Practice Problems for Discrete Structures

1. Evaluate the following expressions at the given value of the variable(s).

a. $3x^3 - 6x^2 - 2x + 1$ when x = -4.
b. $x^5 - x^4 + x^3 - x^2 + x - 1$ when x = 1.
c. $x^5 - x^4 + x^3 - x^2 + x - 1$ when x = -1.
d. $x\left(x\left(x-3\right)+2\right)$ when x = 3.
e. $x\left(x\left(x-3\right)+2\right)$ when x = -3.
f. $xy^2 - x^2y + xy$ when x = -5 and y = 3
g. $xy^2 - x^2y + xy$ when x = 5 and y = -3

2. Simplify the following expressions.

a. $\left(\dfrac{6x^8y^{-3}}{2x^5y^{-6}}\right)^{-2}$ **b.** $\dfrac{1-\dfrac{1}{x^2}}{1+\dfrac{1}{x^2}}$

c. $\dfrac{x+3+\dfrac{2}{x}}{1-\dfrac{4}{x^2}}$ **d.** $\left(\dfrac{3x^8y^{-3}z}{6\dfrac{x^{-5}y^6}{z^2}}\right)^3$

e. $\dfrac{x^2-y^2}{x+y}$ **f.** $\dfrac{xy+x^3y^2}{x+1}$ when y = -2

3. Find the roots of each polynomial by factoring.

a. $x^2 - 3x + 2$ **b.** $x^2 + 7x + 12$
c. $3x^2 - 7x$ **d.** $2x^2 + 24x - 26$
e. $x^2 - 2x - 8$ **f.** $x^4 - 4x^3 + 3x^2$
g. $x\left(x\left(x-2\right)-8\right)$ **h.** $\left(x-3\right)\left(x\left(x+6\right)+8\right)$

4. Find the roots of each quadratic equation by using the quadratic formula.

a. $x^2 - 5x + 3$ **b.** $x^2 - 3x + 3$

5. a. For what values of t will $x^2 + tx + 3$ have 2 different real roots?
b. For what values of t will $x^2 + 3x + t$ have 2 different real roots?
c. How many real roots does the polynomial $x^4 + 5x^2 + 3$ have?

6. Evaluate the following. You should not need a calculator.

a. 2^9 **b.** $(512)^{2/9}$

c. $(32)^{6/5}$ **d.** $\left(\frac{9}{49}\right)^{-3/2}$

e. $\left(\frac{81}{256}\right)^{-3/4}$ **f.** $\left(\frac{512}{27}\right)^{2/3}$

g. $\left(\frac{81}{256}\right)^{-3/4}\left(\frac{9}{4}\right)^{3/2}$ **h.** $\left(\frac{512\cdot 125}{27000}\right)^{2/3}$

7. **a.** $\log_2 128$ **b.** $\log_2 512$

c. $\log_2(.5)$ **d.** $\log_2\left(\frac{1}{16}\right)$

e. $4(\log_2 3)-(\log_2 81)$ **f.** $5\log_2\left(2^{-9/5}\right)$

g. $\log_2(32!)-\log_2(31!)$ **h.** $\lceil \log_2(100!)-\log_2(99!)\rceil$

i. $\lceil \log_2(100!)-\log_2(98!)\rceil$ **j.** $\log_2(16!)-\log_2(13!)-\log_2(105)$

8. Let $y = a2^{bx}$. For each of the following find the values of *a* and *b*.

a. $y = 2$ when $x = 0$ and $y = 8$ when $x = 1$.
b. $y = 2$ when $x = 1$ and $y = 6$ when $x = 2$.
c. $y = 2$ when $x = 0$ and $y = 2$ when $x = 1$.

9. Evaluate the following.

a. $\lfloor 7.234\rfloor$ **b.** $\lfloor -7.234\rfloor$

c. $\lceil 7.234\rceil$ **d.** $\lceil -7.234\rceil$

e. $\lfloor\lceil 7.234\rceil\rfloor$ **f.** $\lceil\lfloor -7.234\rfloor\rceil$

g. $\lceil 7.234\rceil$ **h.** $\lceil -7.234\rceil$

i. $5\lceil 7.234\rceil$ **j.** $\lceil 5\cdot 7.234\rceil$

10. Sketch graphs of the following for $-3 \le x \le 3$. Use solid dots and open circles to show what happens at the endpoints.

a. $y = abs(\lceil x\rceil)+\lceil x\rceil$ **b.** $y = \lceil abs(x)+x\rceil$

c. $y = 2\lfloor x\rfloor$ **d.** $y = \lfloor 2x\rfloor$

e. $y = \left\lceil \frac{x}{2}\right\rceil$ **f.** $y = abs(x)+\lceil x\rceil$

Which of the functions above have some *y* values that are not integers?

11. Find the prime factorization of each of these integers. Write your answer in exponential form with the prime factors in increasing order, e.g. $12 = 2^2 \cdot 3$.

a. 99 **b.** 99000 **c.** 9!

c. $\frac{99!}{98!}$ **d.** (* 111 93) **e.** $\frac{256*125}{100}$

12. Find the gcd or lcm, as indicated.

a. gcd(154, 286) **b.** gcd(25, 396)

c. gcd(42, 196) **d.** lcm(42, 196)

e. gcd(14!, 13!) **f.** lcm(14!, 13!)

g. $\gcd\left(\frac{12!}{6!}, 6!\right)$ **h.** $\operatorname{lcm}\left(2^3 3^2 5^6 7, 2^5 3^3 5^2 7^3\right)$

13. Evaluate these quantities.

a. 2 **mod** 5 **b.** -2 **mod** 5
c. 5 **mod** 2 **d.** -5 **mod** 2
e. 987654321 **mod** 100000 **f.** 44 **mod** 13
g. **[(**44 **mod** 13)*(44 **mod** 13)] **mod 13**
h. **[(**44 **mod** 13) + (44 **mod** 13)] **mod 13**
i. (25 mod 27) mod 17 **j.** (3^{128})mod 7
j. (3^{512})mod 7 **k.** (5^{128})mod 7