CS 4300
Computer Graphics
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Fall 2012
Lecture 24 – October 31, 2012
Today’s Topics

• Ray Casting
Ray Tracing
a World of Spheres
What is a Sphere

Vector3D center; // 3 doubles
double radius;
double R, G, B; // for RGB colors between 0 and 1
double kd; // diffuse coefficient
double ks; // specular coefficient
(double ka; // ambient light coefficient)
-01 .01 500 800 // transform theta phi mu distance
1 // antialias
1 // numlights
100 500 800 // Lx, Ly, Lz
9 // numspheres

//cx  cy  cz  radius  R  G  B  ka  kd  ks  specExp  kgr  kt  pic
-100  -100  0  40  .9  0  0  .2  .9  .0  4  0  0  0
-100   0  0  40  .9  0  0  .2  .8  .1  8  .1  0  0
-100  100  0  40  .9  0  0  .2  .7  .2  12  .2  0  0
 0    -100  0  40  .9  0  0  .2  .6  .3  16  .3  0  0
 0      0  0  40  .9  0  0  .2  .5  .4  20  .4  0  0
 0  100  0  40  .9  0  0  .2  .4  .5  24  .5  0  0
100   -100  0  40  .9  0  0  .2  .3  .6  28  .6  0  0
100     0  0  40  .9  0  0  .2  .2  .7  32  .7  0  0
100  100  0  40  .9  0  0  .2  .1  .8  36  .8  0  0
World of Spheres

Vector3D VP;          // the viewpoint
int numLights;
Vector3D theLights[5];  // up to 5 white lights
double ka;            // ambient light coefficient
int numSpheres;
Sphere theSpheres[20]; // 20 sphere max
int ppmT[3];          // ppm texture files
View sceneView;       // transform data
double distance;      // view plane to VP
bool antialias;       // if true antialias
Simple Ray Tracing for Detecting Visible Surfaces

select window on viewplane and center of projection
for (each scanline in image) {
    for (each pixel in the scanline) {
        determine ray from center of projection through pixel;
        for (each object in scene) {
            if (object is intersected and is closest considered thus far)
                record intersection and object name;
        }
        set pixel’s color to that of closest object intersected;
    }
}
Ray Trace 1
Finding Visible Surfaces
Ray-Sphere Intersection

- Given
  - Sphere
    - Center \((c_x, c_y, c_z)\)
    - Radius, \(R\)
  - Ray from \(P_0\) to \(P_1\)
    - \(P_0 = (x_0, y_0, z_0)\) and \(P_1 = (x_1, y_1, z_1)\)
  - View Point
    - \((V_x, V_y, V_z)\)
- Project to window from \((0,0,0)\) to \((w,h,0)\)
Sphere Equation

Center C = \((c_x, c_y, c_z)\)

Radius R

\[
(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = R^2
\]
Ray Equation

$P_0 = (x_0, y_0, z_0)$ and $P_1 = (x_1, y_1, z_1)$

The ray from $P_0$ to $P_1$ is given by:

$P(t) = (1 - t)P_0 + tP_1 \quad 0 \leq t \leq 1$

$= P_0 + t(P_1 - P_0)$
Intersection Equation

\[ P(t) = P_0 + t(P_1 - P_0) \quad 0 \leq t \leq 1 \]

is really three equations

\[ x(t) = x_0 + t(x_1 - x_0) \]
\[ y(t) = y_0 + t(y_1 - y_0) \]
\[ z(t) = z_0 + t(z_1 - z_0) \quad 0 \leq t \leq 1 \]

Substitute \( x(t), y(t), \) and \( z(t) \) for \( x, y, z, \) respectively in

\[
(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = R^2
\]

\[
((x_0 + t(x_1 - x_0)) - c_x)^2 + ((y_0 + t(y_1 - y_0)) - c_y)^2 + ((z_0 + t(z_1 - z_0)) - c_z)^2 = R^2
\]
Solving the Intersection Equation

\[
((x_0 + t(x_1 - x_0)) - c_x)^2 + ((y_0 + t(y_1 - y_0)) - c_y)^2 + ((z_0 + t(z_1 - z_0)) - c_z)^2 = R^2
\]

is a quadratic equation in variable \( t \).

