Big Data Summarization: The Role of Submodularity

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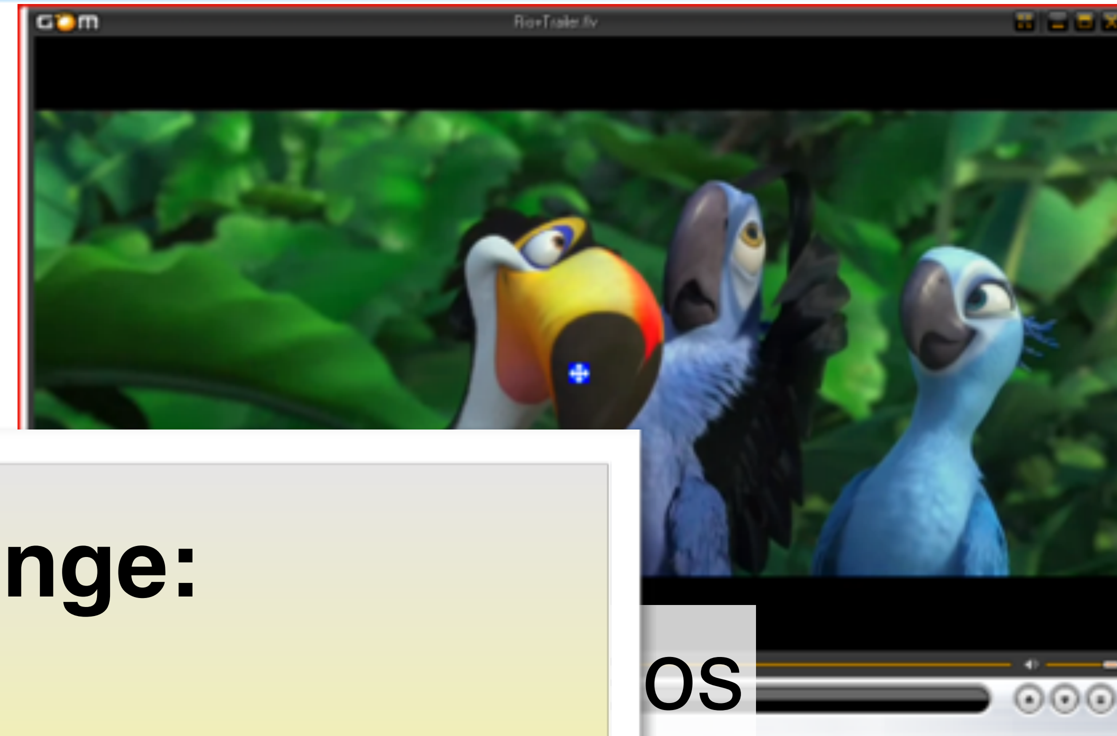
Yale

From Big Data to Small Data

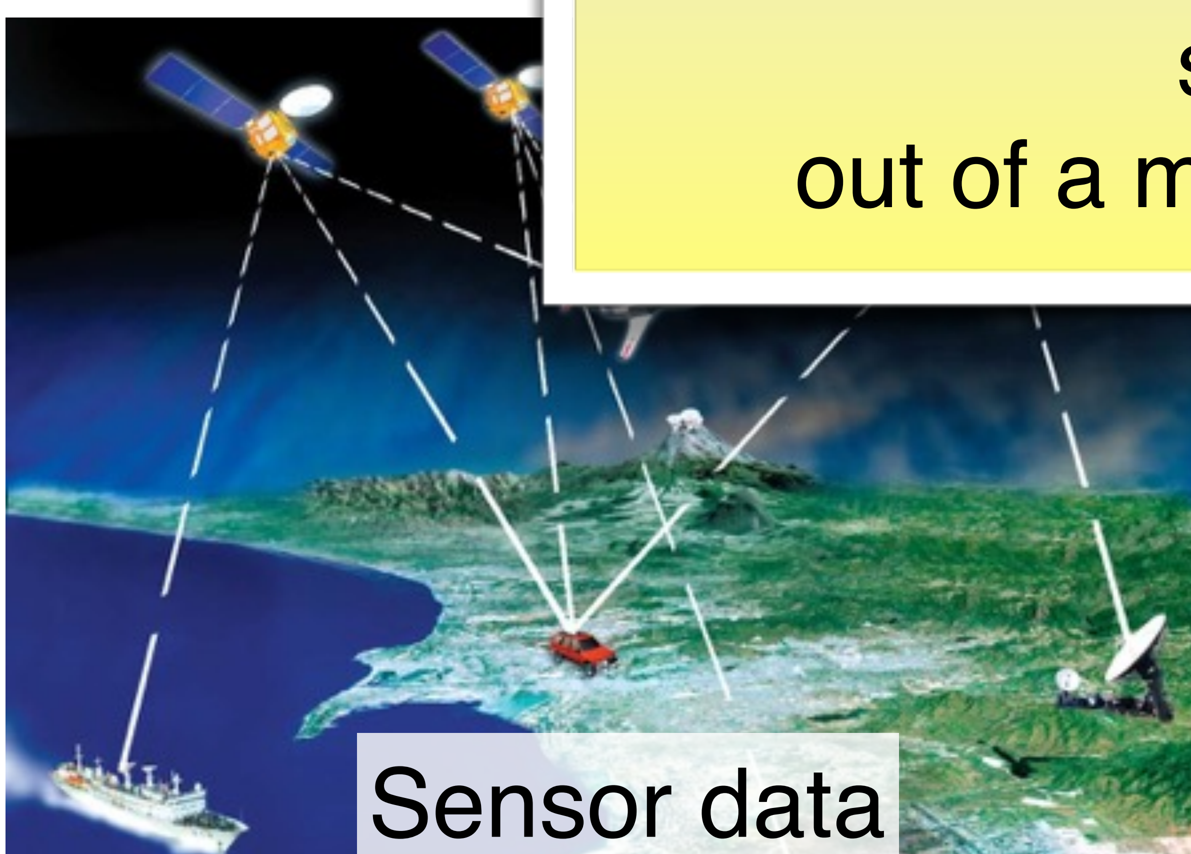
- Data Summarization: What, Why, **How**.
- Summarization through the lens of submodularity
- Efficient algorithms for data summarization

Summarization



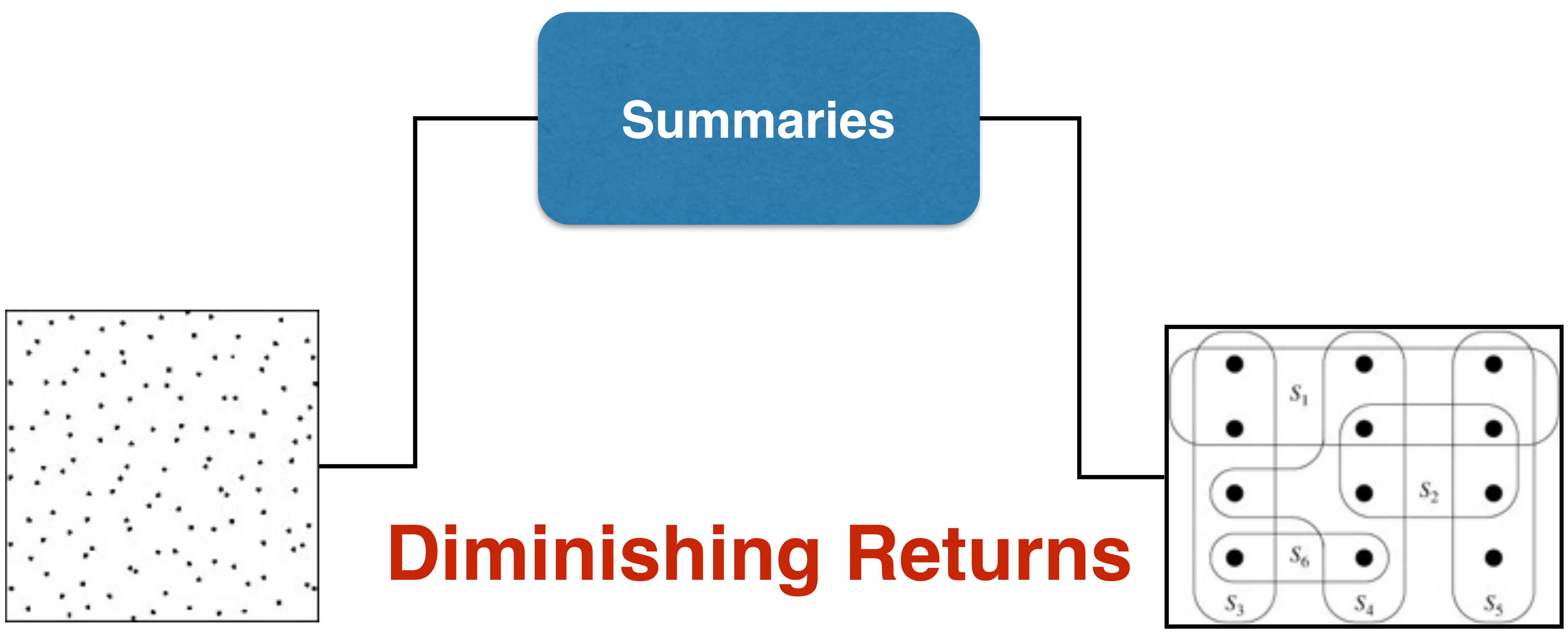
Key challenge:

Extract small, representative subset
out of a massive data set



Summarization: Benefits

- Remove redundancy
- Run the algorithms efficiently AGAIN
- Obtain more accurate results

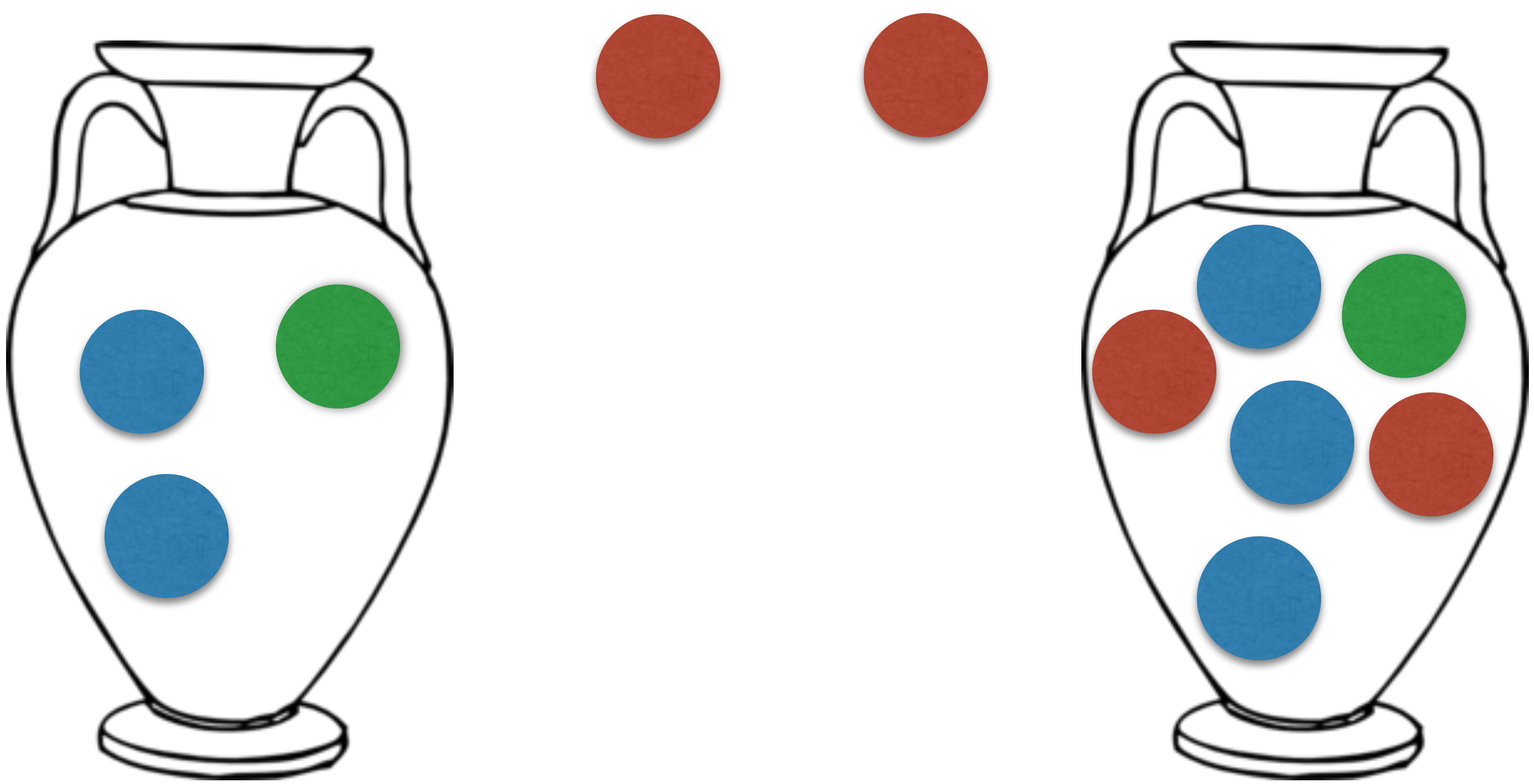


Diversity
(Essential information)

Coverage
(No loss of information)

Balls in Urns (Toy Example)

Utility: number of distinct colors



Initial Value: 2

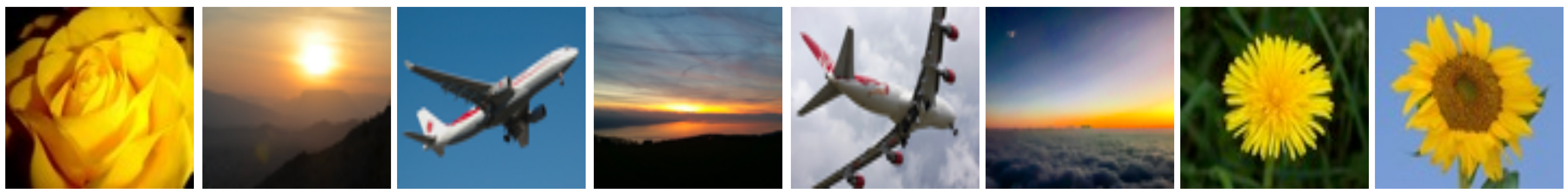
New Value: 3

Initial Value: 3

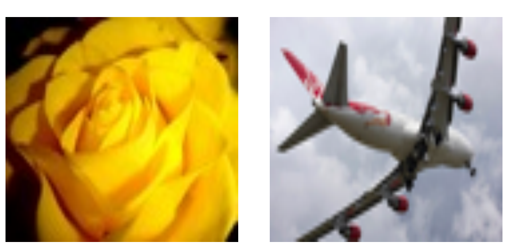
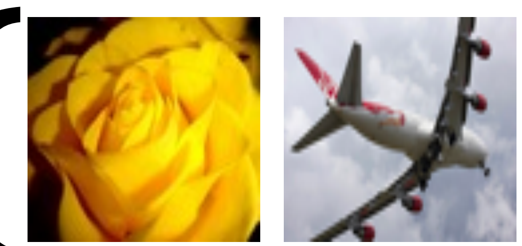
New Value: 3

Submodularity

- Diminishing returns property for set functions.

