SOLUTION
College of Computer and Information Science **CS U200 Discrete Structures**
Professor Harriet Fell **Second Hour Exam –Section 2** **October 20, 2004**

1. (12 points) Simplify the following expressions. Show your work.

a. $2x^3 - x^2 - 7x + 5$ when $x = -3$.
$= 2(-3)^3 - (-3)^2 - 7(-3) + 5 = -54 - 9 + 21 + 5 = -37$

b. $\dfrac{2x^3y^2 - x^2 - 7xy + 5}{x^3 - x}$ when $x = -2$.

$$= \frac{2(-2)^3 y^2 - (-2)^2 - 7(-2)y + 5}{(-2)^3 - (-2)} = \frac{-16y^2 - 4 + 14y + 5}{-8 + 2} = \frac{-16y^2 + 14y + 1}{-6}$$

c.
$$\left(\frac{35x^4y^{-3}z}{7x^7y^{-2}z^{-2}}\right)^{-2} = \left(\frac{35}{7}\right)^{-2}\left(\frac{x^4}{x^7}\right)^{-2}\left(\frac{y^{-3}}{y^{-2}}\right)^{-2}\left(\frac{z}{z^{-2}}\right)^{-2} = \frac{1}{5^2}\left(x^3\right)^2(y)^2\left(\frac{1}{z^3}\right)^2 = \frac{x^6y^2}{25z^6}$$

d. $2(\log_2 4) - 3(\log_2 2) + \left(\log_2 \frac{1}{2}\right) = 2 \cdot 2 - 3 \cdot 1 - 1 = 0$

2. (7 points) Find the roots of each polynomial

a.
$2x^3 - 15x^2 + 26x = x(2x^2 - 15x + 26)$

$x = 0$ or $x = \dfrac{15 \pm \sqrt{225 - 208}}{4} = \dfrac{15 \pm \sqrt{17}}{4}$

b. For what values of t will $tx^2 - 5x + 3$ have 2 different real roots?
There will be 2 real roots when $b^2 - 4ac > 0$.

That is, when $25 - 12t > 0$ or $\dfrac{25}{12} > t$.

3. (9 points) a. Fill in this table:

x	$y = abs(\lfloor x \rfloor) + \lceil x \rceil$
-2	2 + (-2) = 0
-1.5	2 + (-1) = 1
-1	1 + (-1) = 0
-.5	1 + 0 = 1
0	0 + 0 = 0
.5	0 + 1 = 1
1	1 + 1 = 2
1.5	1 + 2 = 3
2	2 + 2 = 4

4. (9 points) Find the prime factorization of each of these integers. Write your answer in exponential form with the prime factors in increasing order, e.g. $12 = 2^2 \cdot 3$.

a. $14400 = 12*12*10*10 = 2^6 \cdot 3^2 \cdot 5^2$

b. $\frac{15!}{9!} = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 = 3 \cdot 5 \cdot 2 \cdot 7 \cdot 13 \cdot 2 \cdot 2 \cdot 3 \cdot 11 \cdot 2 \cdot 5 = 2^4 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$

c. $\frac{18!}{(14!)(4!)} = \frac{18 \cdot 17 \cdot 16 \cdot 15}{4 \cdot 3 \cdot 2} = \frac{6 \cdot 17 \cdot 2 \cdot 15}{1} = 2 \cdot 3 \cdot 17 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3^2 \cdot 5 \cdot 17$

5. (12 points) Find the gcd or lcm, as indicated. Your answers to c and f may be in prime-factorization form.

a. gcd(256, 1024) = 256

b. lcm(1000, 1024) = $2^{10}5^3 = 128000$

c. $\gcd\left(2^3 3^5 5^7 7^{11}, 2^{11} 3^2 7^5 11^2\right) = 2^3 3^2 7^5$

d. $\text{lcm}\left(2^3 3^5 5^7 7^{11}, 2^{11} 3^2 7^5 11^2\right) = 2^{11} 3^2 5^7 7^{11} 11^2$

6. (12 points) Evaluate these quantities. Show your work.

a. [(55 mod 13)*(23 mod 13)] mod 13 = (3*10) mod 13 = 30 mod 13 = 4

b. [(55 mod 13) + (23 mod 13)] mod 13 = (3+10) mod 13 = 0

c. (5^2) mod 9 = 25 mod 9 = 7

d. (5^{16}) mod 9 = 4

(5^4) mod 9 = (7*7) mod 9 = 49 mod 9 = 4

(5^8) mod 9 = (4*4) mod 9 = 7

(5^{16}) mod 9 = (7*7) mod 9 = 4

7. (18 points) Fill in this table of decimal, binary, octal, hexadecimal equivalents.

Decimal	Binary	Octal	Hexadecimal
228	11100100	344	E4
217	11011001	331	D9

8. (12 points) Insert the following numbers (keys), in the order they are listed, into the hash table. Use the hash function

h(key) = key mod 7

followed by linear probing if a collision occurs.

List of keys:	65	82	38	93	46	21
key mod 7	2	5	3	2	4	0

0	21
1	
2	65
3	38
4	93
5	82
6	46

9. (9 points) Assume that each of these sequence starts with the term a_1. Give the next term of each sequence and a formula for a_n. in terms of *n*.

a. (arithmetic) 7 13 19 25 31 37 [43] next term

a_n = 6*n* + 1

b. (geometric) 7 21 63 189 567 1701 [5103] next term

a_n = $7*3^{(n-1)}$

c. (quadratic) 7 13 21 31 43 57 [73] next term

a_n = $a_n = n^2 + 3n + 3$

The work:

$a_n = an^2 + bn + c$

$a_1 = a + b + c = 7$

$3a + b = 6$

$a_2 = 4a + 2b + c = 13$ $2a = 2$ So $a = 1$, $b = 3$, and $c = 3$.

$5a + b = 8$

$a_3 = 9a + 3b + c = 21$