Clipping Lines
We know how to find the intersections of a line segment

$$P + t(Q-P)$$

with the 4 boundaries

- $x = \text{xmin}$
- $x = \text{xmax}$
- $y = \text{ymin}$
- $y = \text{ymax}$
1. Assign a 4 bit *outcode* to each endpoint.

2. Identify lines that are trivially accepted or trivially rejected.

   if (outcode(P) = outcode(Q) = 0) accept
   else if (outcode(P) & outcode(Q)) ≠ 0) reject
   else test further

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<table>
<thead>
<tr>
<th>outcode</th>
<th>Accept</th>
<th>Reject</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td></td>
<td>1000</td>
</tr>
<tr>
<td>1001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0100</td>
<td>0000</td>
<td>0001</td>
</tr>
<tr>
<td>0110</td>
<td>0010</td>
<td>0011</td>
</tr>
</tbody>
</table>

above left below right
Cohen-Sutherland continued

Clip against one boundary at a time, top, left, bottom, right.

Check for trivial accept or reject.

If a line segment PQ falls into the “test further” category then

if (outcode(P) & 1000 \neq 0)
  replace P with PQ intersect y = top
else if (outcode(Q) & 1000 \neq 0)
  replace Q with PQ intersect y = top

go on to next boundary
Liang-Barsky Clipping

Clip window interior is defined by:

\[ x_{\text{left}} \leq x \leq x_{\text{right}} \]

\[ y_{\text{bottom}} \leq y \leq y_{\text{top}} \]
Liang-Barsky continued

\[ V_0 = (x_0, y_0) \]
\[ V_1 = (x_1, y_1) \]

\[ x = x_0 + t\Delta x \quad \Delta x = x_1 - x_0 \]
\[ y = y_0 + t\Delta y \quad \Delta y = y_1 - y_0 \]

\[ t = 0 \text{ at } V_0 \quad t = 1 \text{ at } V_1 \]
Liang-Barsky continued

Put the parametric equations into the inequalities:

\[ x_{\text{left}} \leq x_0 + t\Delta x \leq x_{\text{right}} \]
\[ y_{\text{bottom}} \leq y_0 + t\Delta y \leq y_{\text{top}} \]

\[ -t\Delta x \leq x_0 - x_{\text{left}} \]
\[ -t\Delta y \leq y_0 - y_{\text{bottom}} \]
\[ t\Delta x \leq x_{\text{right}} - x_0 \]
\[ t\Delta y \leq y_{\text{top}} - y_0 \]

These describe the interior of the clip window in terms of \( t \).
Liang-Barsky continued

\[-t\Delta x \leq x_0 - x_{\text{left}}\]
\[-t\Delta y \leq y_0 - y_{\text{bottom}}\]
\[t\Delta x \leq x_{\text{right}} - x_0\]
\[t\Delta y \leq y_{\text{top}} - y_0\]

• These are all of the form
  \[tp \leq q\]

• For each boundary, we decide whether to accept, reject, or which point to change depending on the sign of \(p\) and the value of \(t\) at the intersection of the line with the boundary.
\[ x = x_{\text{left}} \]

\[ p = -\Delta x \]

\[ t \leq \frac{x_0 - x_{\text{left}}}{-\Delta x} = \frac{q}{p} \]

\[ t = \frac{x_0 - x_{\text{left}}}{x_0 - x_1} \]

is between 0 and 1.

\[ \Delta x = x_1 - x_0 > 0 \text{ so } p < 0 \]

so replace \( V_0 \)

\[ t = \frac{x_0 - x_{\text{left}}}{x_0 - x_1} \]

is between 0 and 1.

\[ \Delta x = x_1 - x_0 < 0 \text{ so } p > 0 \]

so replace \( V_1 \)
\( x = t_{\text{left}} \)

\[ p = -\Delta x \]

\[ p < 0 \] so might replace \( V_0 \)

but

\[ t = \frac{x_0 - x_{\text{left}}}{x_0 - x_1} < 0 \]

so no change.

\[ p > 0 \]

\[ t > 1 \]

\[ p < 0 \] so might replace \( V_0 \)

but

\[ t = \frac{x_0 - x_{\text{left}}}{x_0 - x_1} > 1 \]

so reject.

\[ p > 0 \] so might replace \( V_1 \)

but

\[ t = \frac{x_0 - x_{\text{left}}}{x_0 - x_1} < 0 \]

so reject.
Liang-Barsky Rules

- $0 < t < 1, \ p < 0$ replace $V_0$
- $0 < t < 1, \ p > 0$ replace $V_1$
- $t < 0, \ p < 0$ no change
- $t < 0, \ p > 0$ reject
- $t > 1, \ p > 0$ no change
- $t > 1, \ p < 0$ reject