1. The ISBN (International Standard Book Number) is a 10 digit code $x_1x_2\cdots x_{10}$ that is assigned to a book and appears on the back cover of most new books. The ISBN for "Discrete Mathematics, second edition" by Hein is 0-7637-2210-3. In general, the 10 digits consist of a block identifying the language, the publisher, the particular book, and a 1-digit check digit that is a digit or the letter X which stands for 10. The check digit is picked so that the sum
\[(1\cdot x_1 + 2\cdot x_2 + 3\cdot x_3 + 4\cdot x_4 + 5\cdot x_5 + 6\cdot x_6 + 7\cdot x_7 + 8\cdot x_8 + 9\cdot x_9 + 10\cdot x_{10}) \text{mod} 11 = 0.\]
The check digit is used to check errors in the digits or misplaced digits.

For the Hein book, we have
\[1\cdot 0 + 2\cdot 7 + 3\cdot 6 + 4\cdot 3 + 5\cdot 7 + 6\cdot 2 + 7\cdot 2 + 8\cdot 1 + 9\cdot 0 + 10\cdot 3\]
\[= (0+14+18+12+35+12+14+8+0+30) \text{mod} 11 = (3+7+1+2+1+3+8) \text{mod} 11 = 0.\]

a) Give the name and ISBN number for 2 other books and check (showing your work as I did above) that the check sum is correct.

b) The first 9 digits of the ISBN for "Signal Processing in C" by Reid and Passin are 0-471-52713-. What is the check digit?

c) The ISBN for my well-work copy of "The Diamond Age" by Neal Stephenson is 0-553-5D331-4. The D stands for a digit that got rubbed out. What is this digit?

(Note: This problem is based on problems 54, 55, 56, 57 on page 168 of "Discrete Mathematics and its Applications," by Rosen.)

2. The equation $ax \text{ mod } n = b$ has a solution $x$ if and only if $\text{gcd}(a,n)$ divides $b$.

Use this fact to prove that:
If $n > 1$ and we define the function $f(x) = (ax + b) \text{ mod } n$ for all $x$ in $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ then $f(x) \neq x$ for any $x$ in $\mathbb{Z}_n$ if and only if if $\text{gcd}(a-1, n)$ does not divide $b$.

3. If we let the letters A, . . ., Z correspond to the integers, 0, . . ., 25, we can create a simple cipher of the form $c(x) = (ax + b) \text{ mod } 26$.

a. Use the statement at the start of problem 2 to prove that the values of $c(x)$ will be distinct if and only if $\text{gcd}(a, 26) = 1$.

b. For each value of $b$ (0 ≤ $b$ ≤ 25) find all values of $a$ (0 ≤ $a$ ≤ 25) that insure that all the values $c(x)$ for 0 ≤ $x$ ≤ 25 are distinct and $c(x) \neq x$ for any $x$ (0 ≤ $x$ ≤ 25).