$V = \{$  $\}$

$f\Big(\{$  $\}\Big) - f\Big(\{$  $\}\Big) \geq$

$f\Big(\{$  $\}\Big) - f\Big(\{$  $\}\Big)$

$\forall\, A \subseteq B \subseteq V$ and $x \notin$

$f(A \cup \{x\}) - f(A) \geq f(B \cup \{x$

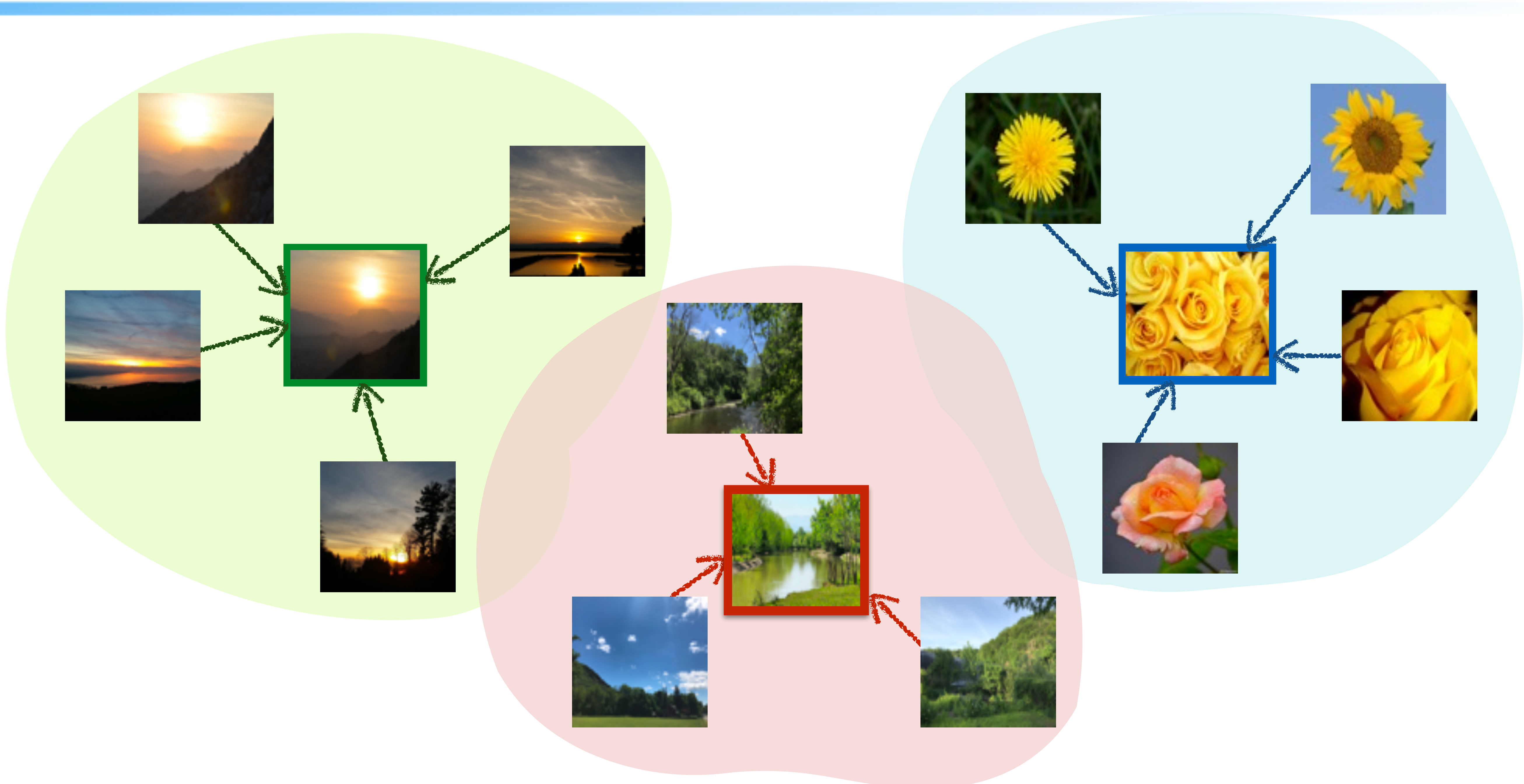
$\forall\, A \subseteq B$

$f(B) \geq f(A)$

Image Summarization

- Given a huge set of images, summarise it to a few **exemplars.**

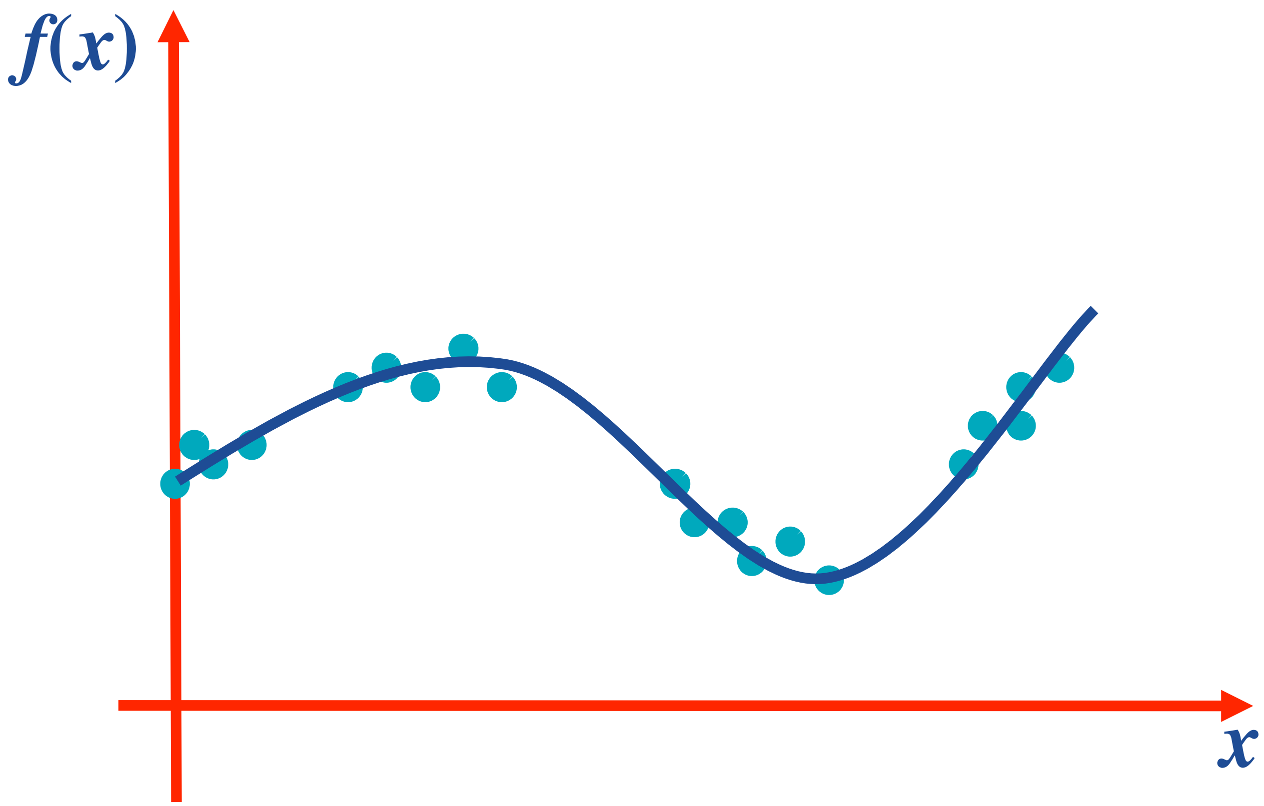
Example: Exemplar Based Clustering



$$L(A) = \frac{1}{V} \sum_{s \in V} \min_{c \in A} d(x_s, x_c)$$

$$f(S) = L(\{e_0\}) - L(S \cup \{e_0\})$$

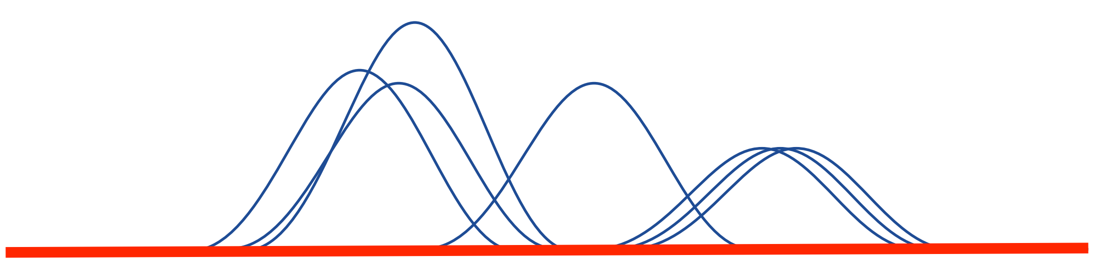
Example: Nonparametric learning in massive data sets

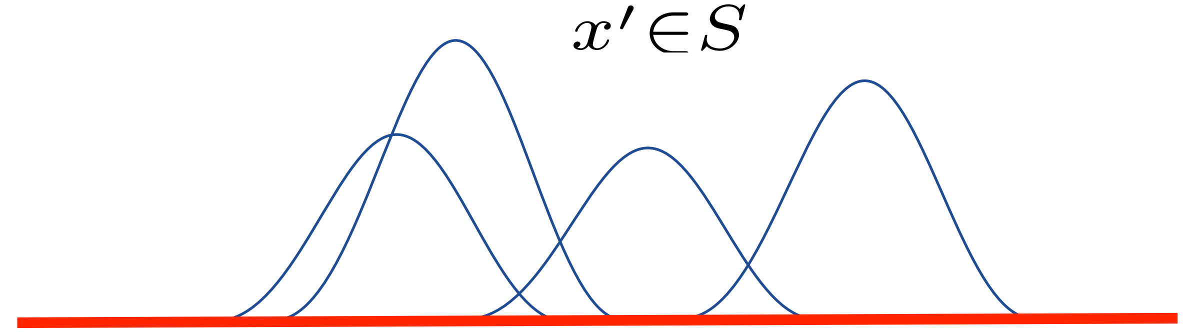


$$S \subseteq V, |S| \ll |V|$$

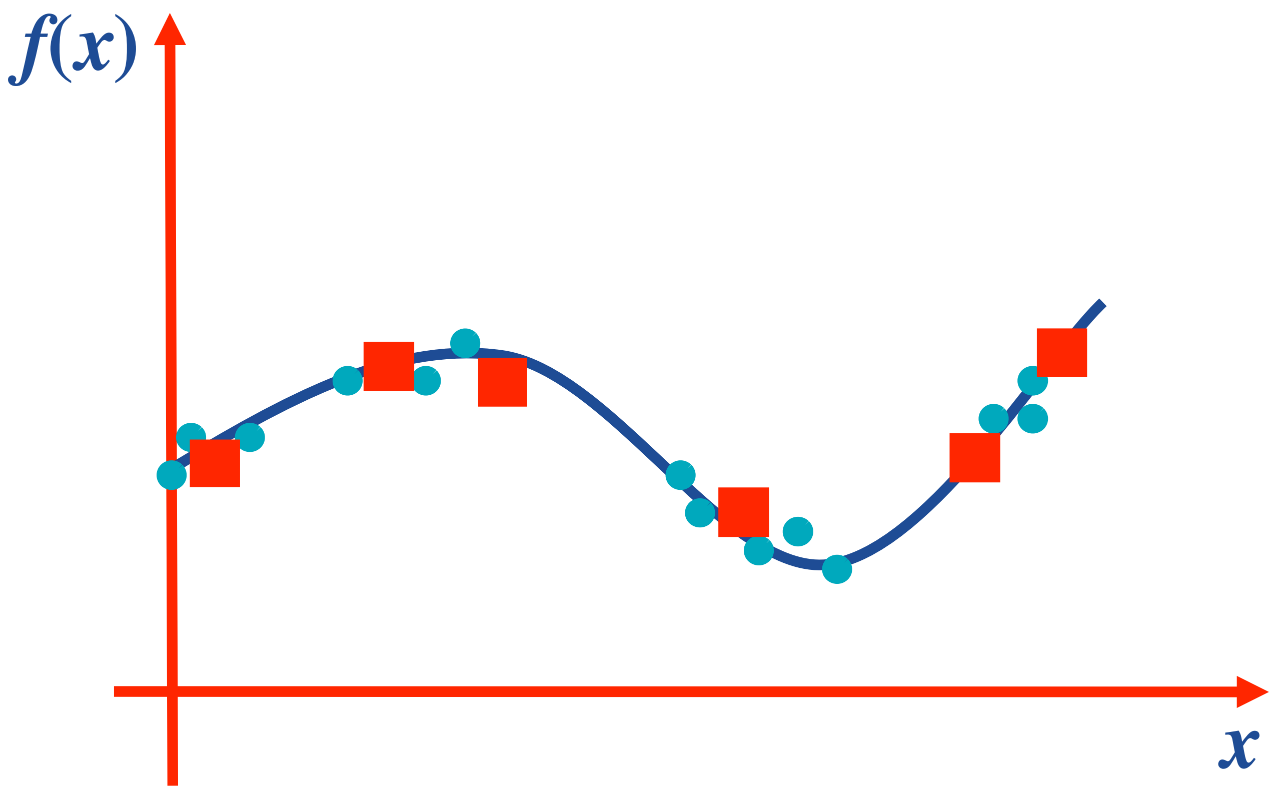
$$f_V(x) = \sum_{x' \in V} \alpha_x k(x', x)$$

$$\approx f_S(x) = \sum_{x' \in S} \hat{\alpha}_x k(x', x)$$





Example: Nonparametric learning in massive data sets



Pick active set to maximally capture variation.

