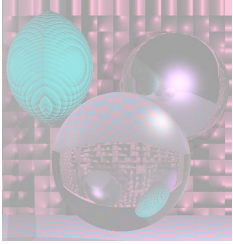# CS 4300
Computer Graphics

Prof. Harriet Fell

Fall 2011
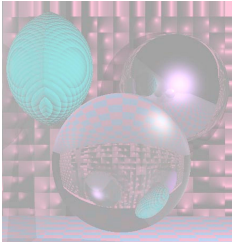
Lecture 28 – November 10, 2011

Today's Topics

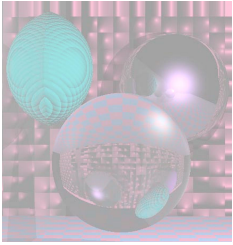- Fractals
  - Mandelbrot Set
    - Julia Sets
 - L-Systems

Fractals

The term *fractal* was coined in 1975 by Benoît Mandelbrot, from the Latin *fractus*, meaning "broken" or "fractured".

(colloquial) a shape that is recursively constructed or self-similar, that is, a shape that appears similar at all scales of magnification.
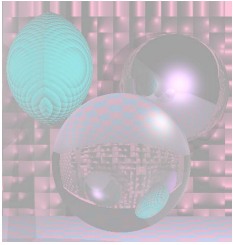
(mathematics) a geometric object that has a Hausdorff dimension greater than its topological dimension.
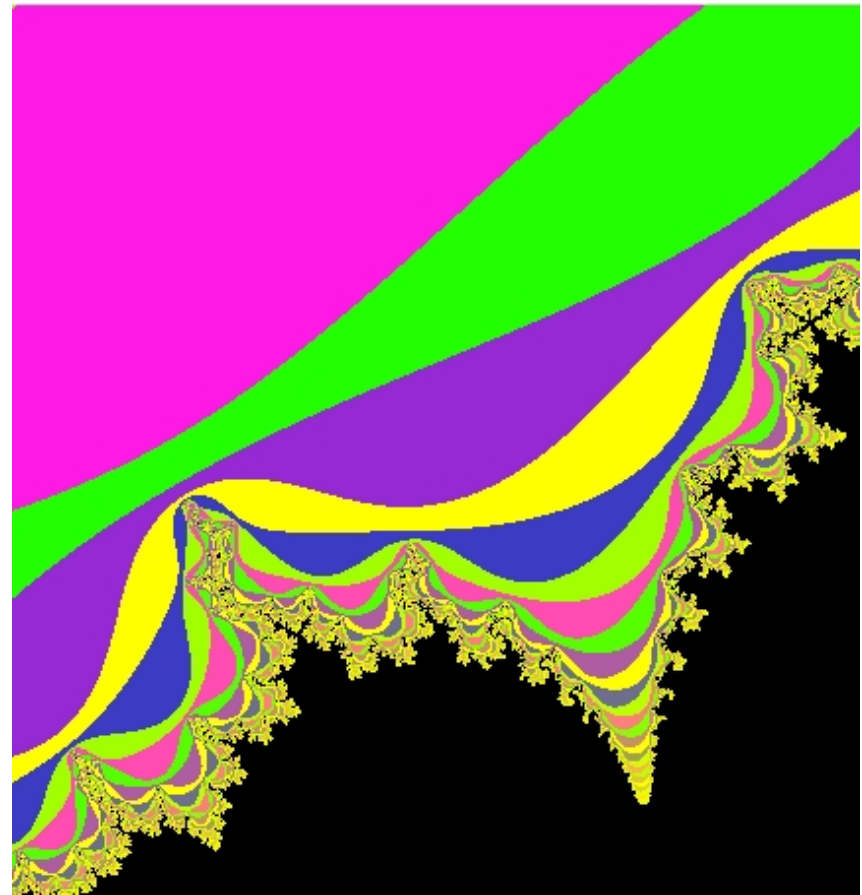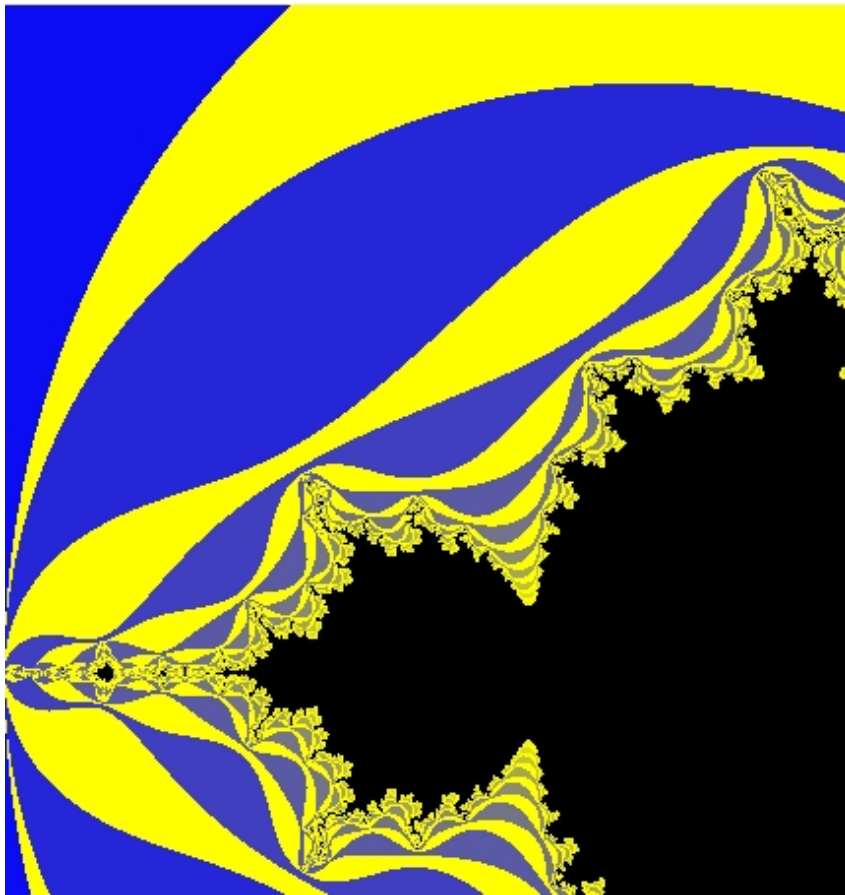
Mandelbrot Set
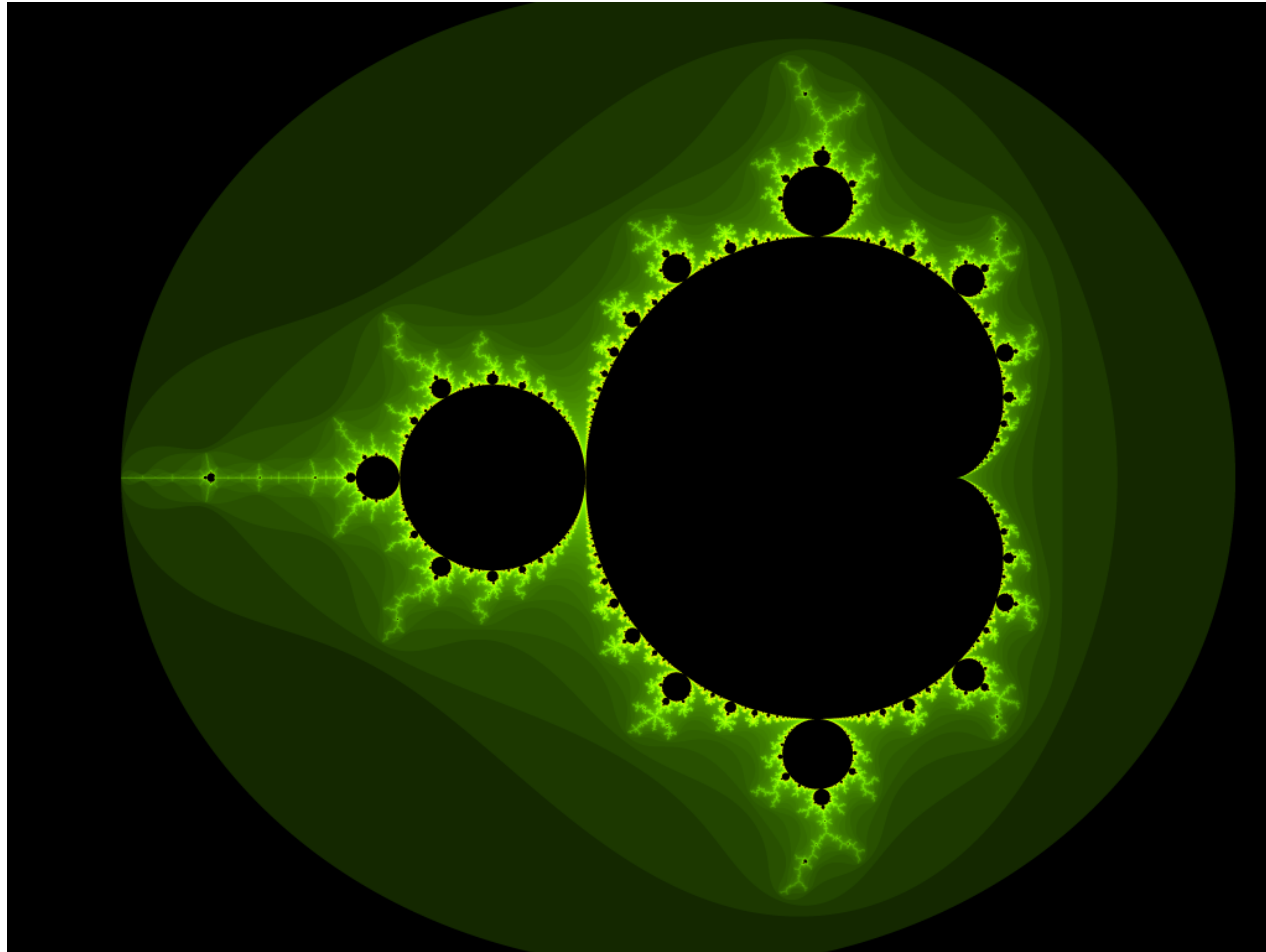
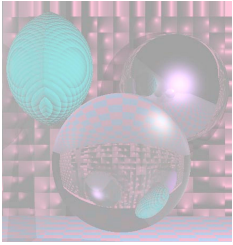

