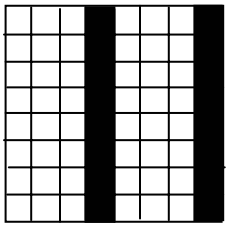
CSU200 Discrete Structures Professor Fell Fall 2004
Written Homework 3 Due: Wednesday, 11/3/2004
at the start of class

1. (12 points) Patterns

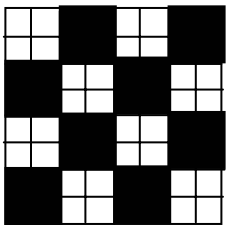
Patterns, like the ones above, are available in most drawing programs for filling regions. A pattern is defined by an 8 by 8 array of bits. So, for example, the pattern on the left is given by

0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1
0 0 0 1 0 0 0 1



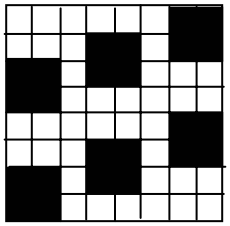
and the second pattern by

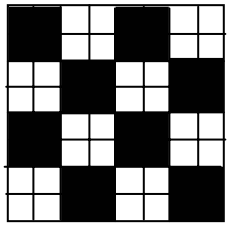
0 0 1 1 0 0 1 1
0 0 1 1 0 0 1 1
1 1 0 0 1 1 0 0
1 1 0 0 1 1 0 0
0 0 1 1 0 0 1 1
0 0 1 1 0 0 1 1
1 1 0 0 1 1 0 0
1 1 0 0 1 1 0 0

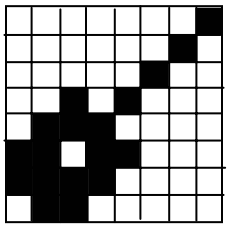


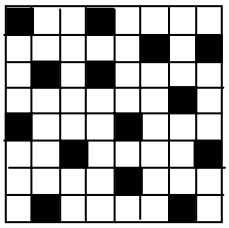
The 0's are where the white is and the 1's are where the black is. Each row is an 8-bit binary number. As we know, a 4-bit binary number can be expressed as a single hex-digit so an 8-bit binary number can be expressed with two hex-digits. Designers specify a pattern by giving eight 2-hex-digit numbers. The two patterns just above are given by 11, 11, 11, 11, 11, 11, 11, 11 and 33, 33, CC, CC, 33, 33, CC, CC.

a. Give the eight 2-hex-digit numbers that represent the following patterns.









b. Use graph paper to show the pattern described by each of the following sequences of eight 2-hex-digit numbers.

BD, A3, DB, 3A, BD, A3, DB, 3A

39, 7B, 42, 88, 88, 24, B7, 93

2. (12 points) Two's Complement

The **two's complement** representation of integers is commonly used in computers to simplify arithmetic. To represent an integer x with $-2^{n-1} \le x \le 2^{n-1} - 1$ for a given n, a total of n bits is used. The leftmost bit is used to represent the sign. A 0 bit in this position is used for positive integers and a 1 bit in this position is used for negative integers. For a positive integer, the remaining bits are just the binary representation of the number. For a negative number, the remaining bits are the binary expansion of $2^{n-1} - |x|$.

SHOW YOUR WORK or you will receive no credit!

a. Using bit string of length 8, give the two's complement representation of these integers.

i. 53 ii. 75 iii -75 iv. -41

b. What integer, in standard base 10 notation, is represented by each of the following 8-bit two's complements numbers?

i. 01010111 ii. 11010111 ii. 01100111 iv. 10011110

3. (12 points) Inverses mod 24.

a. Give all integers from 0 through 23 that have multiplicative inverses mod 24.

b. Use Scheme or Excel to find the multiplicative inverse mod 24 for each number you found in a. Turn in your code or spreadsheet as well as your answers.

4. (4 points) Prove the following by induction.

If n is a positive integer, then

$$2 + 6 + 12 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}..$$