CS U200 Discrete Structures **Review Topics and Problems for the Final Exam**
Professors Harriet Fell and Michael Lipton

Sets that we have met and Set-Builder Notation
N, Z, Q, R, Z^+

1. List the elements of the following sets or give the intervals they describe.
$\{ n \in Z \mid n = 2k \text{ for some } k \in Z \text{ such that } -3 \le k \le 3\}$
$\{ n \in Z \mid n = 2k \text{ for some } k \in Z \text{ such that } -3 \le n \le 3\}$
$\{ n \in Z \mid n = k \bmod 3 \text{ for some } k \in Z \text{ such that } -3 \le k \le 3\}$
$\{ n \in R \mid n = 2k \text{ for some } k \in Z \text{ such that } -3 \le k \le 3\}$
$\{ n \in R \mid n = 2k \text{ for some } k \in R \text{ such that } -3 \le k \le 3\}$
$\{ p/q \in Q \mid -2 \le p/q \le 2 \text{ and } 0 \le q \le 3\}$

Set Operations
$\cup$, $\cap$, complement, A-B, Cartesion Product, Power Set

2. Let A = {1, 2, 5} and B = {2, 3, 4, 5} and the universe U = {1, 2, 3, 4, 5, 6}

List all the elements of
$A \cup B$
$A \cap B$
A-B
$\overline{A}$
A x B
$\mathcal{P}(A)$

What is the cardinality of each of the sets below?
$\mathcal{P}(A)$
$\mathcal{P}(A \times B)$
$\mathcal{P}(A) \times B$

Functions
Know what it means to be a function $f : A \rightarrow B$.
(There will not be questions about 1-1 and onto.)
Functions defined on R that you should know:
$x, x^2, x^3, 2^x, \log_2(x)$ $(x > 0)$, floor(x) = $\lfloor x \rfloor$, ceiling(x) = $\lceil x \rceil$
Functions defined in N that you should know
n!, n mod 6

3. You should be able to calculate values of and sketch any of the above.

$f : A \to B$ **where A and B are finite sets.**

4. **Let A = {1, 2, 3, 4} and B = {0, 1}. Which of the following define functions** $f : A \to B$**?**

a) f(1) = f(2) = f(3) = f(4) = 0.
b) f(1) = 0, f(2) = 1, f(3) = 0, f(4) = 1.
c) f(x) = x.
d) f(x) = $\lfloor x/4 \rfloor$.
e) How many different functions $f : A \to B$ are there?

Prime factorization, gcd, lcm, Euclidean Algorithm, Extended Euclidean Algorithm.
Be able to calculate small examples without a calculator.
Be able to do examples with literals, n!, m = pq^2. . .

5. Give the prime factorization of 72, 100, (11)! in standard form, e.g. $36 = 2^2 3^2$.
Find the **gcd** or **lcm**, as indicated.

gcd(72, 100)
lcm(72, 100)
gcd(64, 205)
gcd(6!, 9)

Find integers A and B such that 76A + 57B = 19.
Assume 0 < p < q, p and q are prime, and $n = p^3$, $m = pq^3$.

gcd(m, n)
lcm(m, n)

Number representations
Be able to fill in a table

decimal	binary	octal	hexadecimal
42			
	1011001		
		101	
			A2

Sequences and Sums
Find the next term, a particular term, or the nth term of

an arithmetic sequence
a geometric sequence
a quadratic sequence

6. Give the next term and the nth term of each of these sequences. Assume the sequences start with term 1 (not term 0).

3, 8, 13, 18, 23, ...
3, 6, 12, 24, 48, ...
0, 2, 7, 12, 20, ...

7. For each of the following, expand the sum* when N = 4 and give formulas for the following sums in terms of N

$$\sum_{k=1}^{N} 4k$$

$$\sum_{k=1}^{N} \frac{1}{3^k}$$

$$\sum_{k=1}^{N} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

* for the first sum, 4 + 8 + 12 + 16

Mathematical Induction
Be able to prove something by simple (not strong) mathematical induction.
Identify P(N):
Basis Step:
Induction Step:

8. Prove using mathematical induction

$3|(n^3 - n)$ for all $n \in Z$, $n > 1$.

$n! > 2^n$ if $n > 3$.

$$\sum_{k=1}^{N} \frac{1}{2^k} = 1 - \frac{1}{2^N} \quad \text{if } N \geq 1.$$

Counting including: Product Rule, Sum Rule, Inclusion-Exclusion, Pigeonhole Principle, Permutations, and Combinations

9. Let A = { a, b, c, d, e, f } and B = { g, h, i, j }.
How many 5 letter strings can be made from the letters in $A \cup B$ if

i) repetitions are allowed?
ii) no repetitions are allowed? (I.e. all the letters in the string a different.)
iii) the first letter is "g" and repetitions are allowed?
iv) the first letter is "a" and no repetitions are allowed?
v) the first letter is "a" and the last letter is "j" and repetitions are allowed?
vi) the first letter is "a" or the last letter is "j" and repetitions are allowed?
vii) the first letter is not "a" and the last letter is not "j" repetitions allowed?
viii) the first three letters are from A and the last 2 from B, no repetitions?
ix) the first, third, and fifth letters are from A, no repetitions?

How many subsets of $A \cup B$

- i) Have 6 letters?
- ii) Have 9 letters?
- iii) Have 3 letters A and 3 letters from B?
- iv) Contain the letters a, b, and c?

10. Suppose a certain course has 12 graders. In an attempt to equalize the grading, students are asked to place their assignments in one of 12 boxes, numbered 0, 1, 2, ..., 11. They are told to take their social security number mod 12 to decide which box to use. How many students must submit an assignment to insure that

some grader gets at least 2 papers to grade?
some grader gets at least 60 papers to grade?
every grader gets at least 1 paper to grade?
every grader gets at least 60 papers to grade?

If 497 students turn in an assignment,

will some grader have at least 30 papers to grade?
will some grader have at least 40 papers to grade?
will some grader have at least 50 papers to grade
will every grader have at least 10 papers to grade?