Mandelbrotset, rendered with Evercat's program.

Mandelbrot Set

What is the Mandelbrot Set?
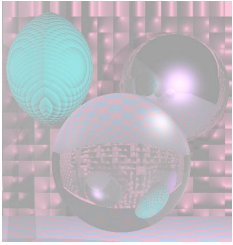
We start with a quadratic function on the complex numbers.

$$f_c : \mathbb{C} \to \mathbb{C}$$

$$z \mapsto z^2 + c$$

The *Mandelbrot Set* is the set of complex c such that

$$f_c^n(0) \not\to \infty$$

where f_c^n is the n-fold composition of f_c with itself.

Example

$$f(z) = z^2 - 1$$

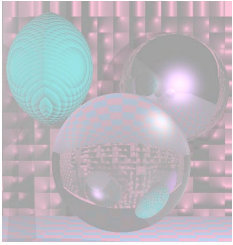$$f(0) = -1 \qquad f^2(0) = f(-1) = 1 - 1 = 0$$

$$f^n(0) = \begin{cases} -1 & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

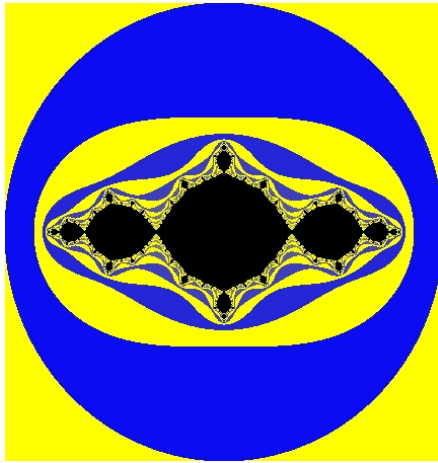$$f(2) = 4 - 1 = 3 \qquad f^2(2) = f(3) = 9 - 1 = 8$$

$f^n(2)$ tend to ∞ as n tends to ∞.
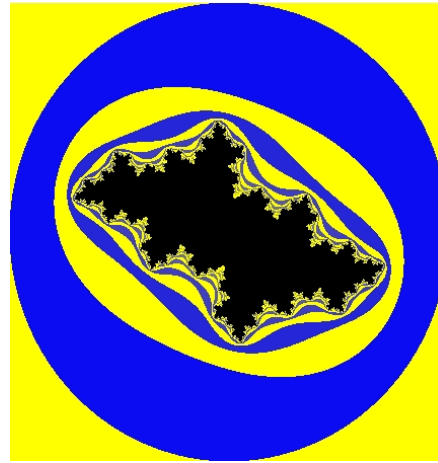
$$f(i) = i^2 - 1 = -2 \qquad f^2(i) = f(-2) = 4 - 1 = 3$$
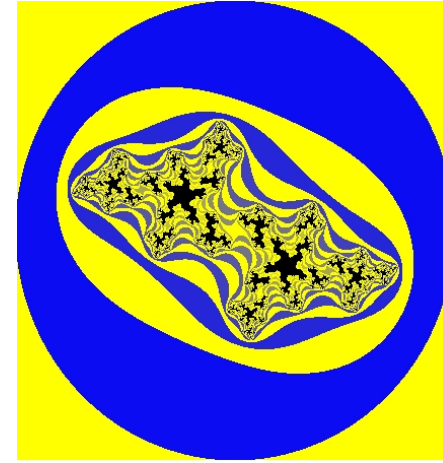
$f^n(i)$ tend to ∞ as n tends to ∞.

