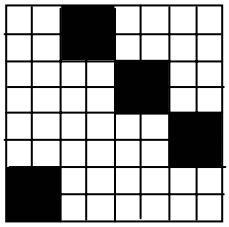
0 0 1 1 0 0 1 1
0 0 1 1 0 0 1 1
1 1 0 0 1 1 0 0
1 1 0 0 1 1 0 0
0 0 1 1 0 0 1 1
0 0 1 1 0 0 1 1
1 1 0 0 1 1 0 0
1 1 0 0 1 1 0 0

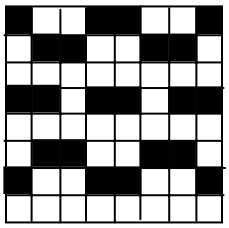


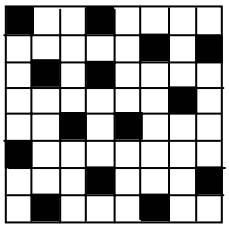
The 0's are where the white is and the 1's are where the black is. Each row is an 8-bit binary number. As we know, a 4-bit binary number can be expressed as a single hex-digit so an 8-bit binary number can be expressed with two hex-digits. Designers specify a pattern by giving eight 2-hex-digit numbers. The two patterns just above are given by 11, 11, 11, 11, 11, 11, 11, 11 and 33, 33, CC, CC, 33, 33, CC, CC.

a. Give the eight 2-hex-digit numbers that represent the following patterns.









b. Use graph paper to show the pattern described by each of the following sequences of eight 2-hex-digit numbers.

CD, A1, DC, 1A, CD, A1, DC, 1A

AA, 55, AA, 55, AA, 55, AA, 55

39, 7B, 42, 88, 88, 24, B7, 93

88, 44. 22. 11. 88. 44. 22. 11