$$F_H(\mathcal{A}) = \log |\mathbf{I} + \mathbf{K}_{\mathcal{A}\mathcal{A}}| \qquad \mathbf{K}_{\mathcal{A}\mathcal{A}} = \begin{pmatrix} k(x_1, x_1) & \dots & k(x_1, x_{|A|}) \\ \vdots & & \vdots \\ k(x_{|A|}, x_1) & \dots & k(x_{|A|}, x_{|A|}) \end{pmatrix}$$

Selecting representative elements via submodular optimization

- Wish to select small, representative subset out of large data set
  - Clustering [NJB ‘05, GK‘ 10]
  - Recommendation [LKGVVG ’06, KSG’11, YG‘12]
  - Document summarization [LB’11, LB‘12]
  - Scaling up non-parametric learning [S‘04, GK‘10]
  - Corpus subset selection [LB‘11]
  - Compostable core-sets [IMMM’14, MZ’15]
- Often, natural utility functions are monotone submodular

$$A^* = \arg\max_{|A| \leq k} F(A)$$

GOAL

- Given a monotone submodular function F, and a cardinality constraint K, find a set A* such that

$$A^* = \arg\max_{|A| \le k} F(A)$$

Centralized Solution

Streaming Solution

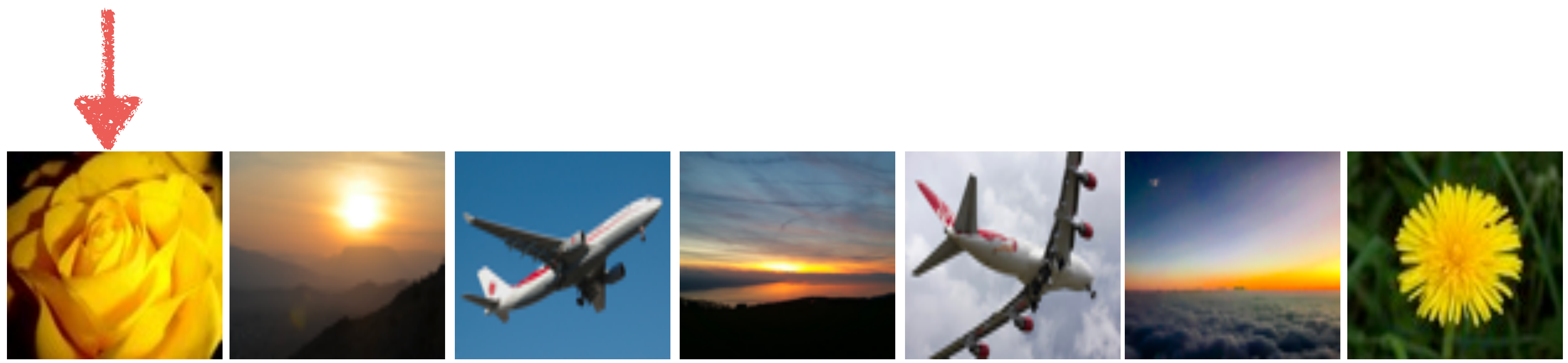
Distributed Solution

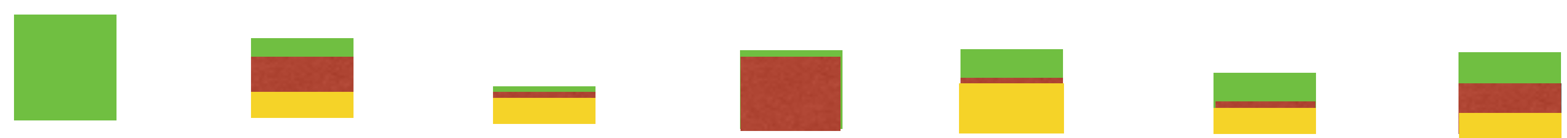
Centralized Solution

Data can be loaded on the main memory

The Greedy Algorithm

- Problem: Find $S^* = \text{argmax } \{F(S) : |S| \leq k\}$



$\Delta f = \{$  $\}$

$$F\left(\left\{ \quad \right\}\right) \longrightarrow \max$$

Greedy algorithm:

Start with $A = \emptyset$

For i = 1 to j

$s^* \leftarrow \arg\max_s F(A \cup \{s\})$

$A \leftarrow A \cup \{s^*\}$

Theorem [Nemhauser, Wolsey & Fisher '78]

For monotonic submodular functions, Greedy algorithm gives constant factor approximation

$$F(S_{\text{greedy}}) \geq (1\text{-}1/e)\ F(S^*)$$

Problem: Greedy can be impractical

- For a ground set V of size n, standard greedy algorithm needs $O(n \cdot k)$ function evaluations to find a summary of size k.

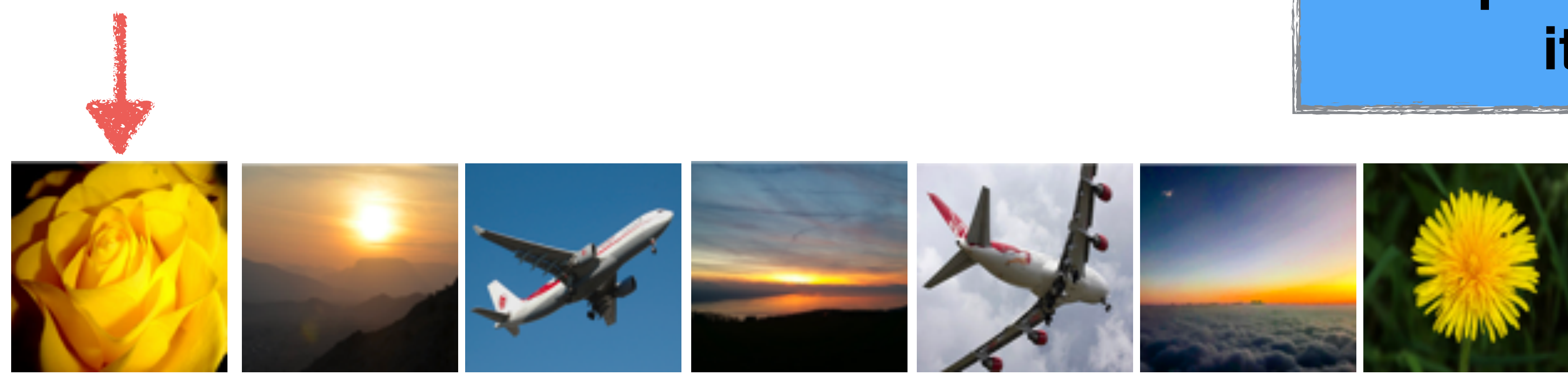
- In many data intensive applications, evaluating f is expensive and running the standard greedy algorithm is infeasible.

Is it possible to have an algorithm that "**does not depend on k**"at all and **scales linearly** with the data size n?

SubSample-Greedy Algorithm

- Problem: Find $S^* = \operatorname{argmax}\ \{F(S) : |S| \le k\}$

Sample n/k points per iteration



$\Delta f = \{$  $\}$

$$f\left(\left\{\qquad\qquad\right\}\right) \longrightarrow \max$$

Subsample Greedy

For monotonic submodular functions, Random Greedy gives constant factor approximation using only ***n*** function evaluation

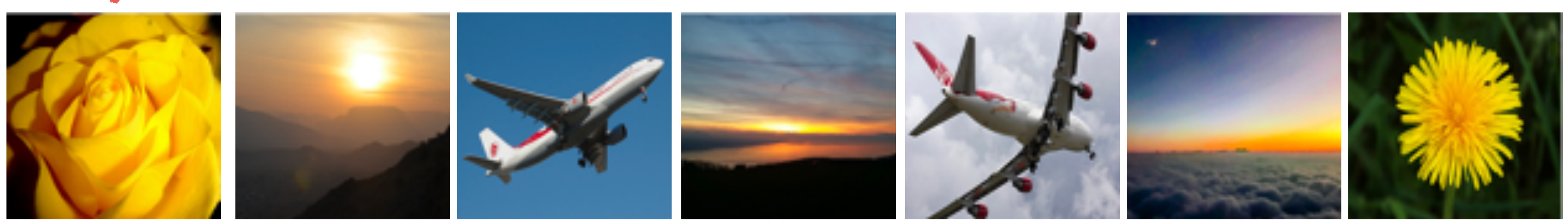
$$E[f(S_{\text{rand greedy}})] \ge (1-1/e)^2 f(S^*)$$

"Differentially Private Submodular Maximization", ICML'17
Mitrovic, Bun, Krause, Karbasi

Random-Greedy Algorithm

- Problem: Find $S^* = \text{argmax } \{F(S) : |S| \leq k\}$

Sample n/k log(1/ε) points per iteration



$\Delta f = \{$

$\}$

$$f\left(\left\{ \qquad \qquad \right\}\right) \longrightarrow \max$$

Random Greedy

For monotonic submodular functions, Random Greedy gives constant factor approximation using only n function evaluation

$$E[f(S_{\text{rand greedy}})] \geq (1\text{-}1/e\text{-}\varepsilon)\, f(S^*)$$

Rand-Greedy Algorithm

Algorithm RAND-GREEDY

Input: $f : 2^V \to \mathbb{R}_+$, $k \in \{1, \dots, n\}$.
Output: A set $A \subseteq V$ satisfying $|A| \le k$.

1: $A \leftarrow \emptyset$.
2: **for** $(i \leftarrow 1; i \le k; i \leftarrow i + 1)$ **do**
3: $R \leftarrow$ a random subset obtained by sampling s_i random elements from $V \setminus A$.
4: $a_i = \text{argmax}_{a \in R} (a \mid A)$.
5: $A \leftarrow A \cup \{a_i\}$.
6: **return** A.

- First centralized algorithm for cardinality-constrained submodular maximization with
  - Constant factor **approximation** guarantee

$$E(f(S)) \geq (1-1/e-\epsilon)\ \texttt{OPT}$$

  - **Time** is linear in the size of the data and
  - is independent of the cardinality constraint, i.e.

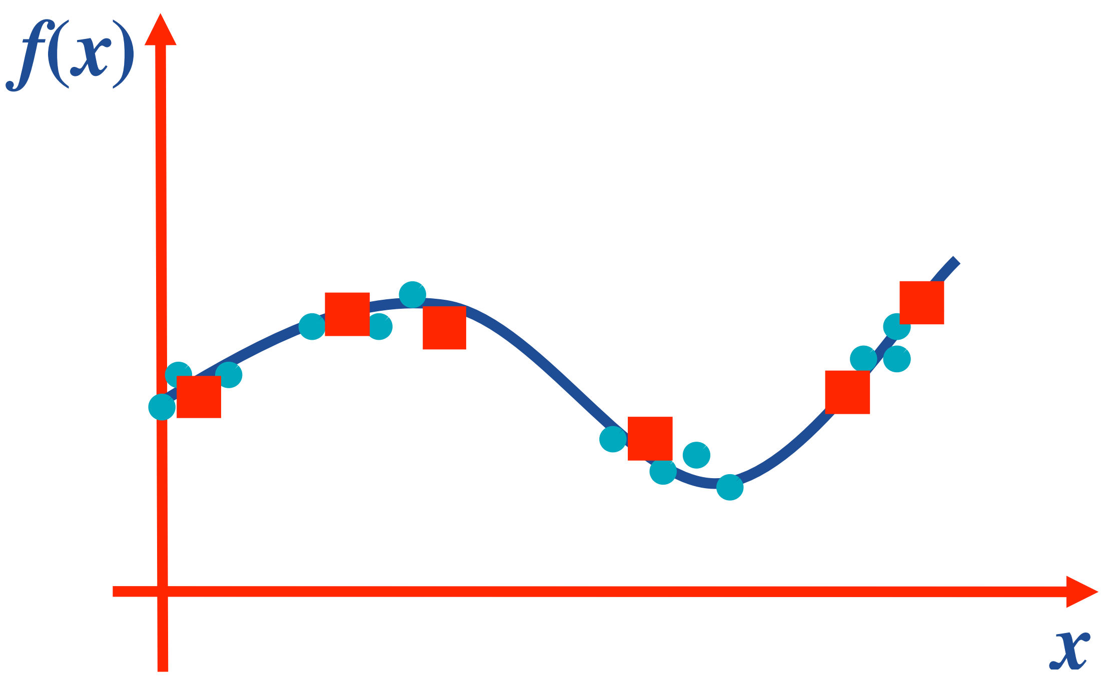
$$O(n \log 1/\epsilon) \text{ function evaluations}$$