(Filled-in) Julia Sets
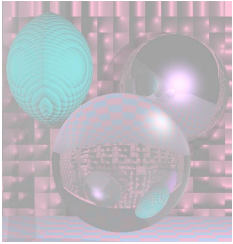

$$c = -1 \qquad c = -.5 + .5i \qquad c = -5 + .5i$$

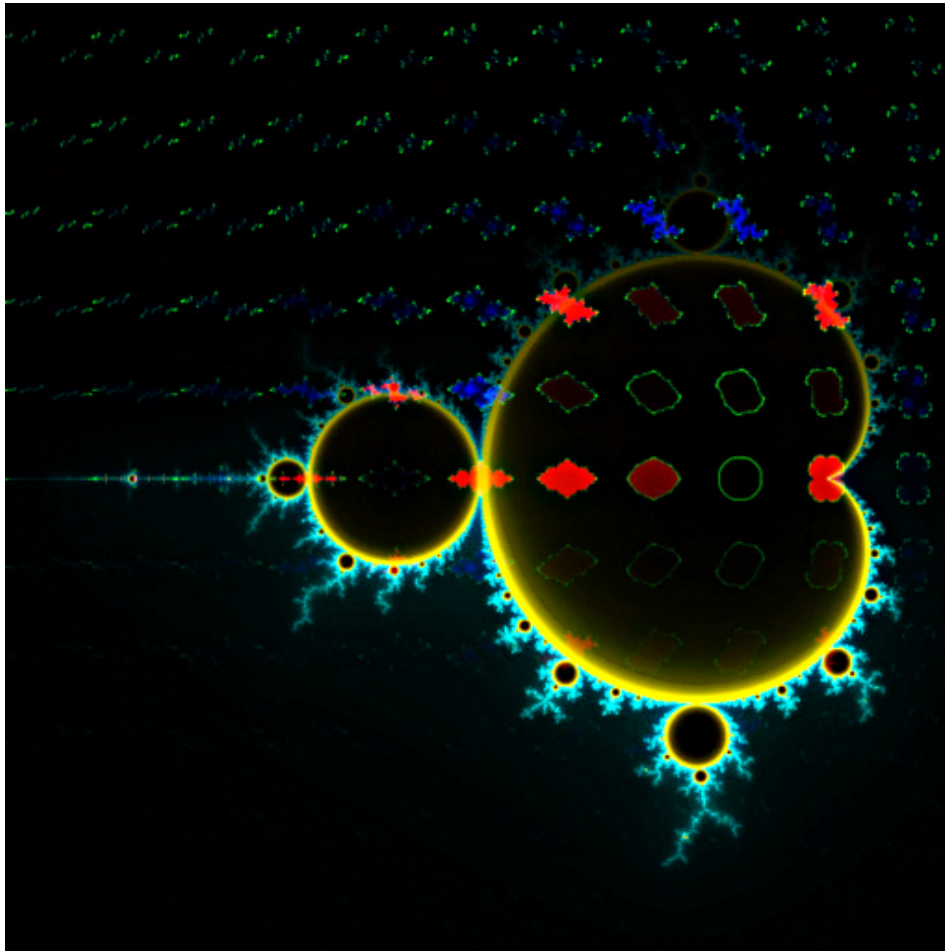$$f_c : \mathbb{C} \to \mathbb{C}$$

$$z \mapsto z^2 + c$$

The *Julia Set* of f_c is the set of points with 'chaotic' behavior under iteration.

The *filled-in Julia set* (or Prisoner Set), is the set of all *z* whos orbits do not tend towards infinity. The "normal" Julia set is the boundary of the filled-in Julia set.

