

## Written Homework 03

**Assigned:** Thu 26 Feb 2009

**Due:** Thu 12 Mar 2009

### Instructions:

- The assignment is due at the *beginning* of class on the due date specified. Late assignments will be penalized 50%, as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.
- We expect that you will study with friends and often work out problem solutions together, but *you must write up your own solutions, in your own words*. Cheating will not be tolerated. Professors, TAs, and peer tutors will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.
- We expect your homework to be neat, organized, legible, and *stapled*. If your handwriting is unreadable, please type. We will *not* accept pages that are ripped from a spiral notebook. Please use 8.5in by 11in loose-leaf or printer paper.

### Problem 1 [20 pts, (4,4,6,6)]: Pseudorandom Number Generators

Pseudorandom number generators are programs that generate sequences of 0s and 1s that “look” random. These sequences have a wide range of applications, including in cryptography, cellular networks, and simulations of large physical systems. Many pseudorandom generators used in practice generate sequences with some specific properties, as discussed below.

Consider a pseudorandom number generator that generates 20-bit sequences; i.e., each sequence is of length 20 and consists of 0s and 1s. For each of the following questions, complete all the calculations and show all your work.

- What is the total number of different 20-bit sequences?
- One criterion that a sequence generator may desire is to have an equal number of 1s and 0s. What number of 20-bit sequences contain an equal number of 0s and 1s?
- Another commonly-used criterion is limiting the length of *runs*. A run is a maximal contiguous sequence of identical bits. That is, a 0-run is a contiguous sequence of 0s, which is preceded by a 1 or is at the start of the sequence, and followed by a 1 or is at the end of the sequence. Similarly, a run of 1s is a contiguous sequence of 1s, which is preceded by a 0 or is at the start of the sequence, and followed by a 0 or is at the end of the sequence. For example, the sequence 001111100001110000111 has a 0-run of length 2, two 0-runs of length 4, one 1-run of length 5, and two 1-runs of length 3.

How many 20-bit sequences contain a 0-run of length 10 or a 1-run of length 10 (or both)?

- How many 20-bit sequences contain at least one 1-run of length 9?

**Problem 2** [20 pts (5,7,8)]: **Basketball**

A local high school basketball team has 14 players. However, only 5 players play at any given time during a game.

- i. In how many ways can the coach choose 5 players?

To be more realistic, the 5 players playing a game normally consist of 2 guards, 2 forwards and 1 center. Assume that there are 6 guards, 3 forwards and 5 centers on the team.

- ii. In how many ways can the coach choose a group of 2 guards, 2 forwards and 1 center?

Now assume that one of the centers can also play forward.

- iii. In how many ways can the coach choose a group of 2 guards, 2 forwards and 1 center?

**Problem 3** [15 pts]: **Binomial Theorem**

Prove that the following statement is true for any number  $n$ :

$$\sum_{j=0}^n \left[ \binom{n}{j} \times (-1)^j \right] = 0$$

(Hint: use the Binomial Theorem.)

**Problem 4** [15 pts]: **Pascal's Triangle**

Write out the numbers in row 11 of Pascal's triangle (assuming that row numbering starts at 0).

**Problem 5** [15 pts]: **Pigeonhole Principle**

New England (consisting of the states MA, VT, ME, NH & CT) is home to over 13 million people. Show that there are at least 3 people in New England with the same three initials who were also born on the same day of the year (though not necessarily in the same *year*).

**Problem 6** [15 pts (4,5,6)]: **Sets**

- i. Let  $A = \{x \mid x \in \mathbb{N} \wedge x^2 + 1 \leq 17\}$ . What are the elements of  $A$ ?
- ii. Let  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$  be the universe. Let  $A = \{1, 4, 5, 7\}$  and  $B = \{1, 3, 6, 7, 8\}$  be two subsets of  $U$ . What are the elements of the set  $\overline{A} \cap B$ ?
- iii. Let  $A = \{7\}$ . What are the elements of  $\mathbb{P}(\mathbb{P}(A))$ ? (Note that  $\mathbb{P}$  is the notation for power set.)