For a fixed pixel, VP, and sphere,

\[
\begin{align*}
x_0, y_0, z_0, & \\
x_1, y_1, z_1, & \\
c_x, c_y, c_z, & \\
\end{align*}
\]

are all constants.

We solve for \( t \) using the quadratic formula.
The Quadratic Coefficients

\[
\left(\left(x_0 + t(x_1 - x_0)\right) - c_x\right)^2 + \left(\left(y_0 + t(y_1 - y_0)\right) - c_y\right)^2 + \left(\left(z_0 + t(z_1 - z_0)\right) - c_z\right)^2 = R^2
\]

Set

\[
\begin{align*}
d_x &= x_1 - x_0 \\
d_y &= y_1 - y_0 \\
d_z &= z_1 - z_0
\end{align*}
\]

Now find the coefficients:

\[
A t^2 + B t + C = 0
\]
Computing Coefficients

\[(x_0 + t(x_1 - x_0) - c_x)^2 + (y_0 + t(y_1 - y_0) - c_y)^2 + (z_0 + t(z_1 - z_0) - c_z)^2 = R^2\]

\[\left((x_0 + t d_x) - c_x\right)^2 + \left((y_0 + t d_y) - c_y\right)^2 + \left((z_0 + t d_z) - c_z\right)^2 = R^2\]

\[
\begin{align*}
(x_0 + td_x)^2 - 2c_x (x_0 + td_x) + c_x^2 + \\
(y_0 + td_y)^2 - 2c_y (y_0 + td_y) + c_y^2 + \\
(z_0 + td_z)^2 - 2c_z (z_0 + td_z) + c_z^2 - R^2 = 0
\end{align*}
\]

\[
\begin{align*}
x_0^2 + 2x_0 td_x + t^2 d_x^2 - 2c_x x_0 - 2c_x td_x + c_x^2 + \\
y_0^2 + 2y_0 td_y + t^2 d_y^2 - 2c_y y_0 - 2c_y td_y + c_y^2 + \\
z_0^2 + 2z_0 td_z + t^2 d_z^2 - 2c_z z_0 - 2c_z td_z + c_z^2 - R^2 = 0
\end{align*}
\]
The Coefficients

\[
\begin{align*}
\mathbf{x}_0^2 + 2\mathbf{x}_0 \mathbf{t}_x + \mathbf{t}_x^2 &+ 2\mathbf{c}_x \mathbf{x}_0 - 2\mathbf{c}_x \mathbf{t}_x + \mathbf{c}_x^2 + \\
\mathbf{y}_0^2 + 2\mathbf{y}_0 \mathbf{t}_y + \mathbf{t}_y^2 &- 2\mathbf{c}_y \mathbf{y}_0 - 2\mathbf{c}_y \mathbf{t}_y + \mathbf{c}_y^2 + \\
\mathbf{z}_0^2 + 2\mathbf{z}_0 \mathbf{t}_z + \mathbf{t}_z^2 &- 2\mathbf{c}_z \mathbf{z}_0 - 2\mathbf{c}_z \mathbf{t}_z + \mathbf{c}_z^2 - \mathbf{R}^2 = 0
\end{align*}
\]

\[
\begin{align*}
\mathbf{A} &= \mathbf{d}_x^2 + \mathbf{d}_y^2 + \mathbf{d}_z^2 \\
\mathbf{B} &= 2\mathbf{d}_x (\mathbf{x}_0 - \mathbf{c}_x) + 2\mathbf{d}_y (\mathbf{y}_0 - \mathbf{c}_y) + 2\mathbf{d}_z (\mathbf{z}_0 - \mathbf{c}_z) \\
\mathbf{C} &= \mathbf{c}_x^2 + \mathbf{c}_y^2 + \mathbf{c}_z^2 + \mathbf{x}_0^2 + \mathbf{y}_0^2 + \mathbf{z}_0^2 + \\
&\quad -2(\mathbf{c}_x \mathbf{x}_0 + \mathbf{c}_y \mathbf{y}_0 + \mathbf{c}_z \mathbf{z}_0) - \mathbf{R}^2
\end{align*}
\]
Solving the Equation