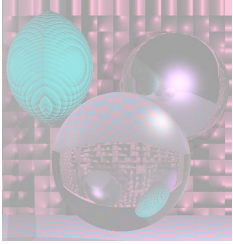# Julia Sets and the Mandelbrot Set

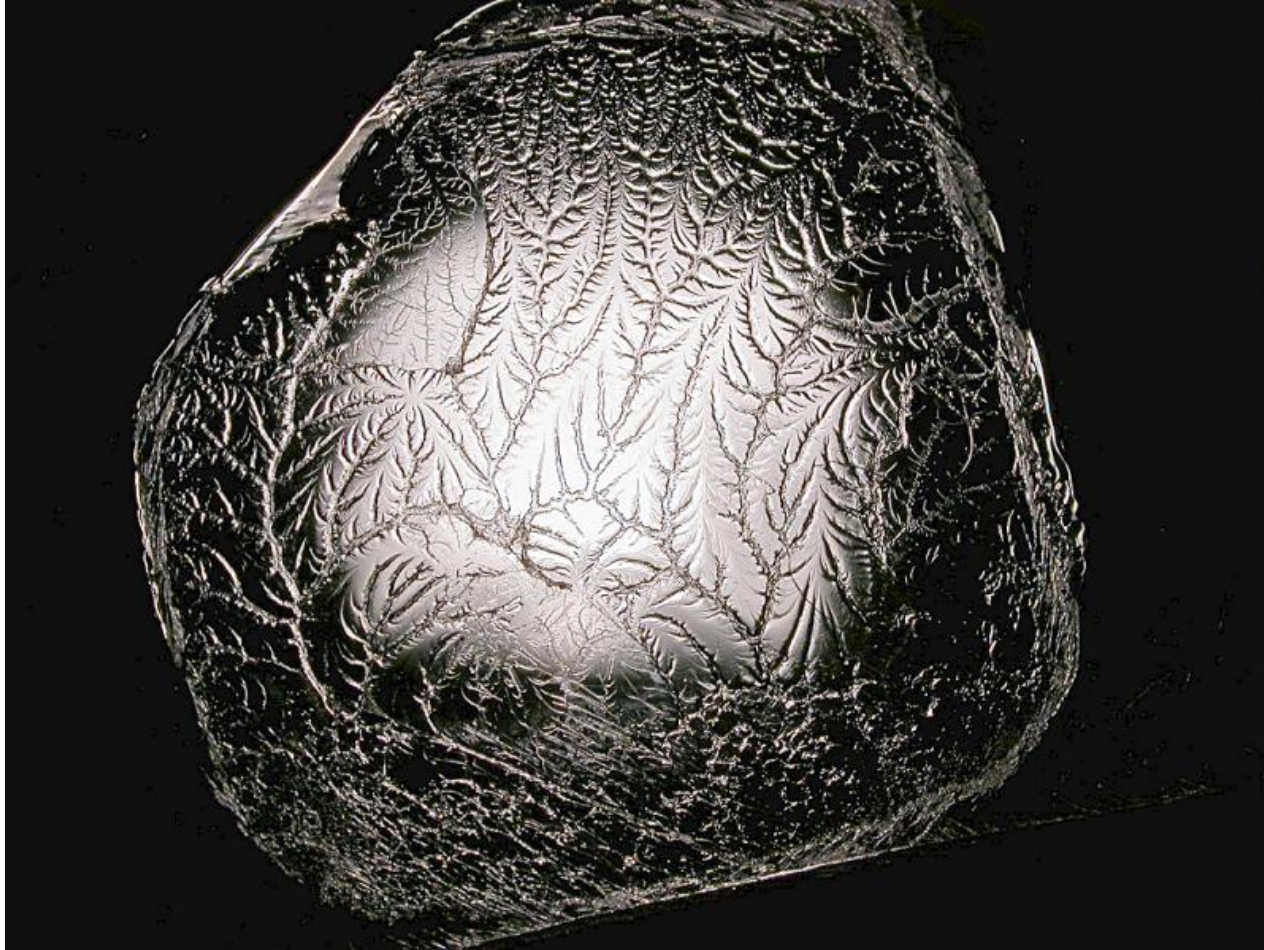Map of 121 Julia sets in position over the Mandelbrot set (wikipedia)
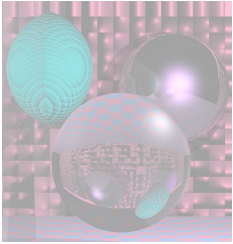
Some Julia sets are connected others are not.

The Mandelbrot set is the set of Complex numbers c for which the Julia set of $f_c(z) = z^2 + c$ is connected.

A fractal is formed when pulling apart two glue-covered acrylic sheets.

Fractal Form of a Romanesco Broccoli
photo by Jon Sullivan
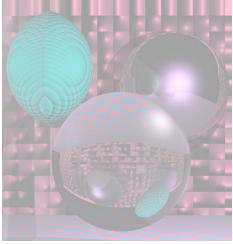
L-Systems

- An *L-system* or *Lindenmayer system*, after Aristid Lindenmayer (1925–1989), is a formal grammar (a set of rules and symbols) most famously used to model the growth processes of plant development, though able to model the morphology of a variety of organisms.