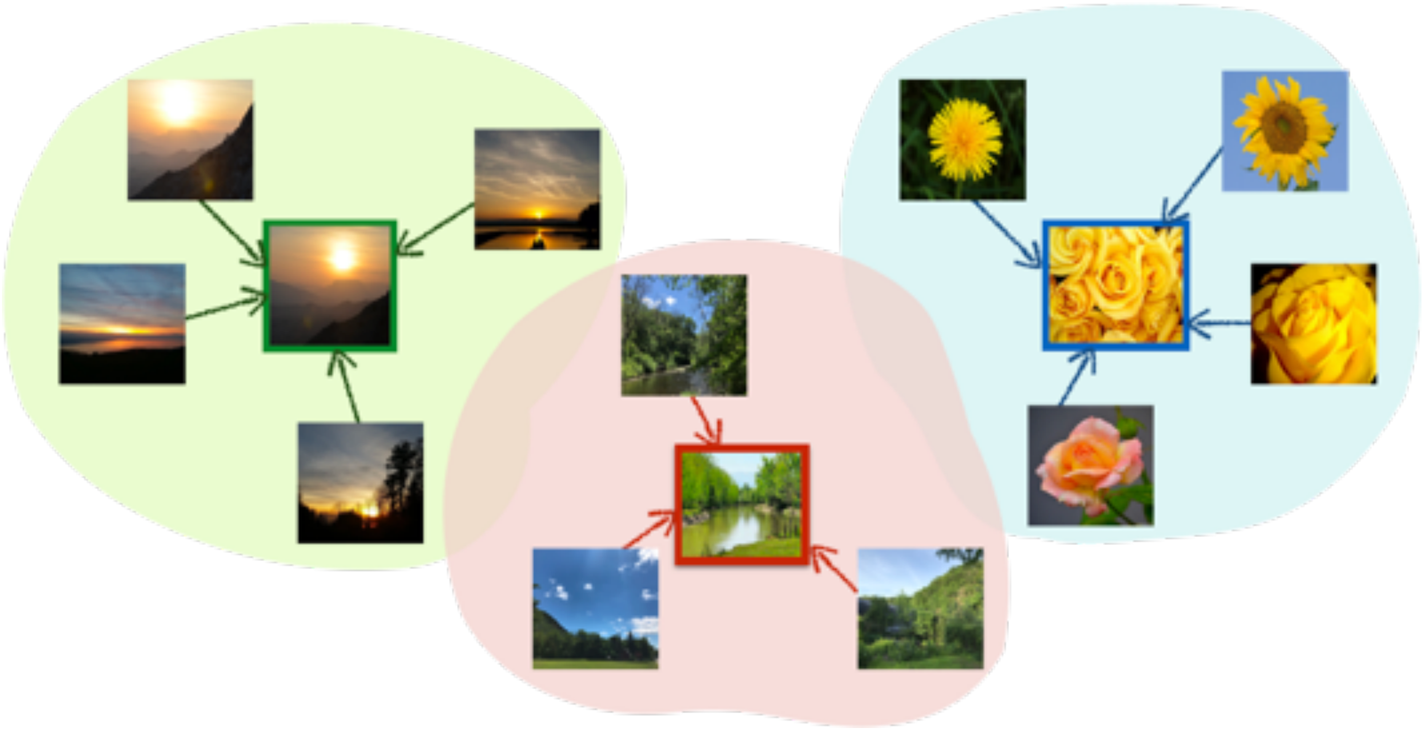
  - Assuming nothing but monotone submodularity

"Lazier Than Lazy Greedy", AAAI'15

(Mirzasoleiman, Badanidiyuru, Karbasi, Vondrak, Krause)

Applications





$$F_H(\mathcal{A}) = \log |\mathbf{I} + \mathbf{K}_{\mathcal{A}\mathcal{A}}|$$

$$L(A) = \frac{1}{V} \sum_{s \in V} \min_{c \in A} d(x_s, x_c)$$

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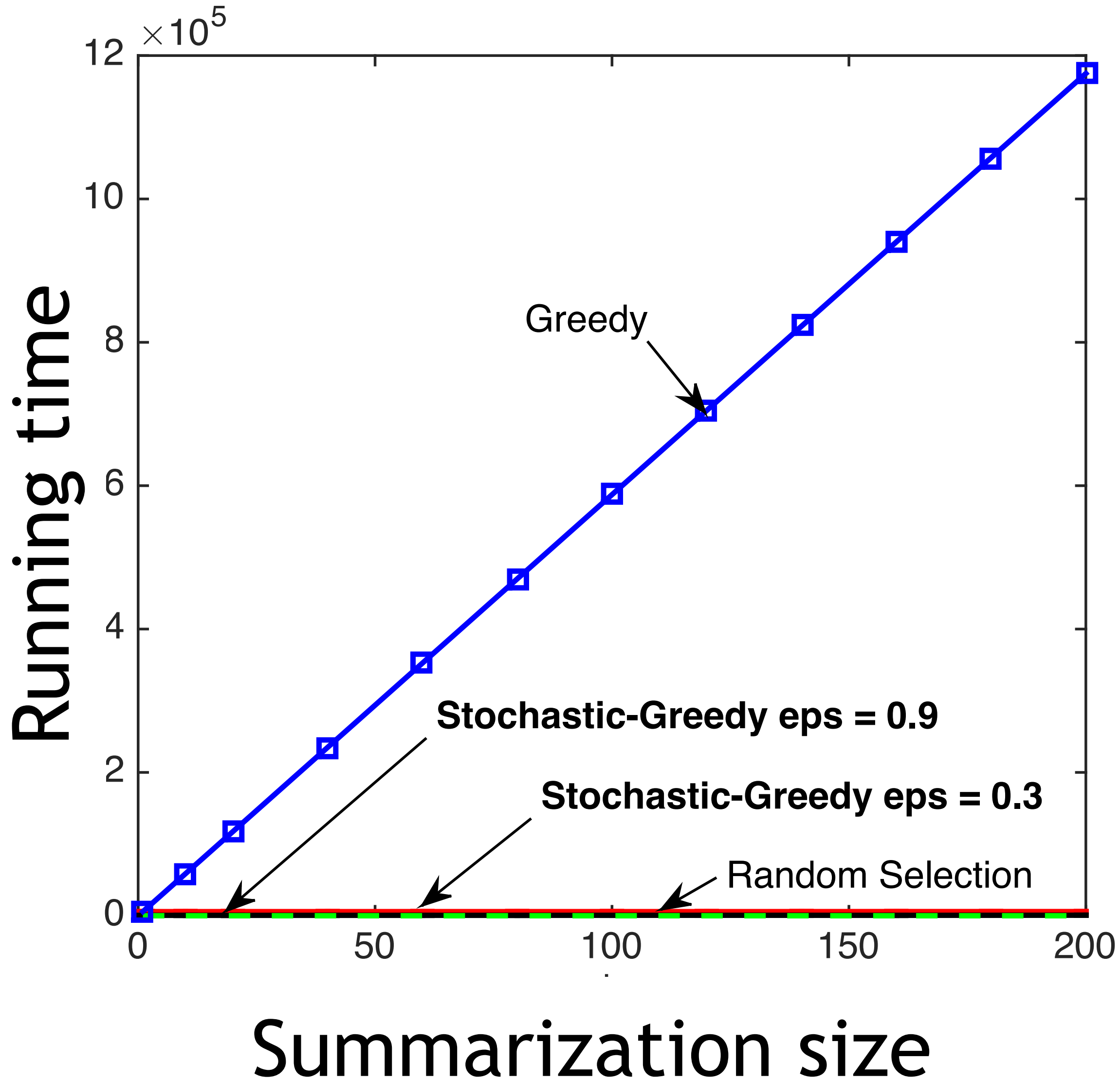
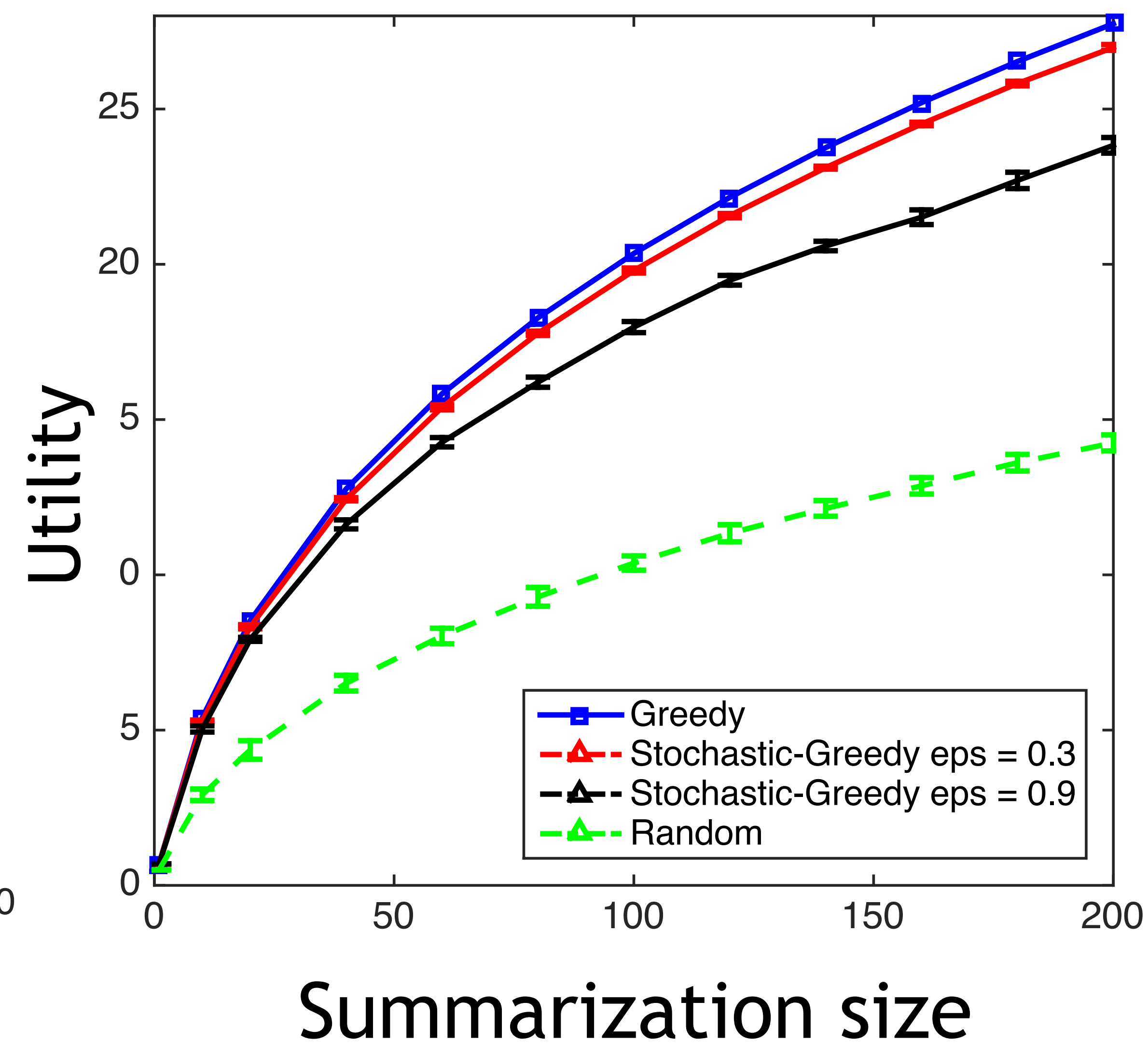
$$f(S) = L(\{e_0\}) - L(S \cup \{e_0\})$$

Nonparametric Regression

Parkinson dataset consists of of 5,875 data points with 22 attributes.