At^2 + Bt + C = 0

discriminant = D(A,B,C) = B^2 - 4AC

\[
\begin{cases}
    < 0 & \text{no intersection} \\
    = 0 & \text{ray is tangent to the sphere} \\
    > 0 & \text{ray intersects sphere in two points}
\end{cases}
\]
The intersection nearest $P_0$ is given by:

$$t = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

To find the coordinates of the intersection point:

$$x = x_0 + td_x$$
$$y = y_0 + td_y$$
$$z = z_0 + td_z$$
First Lighting Model

- Ambient light is a global constant.
  Ambient Light = $k_a (A_R, A_G, A_B)$
  $k_a$ is in the “World of Spheres”
  $0 \leq k_a \leq 1$
  $(A_R, A_G, A_B) = \text{average of the light sources}$
  $(A_R, A_G, A_B) = (1, 1, 1)$ for white light

- Color of object $S = (S_R, S_G, S_B)$

- Visible Color of an object $S$ with only ambient light
  $C_S = k_a (A_R S_R, A_G S_G, A_B S_B)$

- For white light
  $C_S = k_a (S_R, S_G, S_B)$
Visible Surfaces

Ambient Light

View Point: (200, 200, 1000)
Light : (750, 0, 2000)

SPHERES
Center : (100, 100, 100)
Radius : 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50

Center : (150, 200, 300)
Radius : 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00

Center : (350, 220, 150)
Radius : 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50

Center : (250, 300, 400)
Radius : 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50

Only ambient light
Second Lighting Model

• Point source light \( L = (L_R, L_G, L_B) \) at \((L_x, L_y, L_z)\)
• Ambient light is also present.
• Color at point \( p \) on an object \( S \) with ambient & diffuse reflection
  \[ C_p = k_a (A_R S_R, A_G S_G, A_B S_B) + k_d k_p (L_R S_R, L_G S_G, L_B S_B) \]
• For white light, \( L = (1, 1, 1) \)
  \[ C_p = k_a (S_R, S_G, S_B) + k_d k_p (S_R, S_G, S_B) \]
• \( k_p \) depends on the point \( p \) on the object and \((L_x, L_y, L_z)\)
• \( k_d \) depends on the object (sphere)
• \( k_a \) is global
• \( k_a + k_d \leq 1 \)
Diffuse Light

View Point: (200, 200, 1000)
Light: (750, 0, 2000)

SPHERES
Center: (100, 100, 100)
Radius: 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50

Center: (150, 200, 300)
Radius: 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00

Center: (350, 220, 150)
Radius: 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50

Center: (250, 300, 400)
Radius: 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50

Diffuse Lighting one point source
Lambertian Reflection Model
Diffuse Shading

• For matte (non-shiny) objects
• Examples
  ▪ Matte paper, newsprint
  ▪ Unpolished wood
  ▪ Unpolished stones
• Color at a point on a matte object does not change with viewpoint.
Physics of Lambertian Reflection

- Incoming light is partially absorbed and partially transmitted equally in all directions
Geometry of Lambert’s Law

\[ \text{dA} \sin(\theta) \]

\[ \text{dA} \cos(\theta) \]

Surface 1

Surface 2
\[ \cos(\theta) = N \cdot L \]

\[ \text{Surface 2} \]

\[ C_p = k_a \ (\text{SR, SG, SB}) + k_d \ N \cdot L \ (\text{SR, SG, SB}) \]
Finding $N$

$$N = \frac{(x-cx, y-cy, z-cz)}{|(x-cx, y-cy, z-cz)|}$$
Diffuse Light 2
Shadows on Spheres
More Shadows