- L-systems can also be used to generate self-similar fractals such as iterated function systems.

L-System References

- Przemyslaw Prusinkiewicz & Aristid Lindenmayer, "The Algorithmic Beauty of Plants," Springer, 1996.

- http://en.wikipedia.org/wiki/L-System

L-System Grammar

- **G** = {*V, S, ω, P*}, where
  - **V** (the *alphabet*) is a set of variables
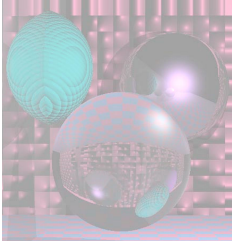 - **S** is a set of constant symbols
  - **ω** (*start, axiom* or *initiator*) is a string of symbols from **V** defining the initial state of the system
  - **P** is a set of *rules* or *productions* defining the way variables can be replaced with combinations of constants and other variables.
    - A production consists of two strings - the *predecessor* and the *successor*.

L-System Examples

- **Koch curve** (from wikipedia)
- A variant which uses only right-angles.
    - **variables** : F
    - **constants** : + −
    - **start** : F
    - **rules** : (F → F+F−F−F+F)
- Here, *F* means "draw forward", + means "turn left 90°", and - means "turn right 90°" (see turtle graphics).
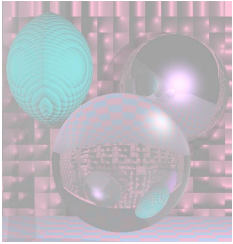
Turtle Graphics

```
class Turtle {
    double angle;      // direction of turtle motion in degrees
    double X;          // current x position
    double Y;          // current y position
    double step;       // step size of turtle motion
    boolean pen;       // true if the pen is down

 public void forward(Graphics g)
// moves turtle forward distance step in direction angle

public void turn(double ang)
// sets angle = angle + ang;

public void penDown(), public void penUp()
// set pen to true  or  false
}
```

My L-System Data Files

Koch Triangle Form   // title
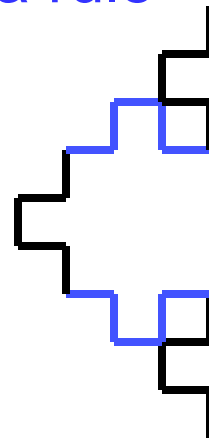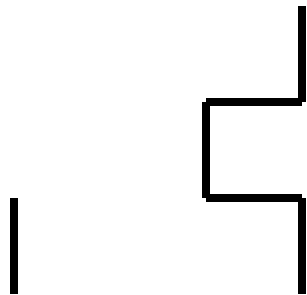4   // number of levels to iterate
90   // angle to turn
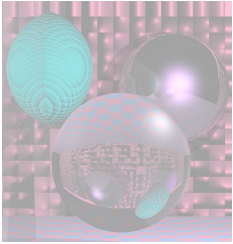F   // starting shape
F:F+F-F-F+F   // a rule

Go to Eclipse

F F+F-F-F+F F+F-F-F+F+F+F-F-F+F-F+F-F-F+F-F+F-F-F+F+F+F-F-F+F

More Variables

Dragon
10
90
L
L:L+R+
R:-L-R

When drawing, treat L and R just like F.

L   L+R+   L+R+ + -L-R +   L+R+ + -L-R + + - L
                                          +R+ - -L-R +

A Different Angle

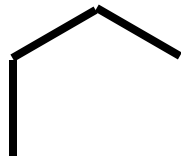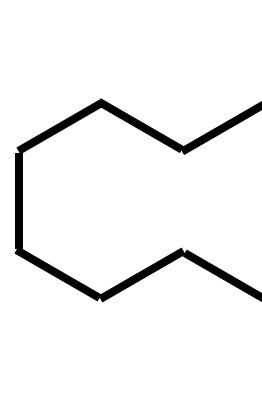Sierpinski Gasket
6
60
R
L:R+L+R
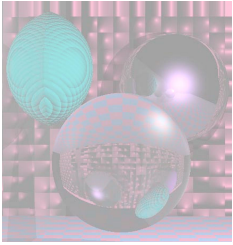R:L-R-L

