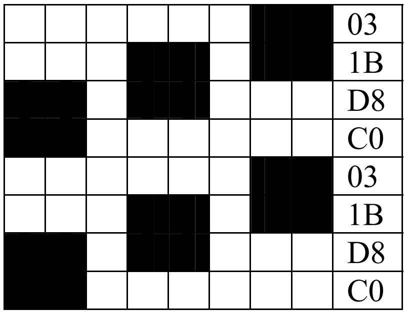
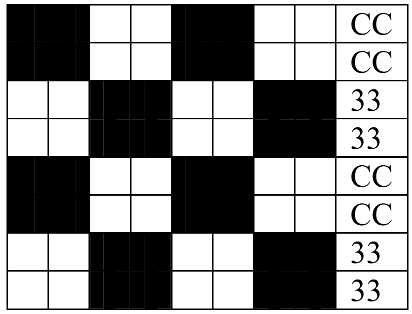
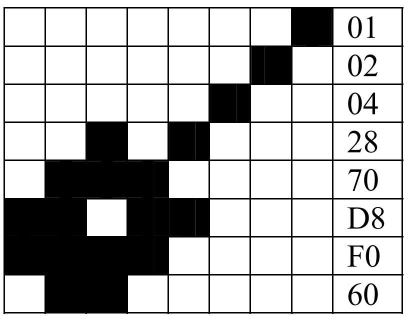
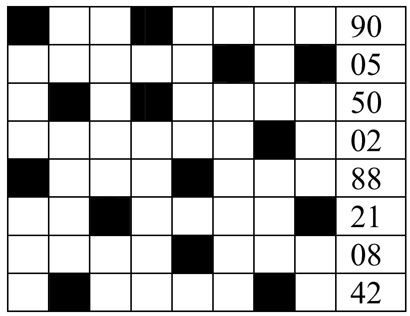
Solutions for Written Homework 3

Problem 1: (12 points) Patterns
(a) Give the eight 2-hex-digit numbers that represent the following patterns.

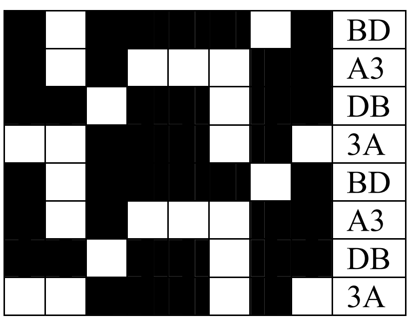


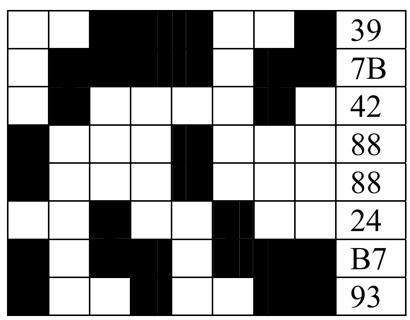






(b) Use graph paper to show the pattern described by each of the following sequences of eight 2-hex-digit numbers.





Problem 2: (12 points) Two's Complement
(a) Using bit string of length 8, give the two's complement representation of these integers.

(i) 53: this number is positive so the leftmost bit is "0" and the rest are the binary representation of 53. So we have 00110101.

(ii) 75: this number is positive so the leftmost bit is "0" and the rest are the binary representation of 75. So we have 01001011.

(iii) -75: this number is negative so the leftmost bit is "1" and the rest are the binary representation of $2^7 - |-75| = 128 – 75 = 53 = 0110101$. So we have 10110101.

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(iv) -41: this number is negative so the leftmost bit is "1" and the rest are the binary representation of 2^7 - |-41| = 128 – 41 = 87 = 1010111. So we have 11010111.

(b) What integer, in standard base 10 notation, is represented by each of the following 8-bit two's complements numbers?

(i) 01010111: this is positive since the leftmost bit is "0", so we just convert this number to decimal. So we get 87.

(ii) 11010111: this is negative since the leftmost bit is "1", so we do the following:
- convert 1010111 to decimal, we get 87
- 2^7 - |87| = 128 – 87 = 41

So the answer is -41.

(iii) 01100111: this is positive since the leftmost bit is "0", so we just convert this number to decimal. So we get 103.

(iv) 10011110: this is negative since the leftmost bit is "1", so we do the following:
- convert 0011110 to decimal, we get 30
- 2^7 - |30| = 128 – 30 = 98

So the answer is -98.

Problem 3: (12 points) Inverses mod 24.
(a) Give all integers from 0 through 23 that have multiplicative inverses mod 24.

A number x in $\mathbf{Z}_{24}$ will have an inverse if gcd(x, 24) = 1. The numbers that satisfy this are 1,5,7,11,13,17,19,23.

(b) Use Scheme or Excel to find the multiplicative inverse mod 24 for each number you found in a. Turn in your code or spreadsheet as well as your answers.

Excel: each internal cell is "=SUM(MOD(top of column*leftmost row entry, 24))"

	1	5	7	11	13	17	19	23
1	1	5	7	11	13	17	19	23
5	5	1	11	7	17	13	23	19
7	7	11	1	5	19	23	13	17
11	11	7	5	1	23	19	17	13
13	13	17	19	23	1	5	7	11
17	17	13	23	19	5	1	11	7
19	19	23	13	17	7	11	1	5
23	23	19	17	13	11	7	5	1

With this table, we see that each number is its own inverse mod 24.

Scheme: There were many ways that this was done.

Problem 4: (4 points) Prove the following by induction.
If n is a positive integer, then 2 + 6 + 12 +…+ n(n+1) = [n(n+1)(n+2)]/3

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We want to show that $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$.

Base Case: Let n = 1, then we have 1(1 + 1) = 2 and $\frac{1(1+1)(1+2)}{3} = \frac{1*2*3}{3} = 2$.

Inductive Step: Assume that this is true for k = n, show that this is true for k = n+1.
So we assume that $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$, and we want to show that
$\sum_{k=1}^{n+1} k(k+1) = \frac{(n+1)((n+1)+1)((n+1)+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$.

So we have, 2 + 6 + 12 + … + n(n+1) + (n+1)(n+1 +2).

From our assumption we have,

$$
\begin{aligned}
2+6+12+\ldots+n(n+1)+(n+1)(n+2) &= \left(\sum_{k=1}^{n} k(k+1)\right)+(n+1)(n+2) \\
&= \frac{n(n+1)(n+2)}{3}+(n+1)(n+2) \\
&= \frac{n(n+1)(n+2)}{3}+\frac{3(n+1)(n+2)}{3} \\
&= \frac{n(n+1)(n+2)+3(n+1)(n+2)}{3} \\
&= \frac{(n+3)(n+1)(n+2)}{3}
\end{aligned}
$$

This result matches the desired formula.

Therefore, $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$ for all n > 0.