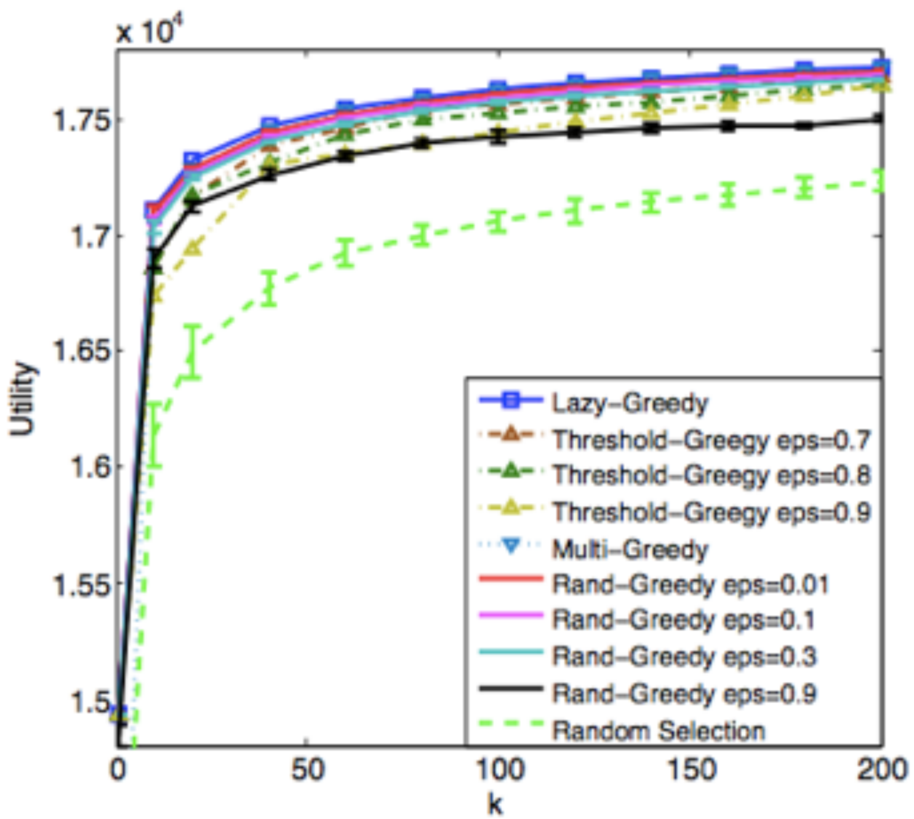
Classic Greedy is too expensive

Experiments

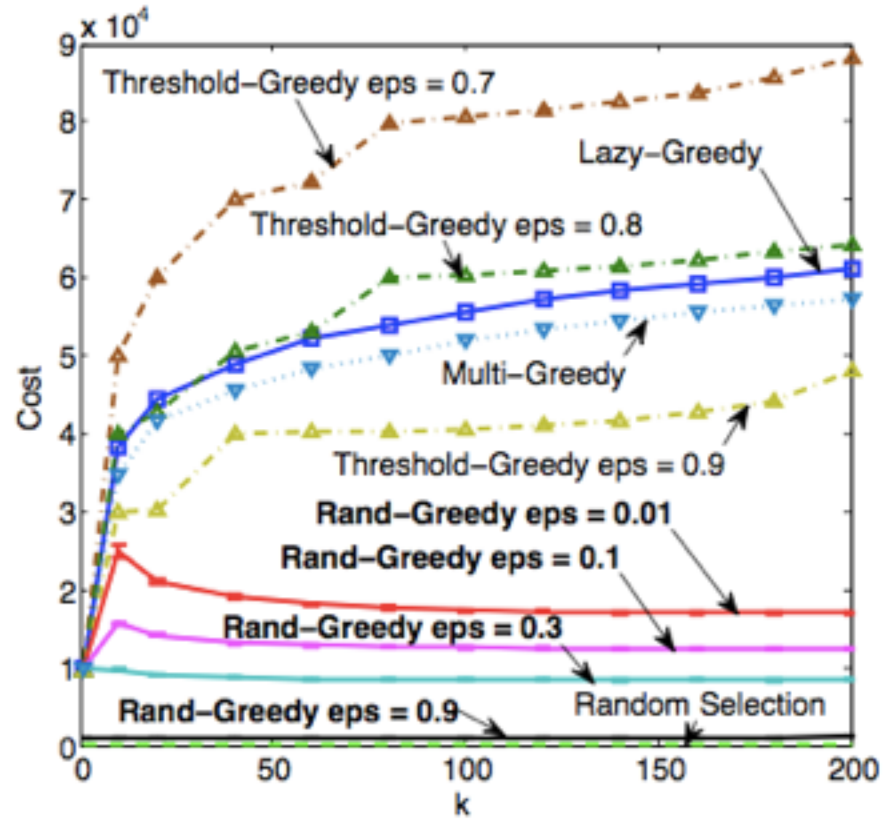
- Benchmarks:
  - **Standard greedy**: the output is the k data points selected by the greedy algorithm. This algorithm is not applicable on large datasets.
  - **Lazy greedy**: the output produced by the accelerated greedy method [M'78]. This algorithm is not applicable in the streaming setting.
  - **Threshold-Greedy**: The output is the k data points provided by Threshold-Greedy [BV'14].
    - Maintains a continuously decreasing threshold and takes elements when their marginal value is above the threshold.
  - **Sample-Greedy**: The output is the k data points produced by applying Lazy-Greedy on a subset of data points parametrized by sampling probability *p*.
  - **Random selection**: The output is k randomly selected data points from V.

Nonparametric Regression

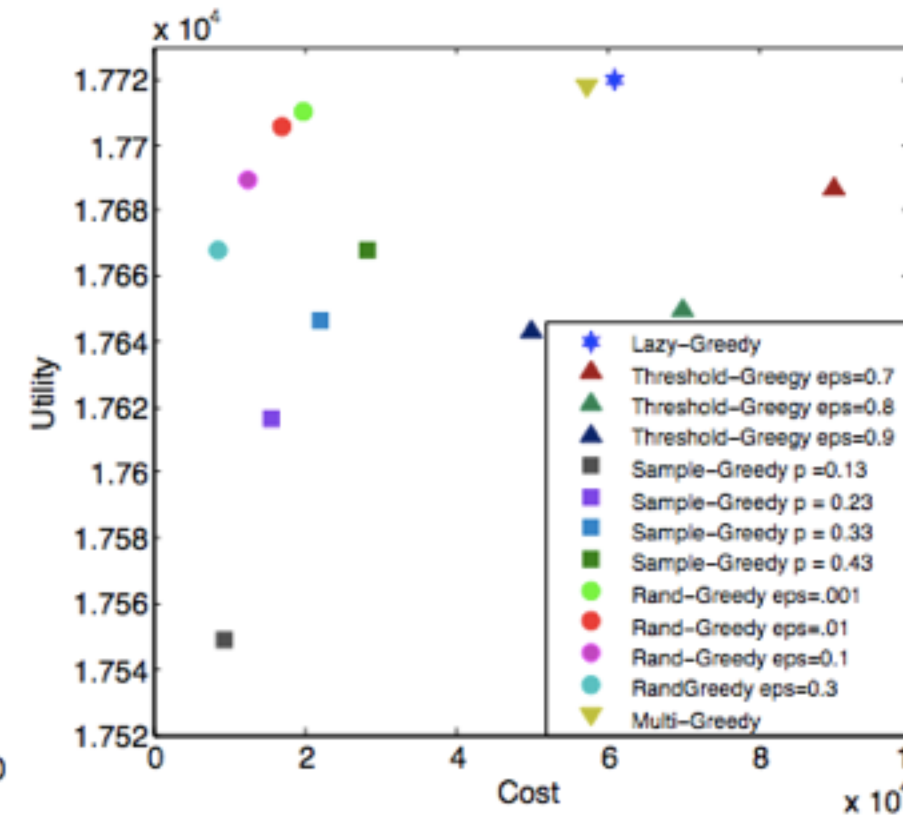
Parkinson dataset consists of of 5,875 data points with 22 attributes.



(d) Parkinsons



(e) Parkinsons

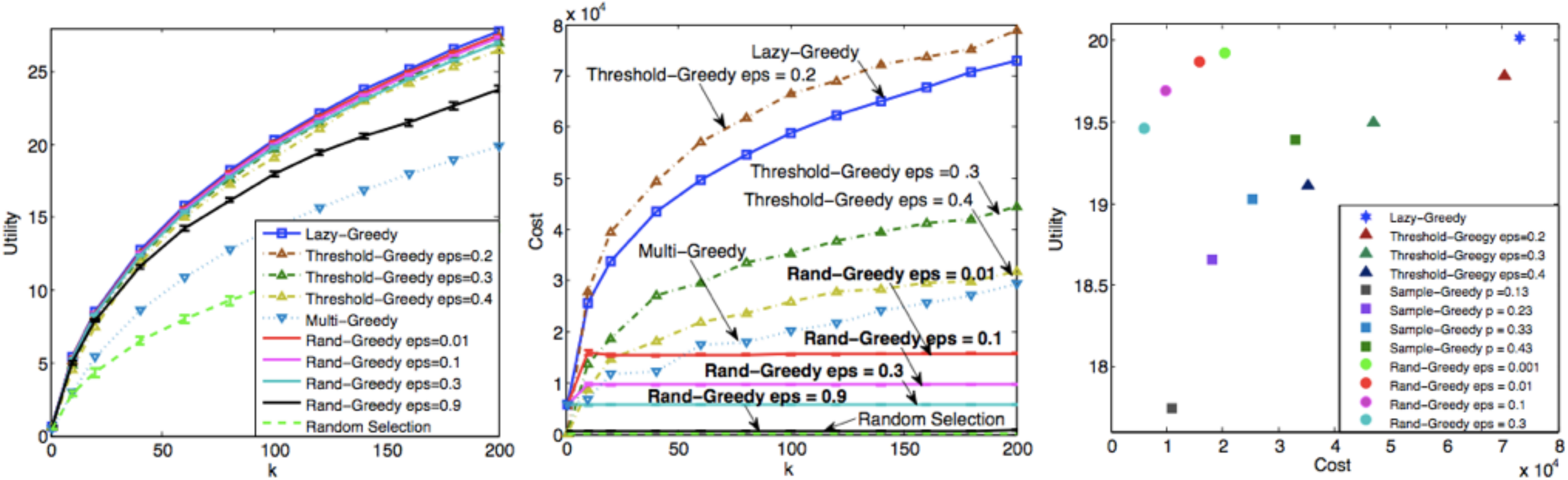


(f) Parkinsons

Similar utility but orders of magnitude faster!

Exemplar Based Clustering

A set of 10,000 Images with 3072 attributes.



(a) Images 10K

(b) Images 10K

(c) Images 10K

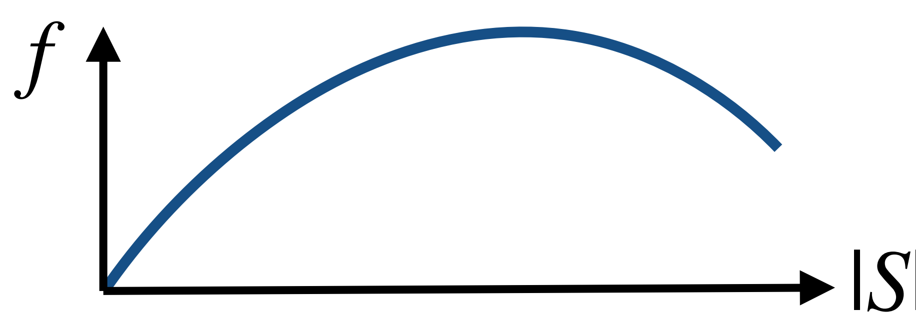
Similar utility but orders of magnitude faster!

Constrained Non-Monotone Submodular Maximization

$$S^* = \arg\max_{S \in \mathcal{I}} f(S)$$

constraints

- When the utility functions f is **(non-monotone) submodular**,



- and there are constraints imposed by the data summarization application.

***p*-system**

matroid intersection

Hypergraph matching

matroid

Matching

cardinality

***l*-knapsack**

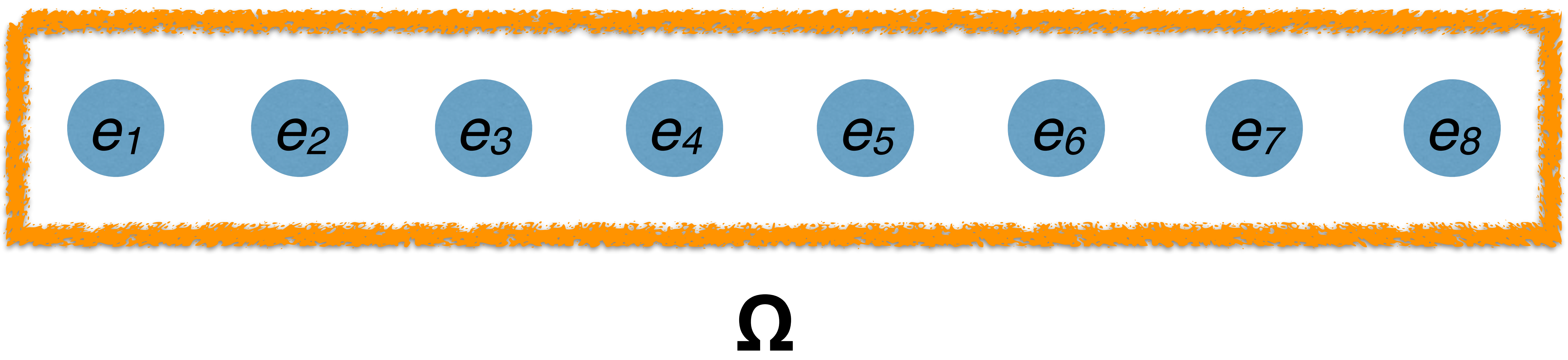
knapsack

Double Greedy: Unconstrained

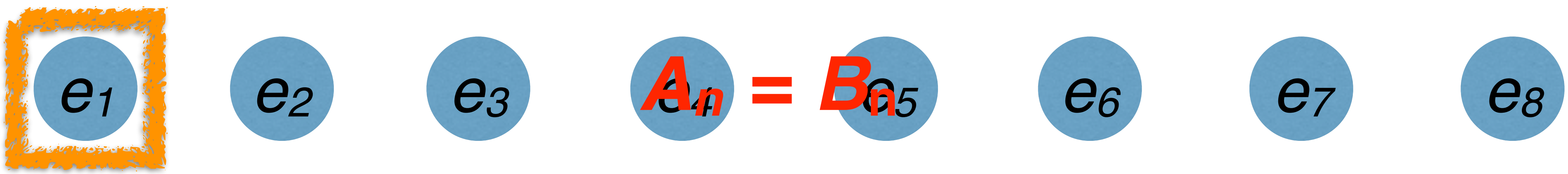
- **Unconstrained non-monotone** submodular maximization:

$$S^* = \arg\max_{S \subseteq \Omega} f(S)$$

- There is no restriction on the choice of *S*



Random Double Greedy: Unconstrained



Random Double Greedy

The random double greedy algorithm, in expectation, provides a 1/2 approximation guarantee for the unconstrained non-monotone submodular maximization problem:

$$E[f(S_{\text{random double greedy}})] \geq (1/2)\, f(S^*)$$

"A Tight Linear Time (1/2)-Approximation for Unconstrained Submodular Maximization", FOCS'12

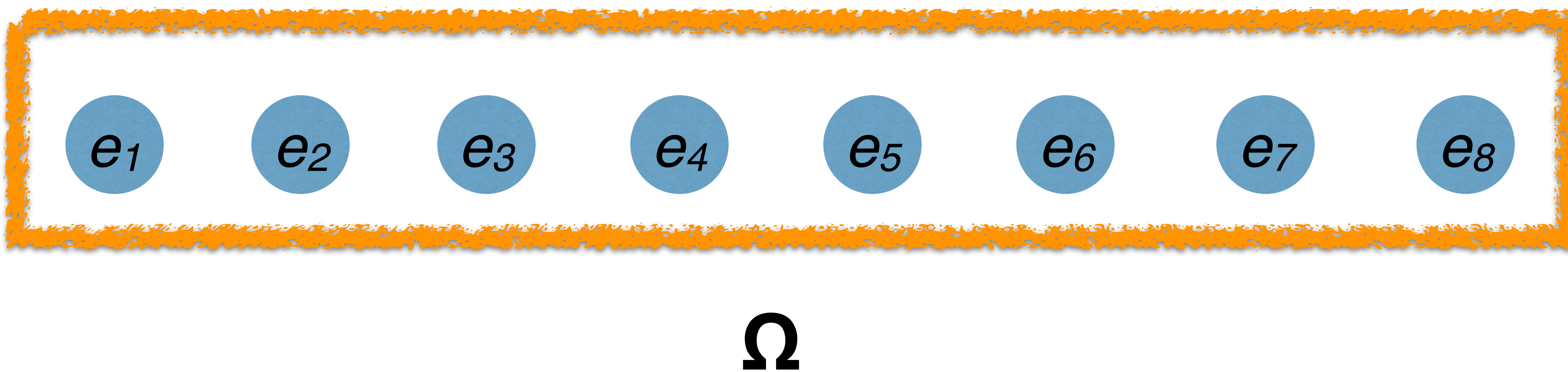
(Buchbinder, Feldman, Naor, Schwartz)