R   L-R-L   R+L+R- L-R-L -R+L+R

Moving with Pen Up

Islands and Lakes
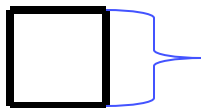
2

90

F+F+F+F

F:F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF
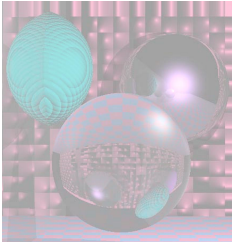
f:ffffff   // f means move forward with the pen up
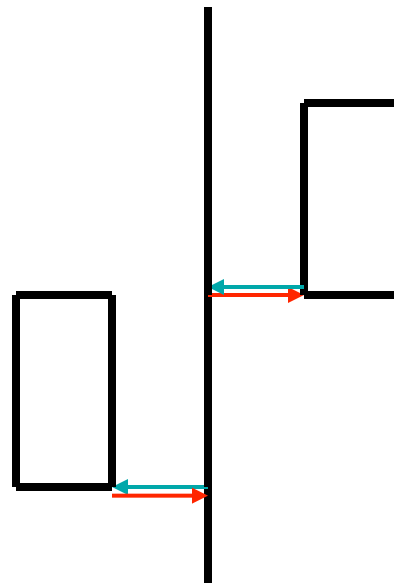
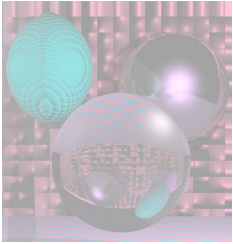F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF

