1. **Set-Builder Notation**
Rewrite each set showing all its elements, for example, \( \{ x \in \mathbb{Z} \mid 1 < x < 5 \} = \{2, 3, 4\} \).

a) \( \{ x \in \mathbb{Z} \mid x \mod 3 = 2 \text{ and } |x| \leq 10 \} \)
\([-10, -7, -4, -1, 2, 5, 8]\)

b) \( \{ x \in \mathbb{Z} \mid 0 \leq x/3 \leq 2 \} \)
\(\{0<1, 2, 3, 4, 5, 6\}\)

2. **Cartesian Product and Power Sets**
Let \( A = \{1, 2\} \) and \( B = \{2, 3, 4\} \).

a) List all the elements of \( A \times B \).
\((1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\)

b) List all the members of \( \mathcal{P}(A) \), the power set of \( A \).
\(\emptyset, \{1\}, \{2\}, \{1, 2\}\)

c) List all the subsets of \( A \times A \) that have two elements.
The elements of \( A \times A \) are \((1, 1), (1, 2), (2, 1), (2, 2)\).
The subsets of \( A \times A \) with two elements are
\(\{(1, 1), (1, 2)\}, \{(1, 1), (2, 1)\}, \{(1, 1), (2, 2)\},\)
\(\{(1, 2), (2, 1)\}, \{(1, 2), (2, 2)\}, \{(2, 1), (2, 2)\}\).

d) How many subsets does \( \mathcal{P}(B) \) have?
\(\mathcal{P}(B) \) has 8 elements so \( \mathcal{P}(B) \) has \(2^8 = 256\) subsets.

e) What is the cardinality of \( A \times \mathcal{P}(B) \)?
\( |A \times \mathcal{P}(B)| = |A| \times |\mathcal{P}(B)| = 2 \times 8 = 16.\)

3. **Summation Notation**

a) Expand each of the following sums as shown in the example.
\[\sum_{k=1}^{4} 2k^2 = 2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2 = 2 + 8 + 18 + 32\]

i) \[\sum_{k=1}^{4} 2^k = 2^1 + 2^2 + 2^3 + 2^4 = 2 + 4 + 8 + 16\]

ii) \[\sum_{k=2}^{5} \frac{k!}{2!} = \frac{1!}{2!} + \frac{2!}{2!} + \frac{3!}{2!} + \frac{4!}{2!} + \frac{5!}{2!} = \frac{1}{2} + 1 + 3 + 12 + 60\]

iii) \[\sum_{k=3}^{5} (k^2 - k + 1) = (3^2 - 3 + 1) + (4^2 - 4 + 1) + (5^2 - 5 + 1) = 7 + 13 + 21\]
4. **Summation Formulas**

For each of the following sums, give a formula in terms of $n$ for the sum, as shown in the example.

$$
\sum_{k=1}^{n} 3k = \frac{3n(n+1)}{2}
$$

i) $$
\sum_{k=1}^{n} 3^k = \frac{3^{n+1} - 3}{2}
$$

Set $S = \sum_{k=1}^{n} 3^k$. Then $3S = \sum_{k=1}^{n+1} 3^k$. Therefore, $2S = \sum_{k=2}^{n+1} 3^k - \sum_{k=1}^{n} 3^k = 3^{n+1} - 3$

ii) $$
\sum_{k=10}^{n} 5k = \frac{(50 + 5n)(n - 9)}{2}
$$

where $n \geq 10$

This is arithmetic. Use the formula $\frac{(first + last) \times (#terms)}{2}$

5. **Permutations and Combinations**

Show the formulas you used as well as the final numbers.

Give the values of each of these quantities:

a) $P(8, 2) = \frac{8!}{6!} = 8 \cdot 7 = 56$

b) $P(7, 3) = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$

c) $C(8, 2) = \frac{8!}{6!2!} = \frac{8 \cdot 7}{2} = 28$

d) $C(7, 3) = \frac{7!}{4!3!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$

e) $C(9, 1) = \frac{9!}{8!1!} = 9$
6. **Counting**  
Show the formulas you used as well as the final numbers.  
If you have 6 books, how many ways can you

a) **arrange 4 of them on a shelf?**  
\[
P(6, 4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360
\]

b) **choose 4 of them to take on a trip?**  
\[
C(6, 4) = \frac{6!}{2!4!} = \frac{6 \cdot 5}{2} = 15
\]

c) **choose 3 of them to leave home?**  
\[
C(6, 3) = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} = 20
\]

d) **choose 1 a day to pick a quote from for the next 3 days, repetition allowed?**  
You have 6 choices on each day so \(6^3 = 216\).

7. **Probability**  
Show your work as well as the final numbers.  
If you roll a pair of dice, one black and one white, what is the probability of

a) **the sum of the numbers rolled is 8?**  
**successful outcomes**  
<table>
<thead>
<tr>
<th>Black die</th>
<th>White die</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

There are 5 successful outcomes in all. There are \(6 \times 6 = 36\) possible outcomes so the probability is \(5/36\).

b) **the sum of the numbers rolled is 2 or 3?**  
**successful outcomes**  
<table>
<thead>
<tr>
<th>Black die</th>
<th>White die</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

There are 3 successful outcomes in all. There are \(6 \times 6 = 36\) possible outcomes so the probability is \(3/36 = 1/12\).
c) the number on the white one is less than the number on the black one?

**successful outcomes**

<table>
<thead>
<tr>
<th>Black die</th>
<th>White die</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5, 4, 3, 2, 1</td>
</tr>
<tr>
<td>5</td>
<td>4, 3, 2, 1</td>
</tr>
<tr>
<td>4</td>
<td>3, 2, 1</td>
</tr>
<tr>
<td>3</td>
<td>2, 1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>no possibilities</td>
</tr>
</tbody>
</table>

There are 15 successful outcomes in all. There are 6×6=36 possible outcomes so the probability is 15/36 = 5/12.

8. Relations

Let A be the set {-3, -2, -1, 0, 1, 2, 3}.

a) Show the pairs (x, y) in the relation \(x^2 = y^2\) by putting an \(\times\) in the corresponding squares:

```
   3  x  x  x  x
   2  x  x  x
   1  x  x  x
   0  x  x
 -1  x  x  x
-2  x  x  x
-3  x  x  x
```

b) Tell whether this relation is

i) Reflexive - Yes, \(x^2 = x^2\). You could also note that the line \(y = x\) is part of the relation.

ii) Symmetric - Yes if \(x^2 = y^2\) then \(y^2 = x^2\).

ii) Transitive - Yes, if \(x^2 = y^2\) and \(y^2 = z^2\) then \(x^2 = z^2\).

**Explain your answers.**