Random Greedy: Constrained

- **Constrained non-monotone** submodular maximization:

$$S^* = \underset{S \subseteq \Omega, |S| \leq k}{\arg\max} \; f(S)$$

- Choose a set *S* of size at most *k*

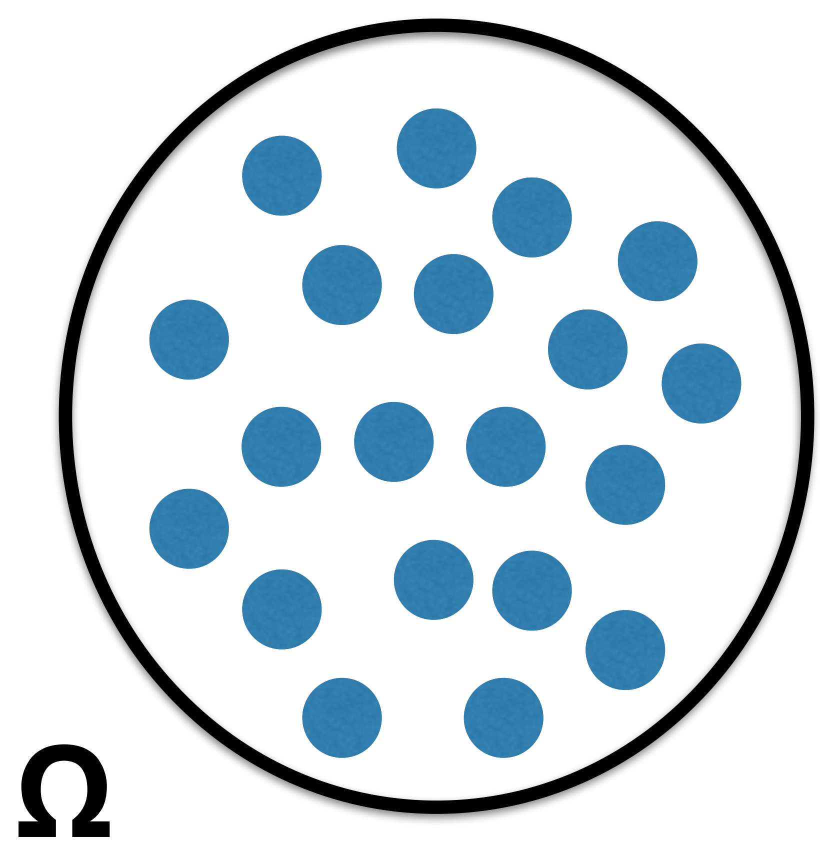


Sample Greedy: Constrained

- **Constrained non-monotone** submodular maximization:

$$S^* = \arg\max_{S \in \mathcal{I}} f(S)$$

- More general constraints such as a ***p*-extendible** system *I*



(Feldman, Harshaw, Karbasi)

Fast Constrained Submodular Maximization
FANTOM

Algorithm FANTOM

Input: Set E, a membership oracle for p-system $I \subset 2^E$, and l knapsack-cost functions c_i: $E \rightarrow [0, 1]$.

Output: Set S satisfying $S \subseteq I$ and $c_i(S) \leq 1 \ \forall i$.

1: $S = \varnothing$.
2: $M = \max_{j \in E} f(j)$, $\gamma = 2pM/(p+1)(2p+1)$, $U = \{\}$.
3: $R = \{\gamma, (1+\varepsilon)\gamma, (1+\varepsilon)\gamma^2, (1+\varepsilon)\gamma^3, \ldots, \gamma n\}$.
4: For $\rho \in R$ do
5: $\Omega = E$.
6: For $i = 1$; $i \leq p+1$; i ++ do
7: S_i = run greedy and at each step pick the element if and only if $f_S(j)/\Sigma^l_{i=1} c_{ij} \geq \rho$
8: $S_i = \operatorname{argmax}(S_i, \operatorname{argmax}_{z \in E} f(z))$
9: S'_i = Unconstrained-Maximization(S_i)
10: $\Omega = \Omega - S_i$
11: $S = \operatorname{argmax}(S, S_i)$
12: **return** S

First efficient algorithm for non-monotone submodular max s.t a ***p*-system** and ***l*-knapsack**

Approximation ratio

$f(S) \geq (1 + \varepsilon)(p + 1)(2p + 2l + 1)/p$ OPT

Running time

$O(nrp \log(n)/\varepsilon)$

"Fast Constrained Submodular Maximization", ICML'16

(Mirzasoleiman, Badanidiyuru, Karbasi)

GOAL

- Given a monotone submodular function F, and a cardinality constraint K, find a set A* such that

