Written Homework 01

Assigned: Thu 15 Jan 2009
Due: Mon 26 Jan 2009

Instructions:

- The assignment is due at the *beginning* of class on the due date specified. Late assignments will be penalized 50%, as stated in the course information sheet. Late assignments *will not be accepted* after the solutions have been distributed.
- We expect that you will study with friends and often work out problem solutions together, but you must write up you own solutions, in your own words. Cheating will not be tolerated. The instructor and TA will be available to answer questions but will not do your homework for you. One of our course goals is to teach you how to think on your own.
- We expect your homework to be neat, organized, and legible. If your handwriting is unreadable, please type. We will *not* accept pages that are ripped from a spiral notebook. Please use 8.5in by 11in loose-leaf or printer paper.

Problem 1 [32 pts, (8,8,8,8)]**:** Negative numbers and two's complement.

In each of the parts below, show your work.

i. Give the 8-bit two's complement representations of the following integers: 42, 89, -71, -115.

ii. Give the integer (in standard base-10 notation) which is represented by each of the following 8-bit two's complement numbers: 11001010, 01011011, 10100101, 11111111.

iii. Compute the following using 8-bit two's complement representations, as shown in class and described in the text: $-71 + 42$, $89 - 71$, and $89 + 42$. In each case, indicate whether the calculation results in an overflow. If it does not result in an overflow, then verify that your answer is correct by converting the result back to standard base-10 notation.

Note: Use the two's complement representations from part i above.

iv. What range of numbers can be represented in (a) 9-bit two's complement and (b) 13-bit two's complement? What are the *minimum* number of bits necessary to represent (c) 1994 and (d) -61 in two's complement?

Problem 2 [20, (4 pts each)]**:** Multiplication.

Perform the following multiplications in binary. For each problem part, you must (1) convert each decimal number to binary, (2) perform the multiplication in binary, and (3) convert the binary result back to decimal. You must show your work. For this problem, use standard binary representation, *not* two's complement representation.

Note: For consistency, place the binary representation of the left multiplicand in the top row of your multiplication and place the binary representation of the right multiplicand on the bottom row of your multiplication. Thus, "4 × 7" would be

$$\begin{array}{rrrr} & 1 & 0 & 0 \\ \times & 1 & 1 & 1 \\ \hline \end{array}$$

while "7 × 4" would be

$$\begin{array}{rrrr} & 1 & 1 & 1 \\ \times & 1 & 0 & 0 \\ \hline \end{array}$$

by this convention.

i. 11×18

ii. 79×39

iii. 18×79

iv. 11×39

Just as in addition, multiplication of two binary numbers may result in overflow if the size of the representation is not large enough.

v. What is the maximum number of bits needed to represent the multiplication of a 12-bit number with an 8-bit number? Justify your answer.

Problem 3 [24, (8 pts each)]**:** IP addresses and their various formats.

IP addresses are 32-bit binary numbers, most commonly expressed in the dotted-decimal format in which the 32 bits are grouped into four bytes of 8 bits each, separated by the dot symbol, and each byte is written out in decimal form. Thus, the following IP address, written in binary,

10000001000010100111010011001000

is written as follows in the dotted decimal notation:

129.10.116.200

The first eight bits 10000001 form the number 129 in decimal notation, the next eight bits 00001010 form the number 10, the next eight bits 01110100 form the number 116, and finally the last eight bits 11001000 form the number 200.

IP addresses can also be represented in other formats, including hexadecimal and decimal. In fact, most browsers will accept IP addresses in these representations as well. These formats are used sometimes for purposes of obfuscation and identity-hiding.

i. Convert the IP address 199.239.136.200 from dotted decimal format to the hexadecimal and the decimal formats.

ii. Convert the IP address AAB3562A from hexadecimal format to the dotted decimal and decimal formats.

iii. Convert the IP address 2164946120 from decimal format to the hexadecimal and dotted decimal formats.

Problem 4 [24, (6,6,12)]**:** Logical completeness.

Every truth table, Boolean formula, and circuit can be implemented using just AND, OR, and NOT gates; hence, the collection $\{\text{AND}, \text{OR}, \text{NOT}\}$ is *logically complete*. In this problem, you will show that the NOR gate, by itself, is logically complete. To do so, you will show how to construct the AND, OR, and NOT gates from NOR gates.

i. Fill in the following truth table:

X	X NOR X
0	?
1	?

What logical operation does X NOR X correspond to?

ii. Fill in the following truth table:

X	Y	$\neg X$ NOR $\neg Y$
0	0	?
0	1	?
1	0	?
1	1	?

What logical operation does $\neg X$ NOR $\neg Y$ correspond to?

iii. Using only NOR gates, draw circuit diagrams corresponding to the AND, OR, and NOT gates. *Hint:* The constructions for two of these circuits are essentially given in parts i and ii above, and the construction of the third should be relatively straightforward given what you've learned above.