next slide

F+F+F+F

Islands and Lakes
One Side of the Box
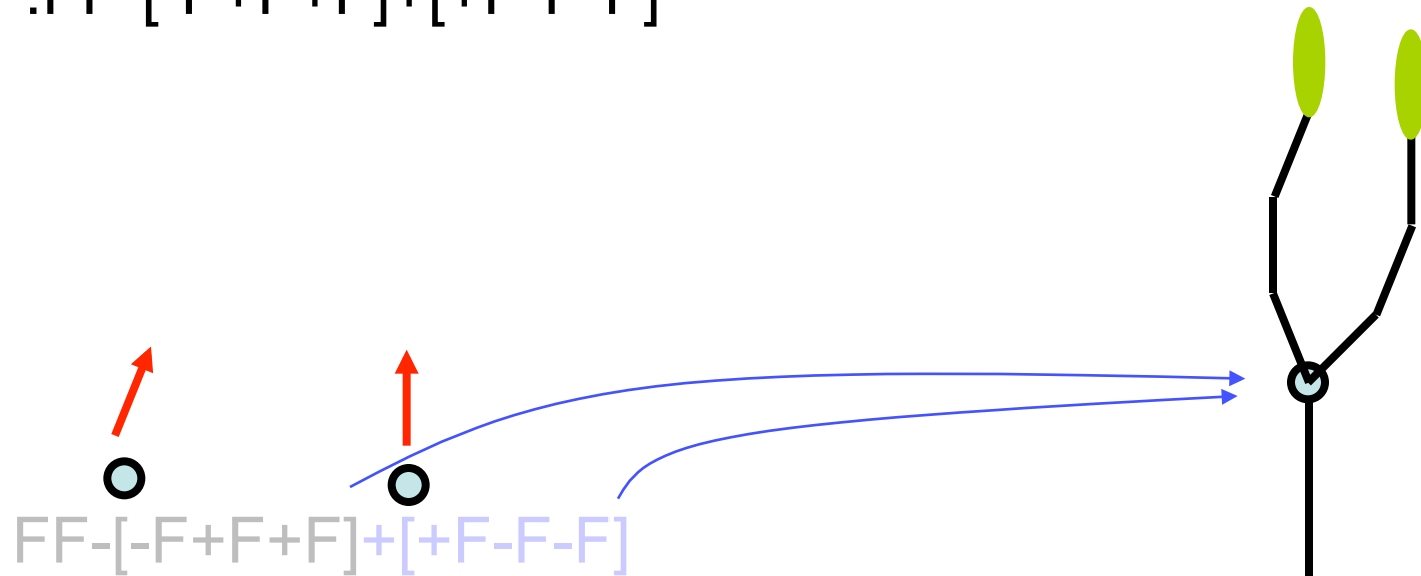

F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF

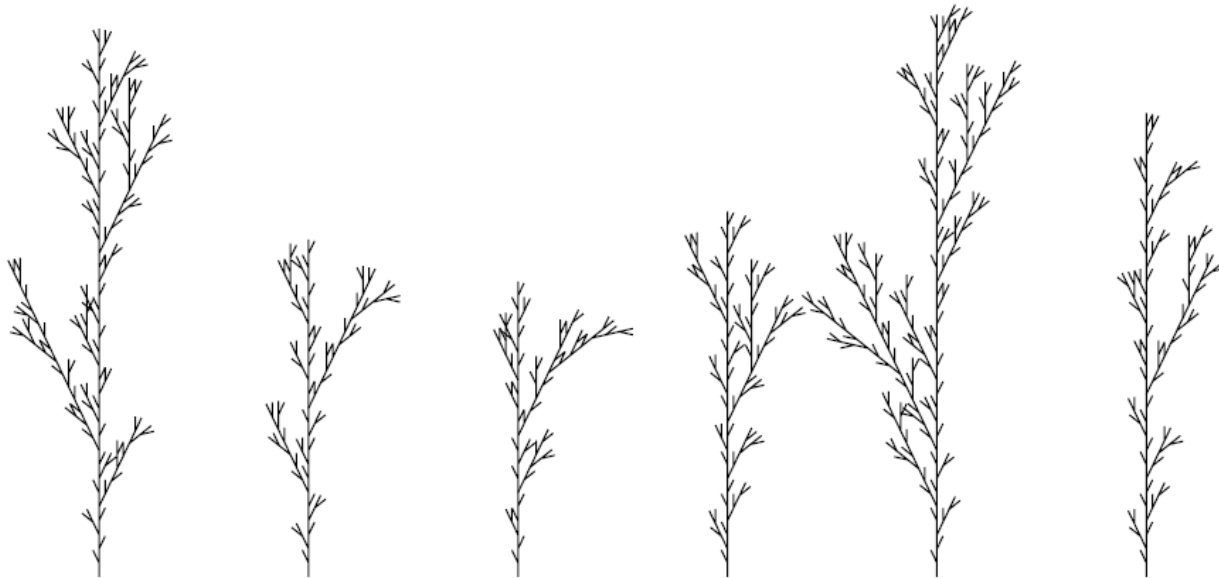# Using a Stack to Make Trees

Tree1   [push the turtle state onto the stack

4   ]   pop the turtle state from the stack

22.5   and I add leaves here

F

F:FF-[-F+F+F]+[+F-F-F]

FF-[-F+F+F]+[+F-F-F]

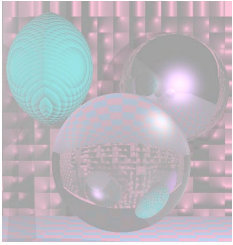# Stochastic L-Systems

seed: 2454 // different seeds for different trees
derivation length: 3
axiom: F
F--> F[+F]F[-F]F : 1/3
F--> F[+F]F : 1/3
F--> F[-F]F : 1/3

3D Turtle Rotations

Heading, Left, or, Up vector tell turtle direction.

$+(\theta)$　　Turn left by angle $\theta°$ around the *U* axis.

$-(\theta)$　　Turn right by angle $\theta°$ around the *U* axis.

$\&(\theta)$　　Pitch down by angle $\theta°$ around the *L* axis.

$\wedge(\theta)$　Pitch up by angle $\theta°$ around the *L* axis.

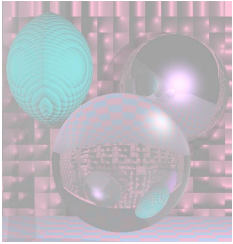$\backslash(\theta)$　　Rollleftbyangle$\theta°$ around the *H* axis.

$/(\theta)$　　Roll right by angle $\theta°$ around the *H* axis.

$|$　　　Turn around $180°$ around the *U* axis.

@v　　Roll the turtle around the *H* axis so that *H* and *U* lie in a common vertical plane with *U* closest to up.
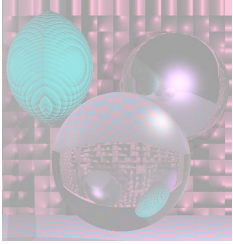
A Mint

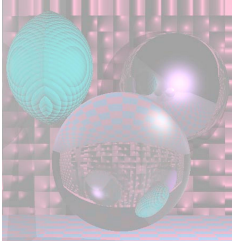A model of a member of the mint family that exhibits a basipetal flowering sequence.

Flow

"Flow means we need a certain amount of time to load our knowledge and skills into our brain RAM. And the more big or small interruptions we have, the less likely we are to ever get there."

> *-- Kathy Sierra*

Slow Down, Brave Multitasker, and Don't Read This in Traffic

- Check e-mail messages once an hour, at most.

- In a recent study, a group of [Microsoft](#) workers took, on average, 15 minutes to return to serious mental tasks, like writing reports or computer code, after responding to incoming e-mail or instant messages.