SOLUTION
College of Computer and Information Science **CS U200 Discrete Structures**
Professor Harriet Fell **Second Hour Exam –Section 1** **October 20, 2004**

1. (12 points) Simplify the following expressions. Show your work.

a. $x^3 - 2x^2 - 7x + 3$ when $x = -5$.

$(-5)^3 - 2(-5)^2 - 7(-5) + 3 = -125 - 50 + 35 + 3 = -137$

b. $\dfrac{3x^2y^3 - x^2 - 7xy^3 + 2}{x^3 - x + 1}$ when $x = -3$.

$$\frac{3(-3)^2 y^3 - (-3)^2 - 7(-3)y^3 + 2}{(-3)^3 - (-3) + 1} = \frac{27y^3 - 9 + 21y^3 + 2}{-27 + 3 + 1} = \frac{48y^3 - 7}{-23} \text{ or } \frac{-48y^3}{23} + \frac{7}{23}$$

c.

$$\left(\frac{5x^7y^{-2}z}{35x^{-4}y^3z^{-2}}\right)^{-2} = \left(\frac{1}{7}\right)^{-2}\left(\frac{x^7}{x^{-4}}\right)^{-2}\left(\frac{y^{-2}}{y^3}\right)^{-2}\left(\frac{z}{z^{-2}}\right)^{-2} = 7^2\left(x^{11}\right)^{-2}\left(y^{-5}\right)^{-2}\left(z^3\right)^{-2} = \frac{49y^{10}}{x^{22}z^6}$$

d. $(\log_2 20) - 3(\log_2 2) + \left(\log_2 \frac{1}{10}\right) = \log_2\left(\frac{20}{2^3 \cdot 10}\right) = \log_2\left(\frac{1}{2^2}\right) = -2$

2. (7 points) Find the roots of each polynomial

a. $3x^3 - 15x^2 + 12x = 3x(x^2 - 5x + 4) = 3x(x-1)(x-4)$

The roots are x = 0, 1, 4.

b. For what values of t will $x^2 - 5tx + 5$ have 2 different real roots?

There will be two distinct roots when $b^2 - 4ac > 0$ $b^2 - 4ac = 25t^2 - 20$

So there will be two distinct roots when

$25t^2 - 20 > 0$

$t > \frac{2}{\sqrt{5}}$ or $t < -\frac{2}{\sqrt{5}}$.

3. (9 points) a. Fill in this table:

x	$y = \lfloor x \rfloor + 2\lceil x \rceil$
-2	-6
-1.5	-4
-1	-3
-.5	-1
0	0
.5	2
1	3
1.5	5
2	6

4. (9 points) Find the prime factorization of each of these integers. Write your answer in exponential form with the prime factors in increasing order, e.g. $12 = 2^2 \cdot 3$.

a. $25600 = 2^8 10^2 = 2^{10} 5^2$

b.
$$\frac{14!}{7!} = 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 2 \cdot 7 \cdot 13 \cdot 2^2 \cdot 3 \cdot 11 \cdot 2 \cdot 5 \cdot 3^2 \cdot 2^3 = 2^7 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$$

c. $$\frac{18!}{(15!)(3!)} = \frac{18 \cdot 17 \cdot 16}{3 \cdot 2} = 3 \cdot 17 \cdot 16 = 2^4 \cdot 3 \cdot 17$$

5. (12 points) Find the gcd or lcm, as indicated. Your answers to c and d may be in prime-factorization form.

a. gcd(512, 128) = 128 **b.** lcm(512, 500) = 64000 or $2^9 \cdot 5^3$

c. $\gcd\left(2^4 3^4 5^4 7^4, 2^{11} 3^2 7^5 11^2\right) = 2^4 3^2 7^4$

d. $\text{lcm}\left(2^4 3^4 5^4 7^4, 2^{11} 3^2 7^5 11^2\right) = 2^{11} 3^4 5^4 7^5 11^2$

6. (18 points) Evaluate these quantities. Show your work.

a. [(55 mod 12)*(23 mod 12)] mod 13
= [7 * 11] mod 13 = 77 mod 13 = 12

b. [(55 mod 12) + (23 mod 12)] mod 12
= [7 + 11] mod 13 = 18 mod 12 = 6

c. (5^2) mod 11 = 25 mod 11 = 3

d. (5^4) mod 11 = (3^2) mod 11 = 9
(5^8) mod 11 = (9^2) mod 11=4

7. (12 points) Fill in this table of decimal, binary, octal, hexadecimal equivalents.

Decimal	Binary	Octal	Hexadecimal
179	10110011	263	B3
221	11011101	335	DD
351	101011111	537	15F
154	10011010	232	9A

8. (12 points) Insert the following numbers (keys), in the order they are listed, into the hash table. Use the hash function

$$h(key) = key \bmod 7$$

followed by linear probing if a collision occurs.

List of keys:	93	46	21	65	82	38
key mod 7	2	4	0	2	5	3

0	21
1	
2	93
3	65
4	46
5	82
6	38

9. (9 points) Assume that each of these sequence starts with the term a_1. Give the next term of each sequence and a formula for a_n. in terms of *n*.

a. (arithmetic) 7 15 23 31 39 47 [55] next term

a_n = 7 + 8(n-1) = 8n - 1

b. (geometric) 7 14 28 56 112 224 [248] next term

a_n = $7*2^{(n-1)}$

c. (quadratic) 7 14 27 46 71 102 [139] next term

a_n = $a_n = 3n^2 - 2n + 6$

The work:

$a_n = an^2 + bn + c$

$a_1 = a + b + c = 7$

$3a + b = 7$

$a_2 = 4a + 2b + c = 14$ $\qquad 2a = 6$

$5a + b = 13$

$a_3 = 9a + 3b + c = 27$

So $a = 3$, $b = -2$, and $c = 6$.