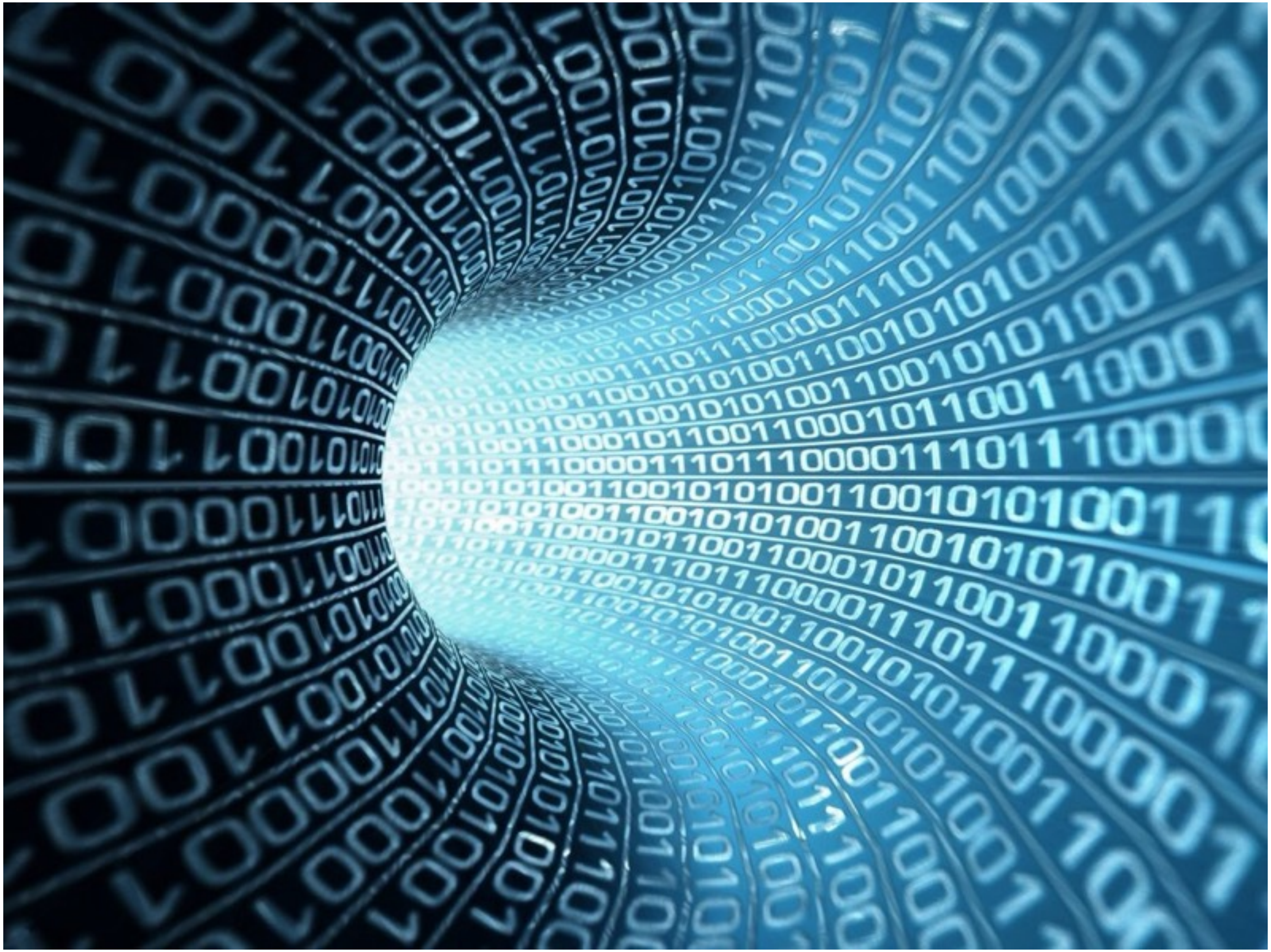
$$A^* = \arg\max_{|A| \leq k} F(A)$$

Centralized Solution

Streaming Solution

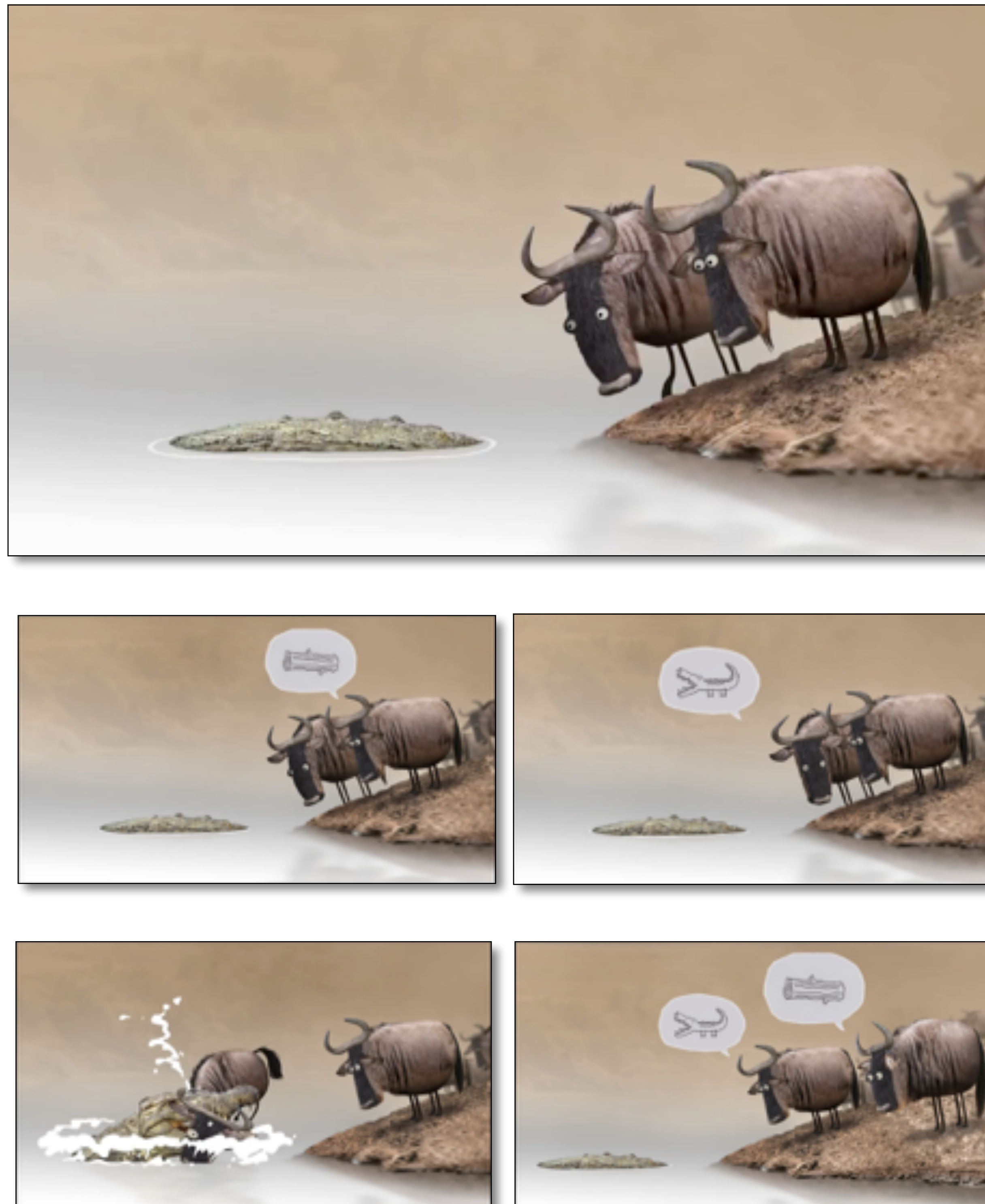
Distributed Solution

Streaming Solution



Data cannot be loaded on the main memory

Data Stream

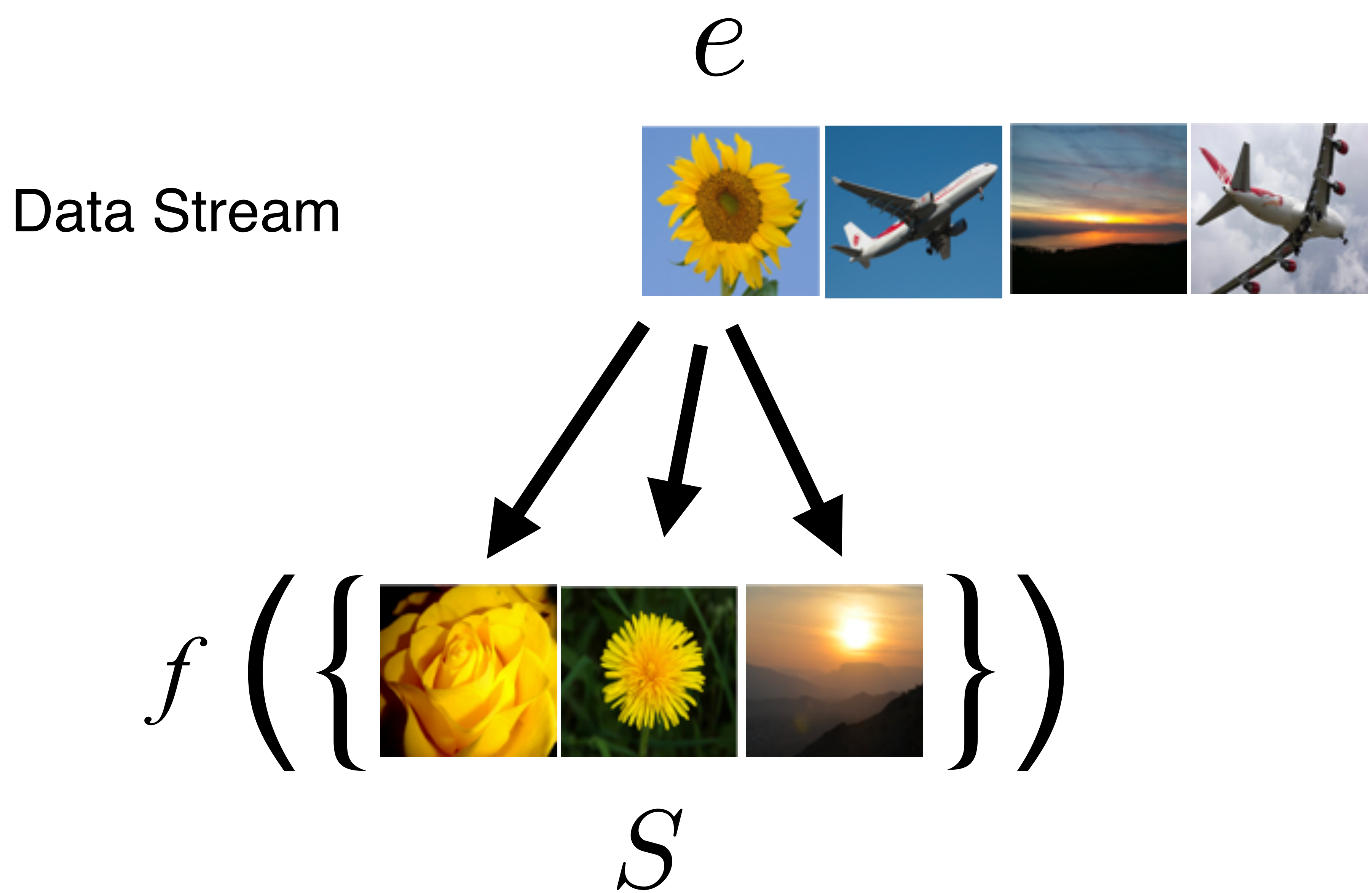


Problem: Greedy can be impractical

- Greedy approaches require random access to the complete data set; but for truly large-scale problems, where data is residing on disk, or arriving over time at a fast pace they are often impractical.

Is it possible to summarize a massive data set "**on the fly**" i.e., at any point of time we have access only to a small fraction of data?

Stream-Greedy



Can we swap e with any elements in S s.t. the utility increases?

$$\exists x \in S \text{ s.t. } f(S \cup \{e\} \setminus \{x\}) > f(S)?$$

Stream-Greedy is Bad!

- Stream-Greedy could be arbitrary poor!

- Example: a set *X* and a collection *V* of subsets of *X:*

$$f(S) = \sum_{x \in \cup_{v \in S} v} w(x)$$

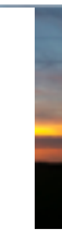
$S^* = \{\{1,2,\ldots,k\},$
$\{k+1,k+2,\ldots,2k\},$
$\{k^2+1,k^2+2,\ldots,k^2+k\}\}$

$$w(1) = \cdots = w(k) = 1$$
$$w(k+1) = \cdots = w(2k) = 1 + \epsilon$$
$$\cdots$$
$$w(k^2+1) = \cdots = w(k^2+k) = 1 + k\epsilon$$
$$\text{and } \epsilon \ll 1$$

$f(S^*) \approx k^2$ and $f(S) \approx k$

S

Streaming with Preemption



For monotonic submodular functions with a cardinality constraint *k*, the streaming algorithm with preemption gives a solution $S_{\text{preemption}}$ such that:

$$f(S_{\text{preemption}}) \geq (1/4)\, f(S^*)$$

"Online Submodular Maximization with Preemption.", SODA'15

(Buchbinder, Feldman, Schwartz)

Choose the one with largest benefit to exchange

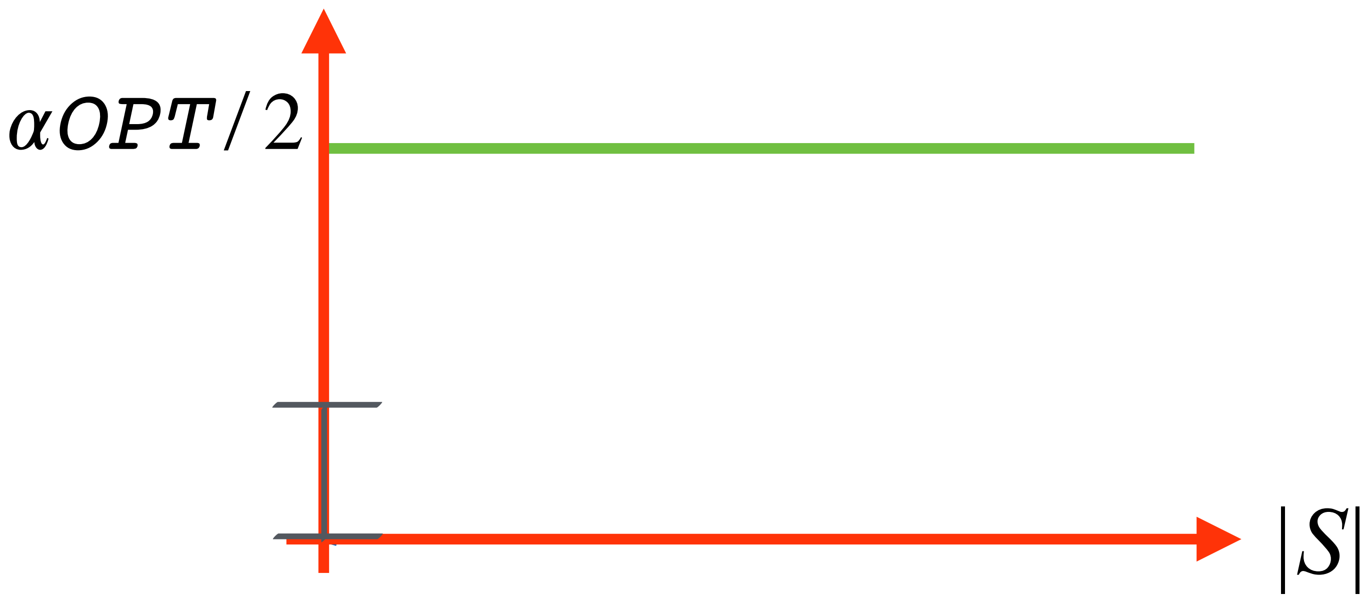
Data Summarization on the Fly!

Data Stream

$$OPT \geq v \geq \alpha OPT$$

$$\Delta_f(e_i|S) \geq (v/2 - f(S))/(k - |S|)$$

$$f\Big(\Big\{ \qquad \Big\}\Big) \longrightarrow \max$$



Properties:

- 1 pass
- $f(S) \geq \alpha OPT/2$
- $O(k)$ memory
- $O(1)$ update time

Knowing max marginal is enough

- But how can we find a good approximation of `OPT`?

Tracking *m* on the fly!

- Obtaining m requires a full pass over the data ☹

Sieve-Streaming

Algorithm SIEVE-STREAMING

1: $O = \{(1+\epsilon)^i \mid i \in Z\}$
2: For each $v \in O$, $S_v := \emptyset$ (maintain the sets only for the necessary v's lazily)
3: $m := 0$
4: **for** $i = 1$ to n **do**
5: $m := \max(m, f(\{e_i\}))$
6: $O_i = \{(1+\epsilon)^i \mid m \leq (1+\epsilon)^i \leq 2.k.m\}$
7: Delete all S_v such that $v \notin O_i$.
8: **for** $v \in O_i$ **do**
9: **if** $\Delta_f(e_i|S_v) \geq \frac{v/2 - f(S_v)}{k - |S_v|}$ and $|S_v| < k$ **then**
10: $S_v := S_v \cup \{e_i\}$
11: **return** $\mathrm{argmax}_{v \in O_n} f(S_v)$

- First streaming algorithm for cardinality-constrained submodular maximization with
  - Constant factor approximation guarantee
    $f(S) \geq (1/2-\epsilon)\ \mathtt{OPT}$
  - Makes no assumptions on the data stream
  - Requires only a single pass
  - Only $O(k \log k)$ memory
  - Only $O(\log k)$ update time
  - Assuming nothing but monotone submodularity

“Streaming Submodular Maximization: Massive Data Summarization on the Fly”, KDD’14

(Badanidiyuru, Mirzasoleiman, Karbasi, Krause)

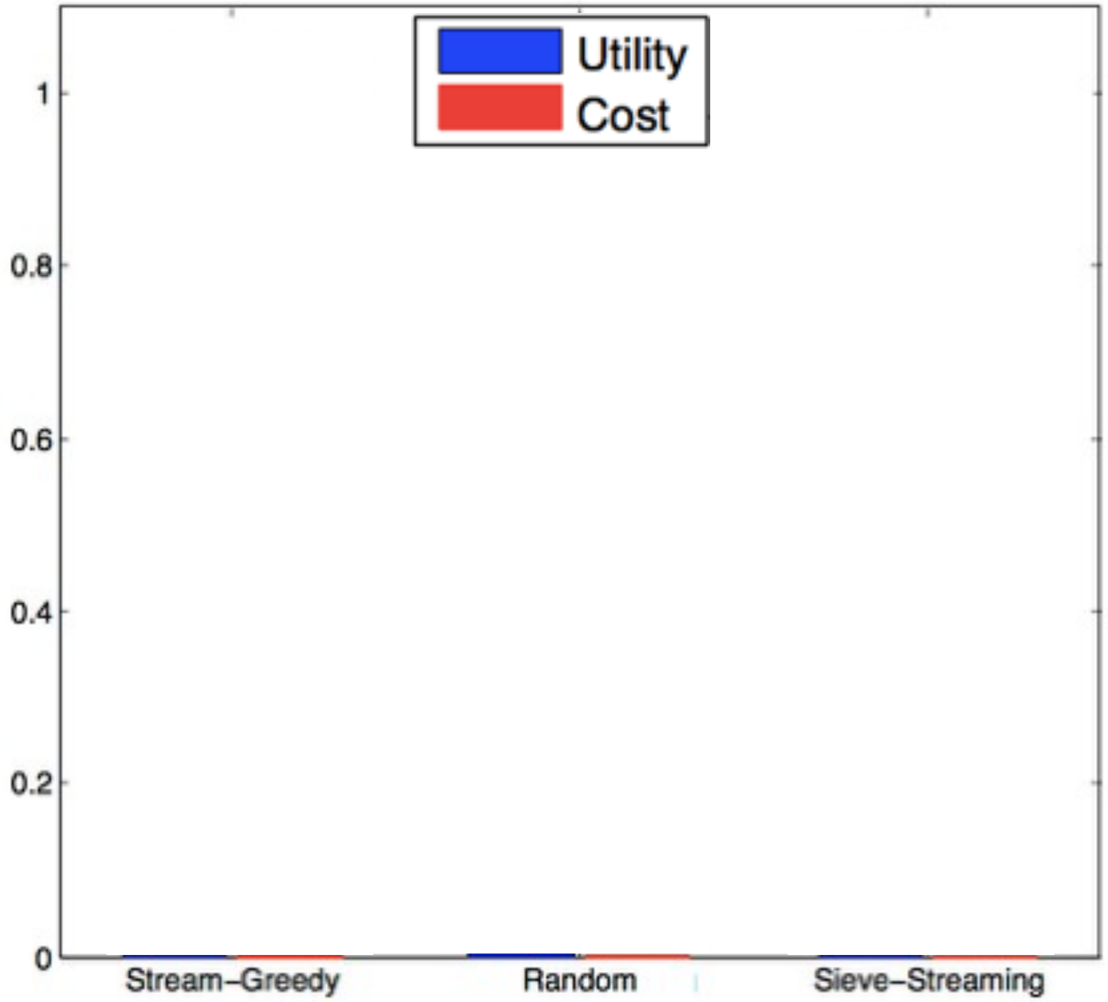
Experiments

- Benchmarks:

  - **Stream-Greedy**: The output is the k data points provided by Stream-Greedy [GK'10].
    - For each new data point, we check whether switching it with an element in S will increase the value of the utility function f.
  - **Random selection**: the output is k randomly selected data points from V.

Exemplar-Based Clustering

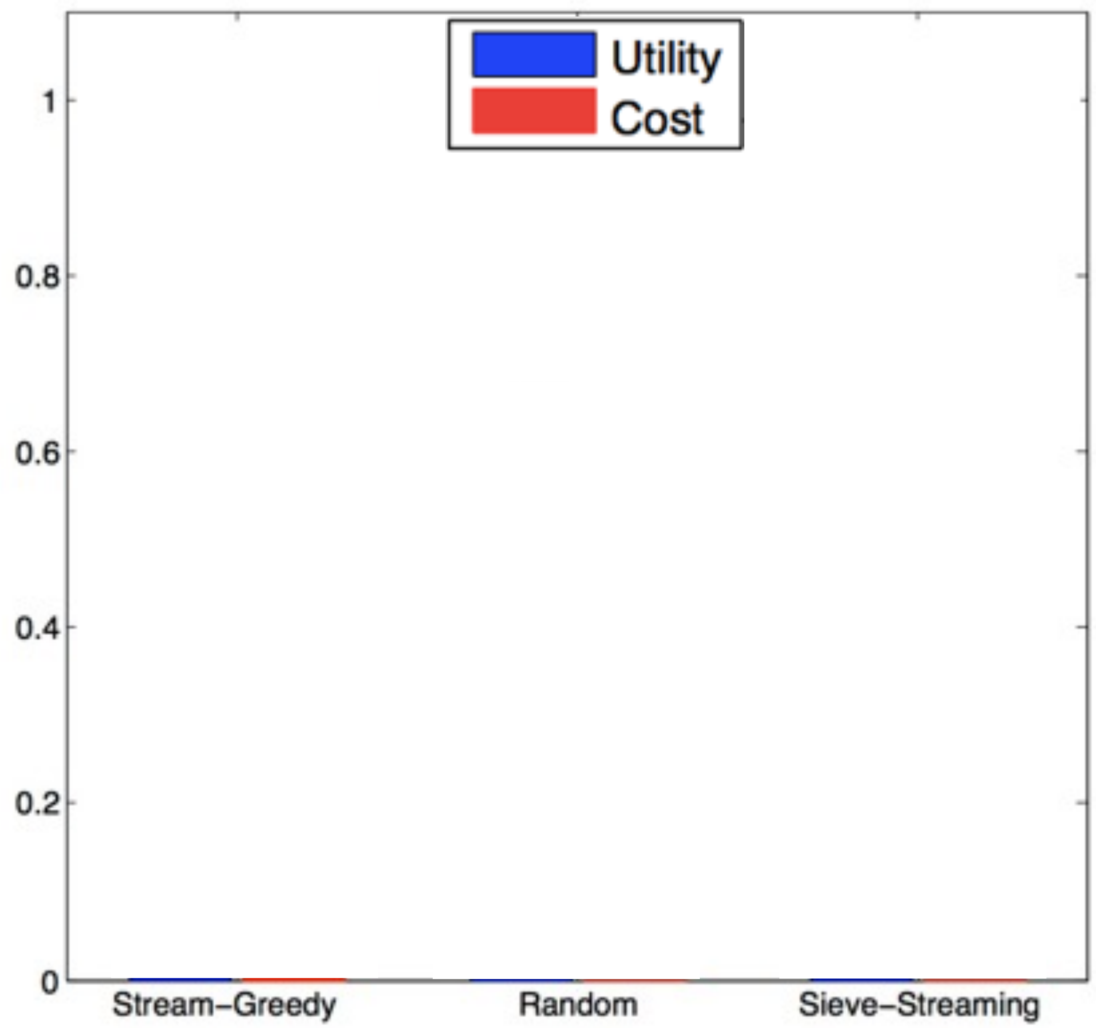
Census data set consists of 2,458,285 data points with 68 attributes.