View Point: (240, 248, 5000)
Light: (240, 600, 2000)

SPHERES
Center: (100, 100, 50)
Radius: 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50

Center: (150, 200, 250)
Radius: 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00

Center: (350, 220, 500)
Radius: 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50

Center: (250, 300, 750)
Radius: 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50

Diffuse Lighting
one point source
shadows on view plane
and spheres
Finding Shadows

Pixel gets shadow color
Shadow Color

• Given
  Ray from P (point on sphere S) to L (light)
  \( P = P_0 = (x_0, y_0, z_0) \) and \( L = P_1 = (x_1, y_1, z_1) \)

• Find out whether the ray intersects any other object (sphere).
  - If it does, P is in shadow.
  - Use only ambient light for pixel.
Shape of Shadows
Different Views
Planets

View Point: (250, 252, 2000)
Light: (700, 700, 2000)

SPHERES
Center: (100, 100, 50)
Radius: 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50
Center: (150, 230, 250)
Radius: 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00
Center: (350, 220, 500)
Radius: 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50
Center: (250, 300, 750)
Radius: 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50
Center: (310, 30, -20)
Radius: 50
RED: 0.50
GREEN: 0.50
BLUE: 0.50

Diffuse Lighting
one point source
shadows on spheres
Starry Skies

View Point: (225, 225, 4000)
Light: (2500, 2500, 6000)

SPHERES

Center: (100, 100, 50)
Radius: 50
RED: 0.50
GREEN: 0.00
BLUE: 0.50

Center: (150, 200, 250)
Radius: 50
RED: 0.50
GREEN: 0.50
BLUE: 0.00

Center: (350, 220, 500)
Radius: 50
RED: 0.00
GREEN: 0.50
BLUE: 0.50

Center: (250, 300, 750)
Radius: 50
RED: 0.25
GREEN: 0.25
BLUE: 0.50

Center: (310, 80, -20)
Radius: 50
RED: 0.50
GREEN: 0.50
BLUE: 0.50

Diffuse Lighting
one point source
shadows on spheres
Shadows on the Plane
Finding Shadows on the Back Plane

Pixel in Shadow

Shadow Ray
Close up
On the Table
Phong Highlight
Phong Lighting Model

- Light
- Normal
- Reflected
- View

The viewer only sees the light when $\alpha$ is 0.

Surface

We make the highlight maximal when $\alpha$ is 0, but have it fade off gradually.
Phong Lighting Model

\[ \cos(\theta) = \mathbf{R} \cdot \mathbf{V} \]

We use \( \cos^n(\theta) \).

The higher \( n \) is, the faster the drop off.

\[ \mathbf{C}_p = k_a (\mathbf{SR}, \mathbf{SG}, \mathbf{SB}) + k_d \mathbf{N} \cdot \mathbf{L} (\mathbf{SR}, \mathbf{SG}, \mathbf{SB}) + k_s (\mathbf{R} \cdot \mathbf{V})^n (1, 1, 1) \]
Powers of \( \cos(\theta) \)
Computing $R$

$L + R = (2 \ L \cdot N) \ N$

$R = (2 \ L \cdot N) \ N - L$
The Halfway Vector

\[ H = \frac{L + V}{|L + V|} \]

Use \( H \cdot N \) instead of \( R \cdot V \).

\( H \) is less expensive to compute than \( R \).

From the picture
\[ \theta + \varphi = \theta - \varphi + \alpha \]
So \( \varphi = \alpha / 2 \).

This is not generally true. Why?

Surface

\[ C_p = ka (SR, SG, SB) + kd \, N \cdot L (SR, SG, SB) + ks \, (H \cdot N)^n (1, 1, 1) \]
Varied Phong Highlights
Varying Reflectivity