Similar utility but orders of magnitude faster!

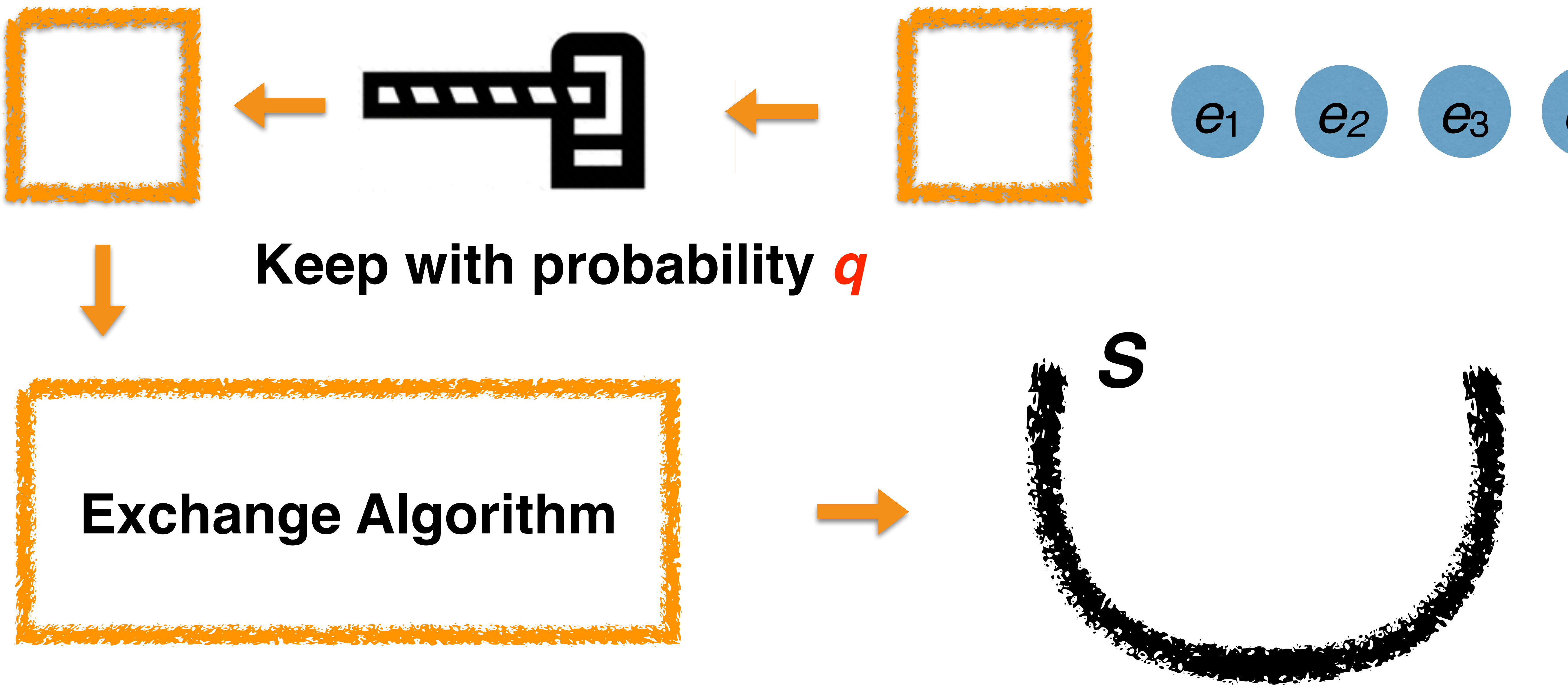
Nonparametric Regression

Yahoo! Webscope data set consists of 45,811,883 user visits from the Featured Tab of the Today Module on the Yahoo! Front Page.



Similar utility but orders of magnitude faster!

Constrained Non-Monotone Submodular Maximization



“Do Less, Get More: Streaming Submodular Maximization with Subsampling”, arXiv’18

(Feldman, Karbasi, Kazemi)

GOAL

- Given a monotone submodular function F, and a cardinality constraint K, find a set A* such that

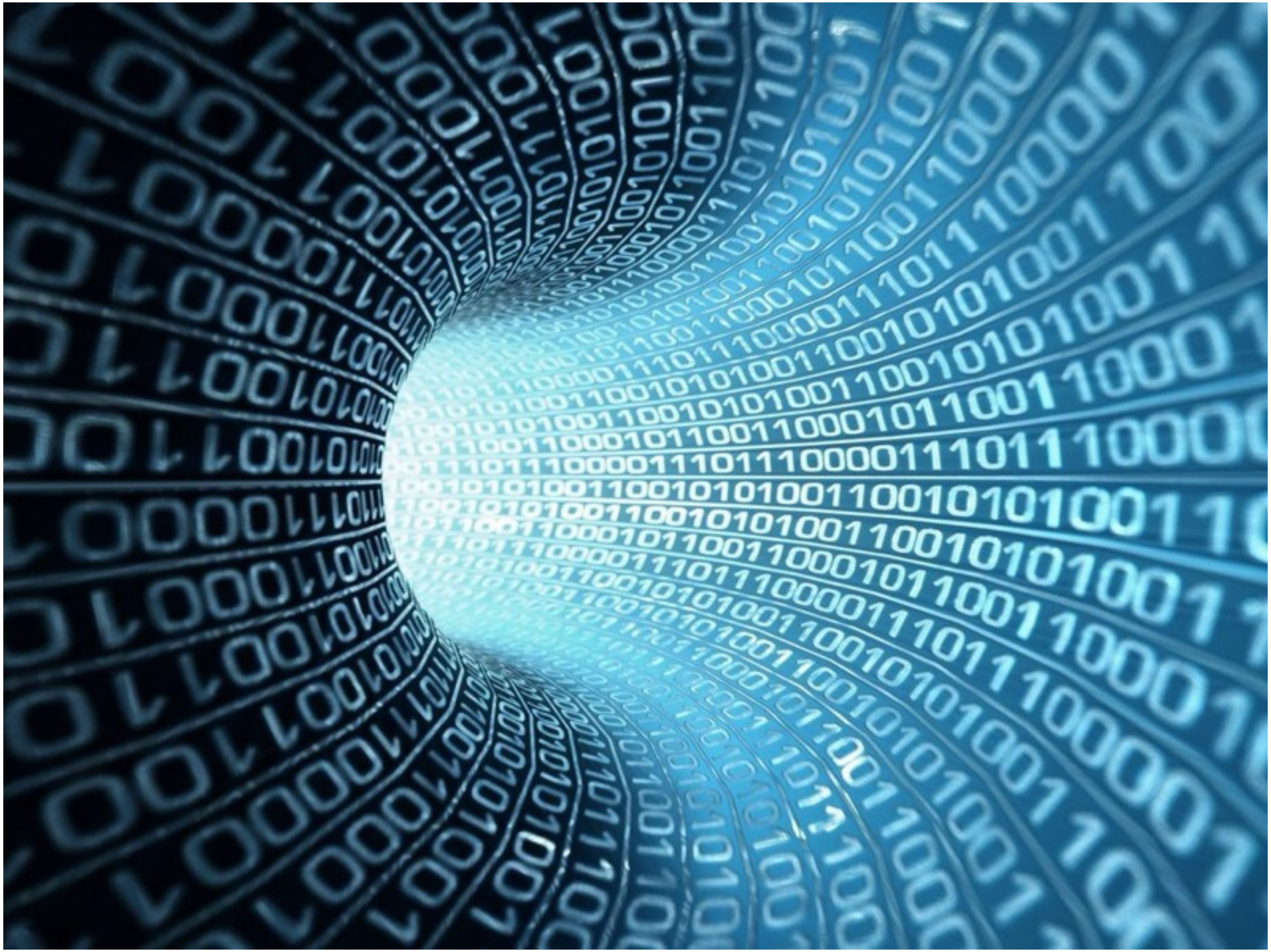
$$A^* = \arg\max_{|A| \leq k} F(A)$$

Centralized Solution

Streaming Solution

Distributed Solution

Distributed Solution



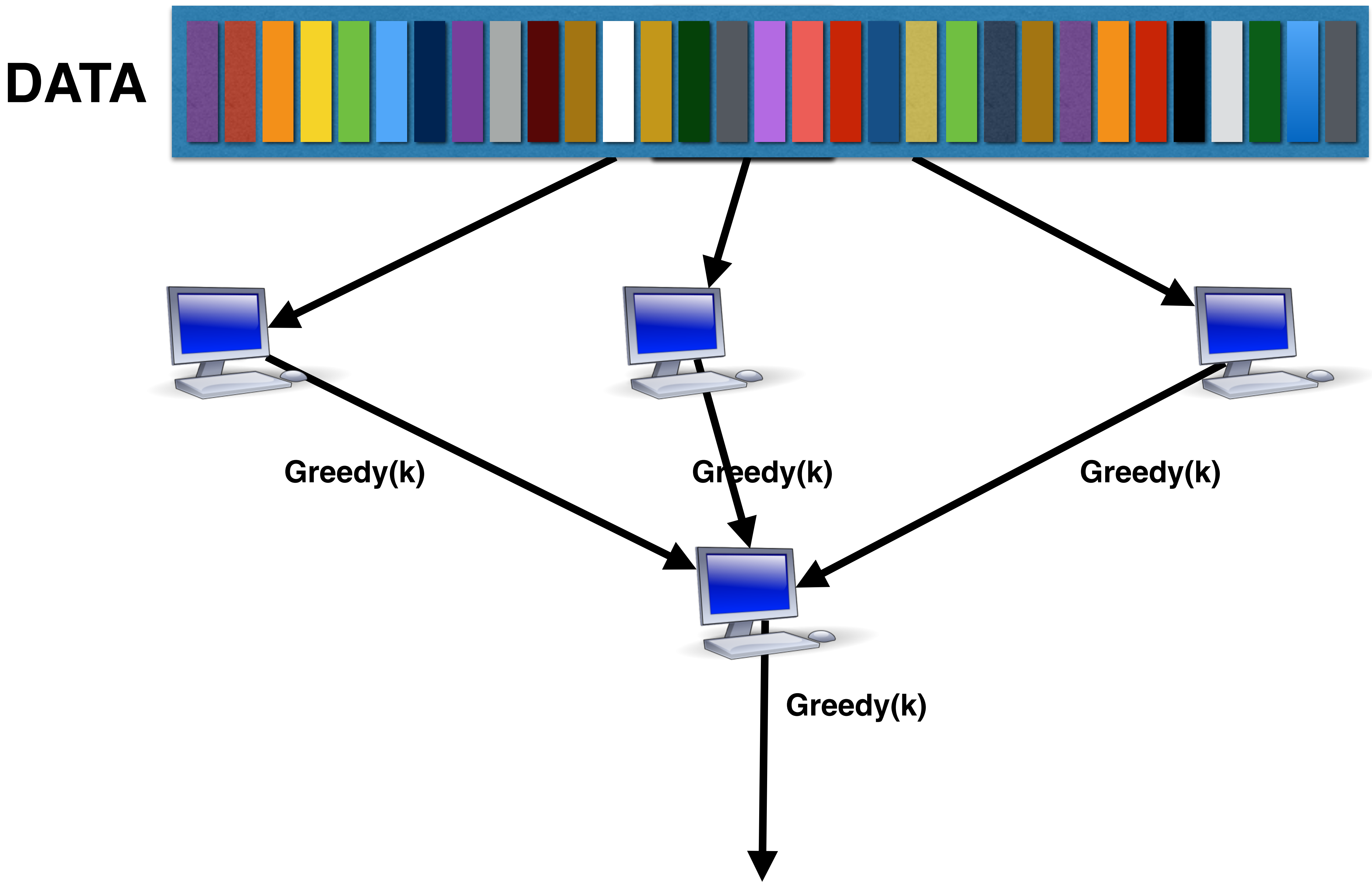
Big data but also many servers

Problem: Scale Up

- On massive data (80,000,000 images) the greedy policies take a few days/weeks to complete

- Can we **parallelise** the greedy approach?

Two-Stage Greedy (GreeDi)



Theoretical Guarantees

- Split data (arbitrarily) among *m* machine

GreeDi

For monotonic submodular functions, GreeDi gives

$$F(A) \geq \frac{1 - 1/e}{\sqrt{\min(m,k)}} OPT$$

Cannot do better in **general**

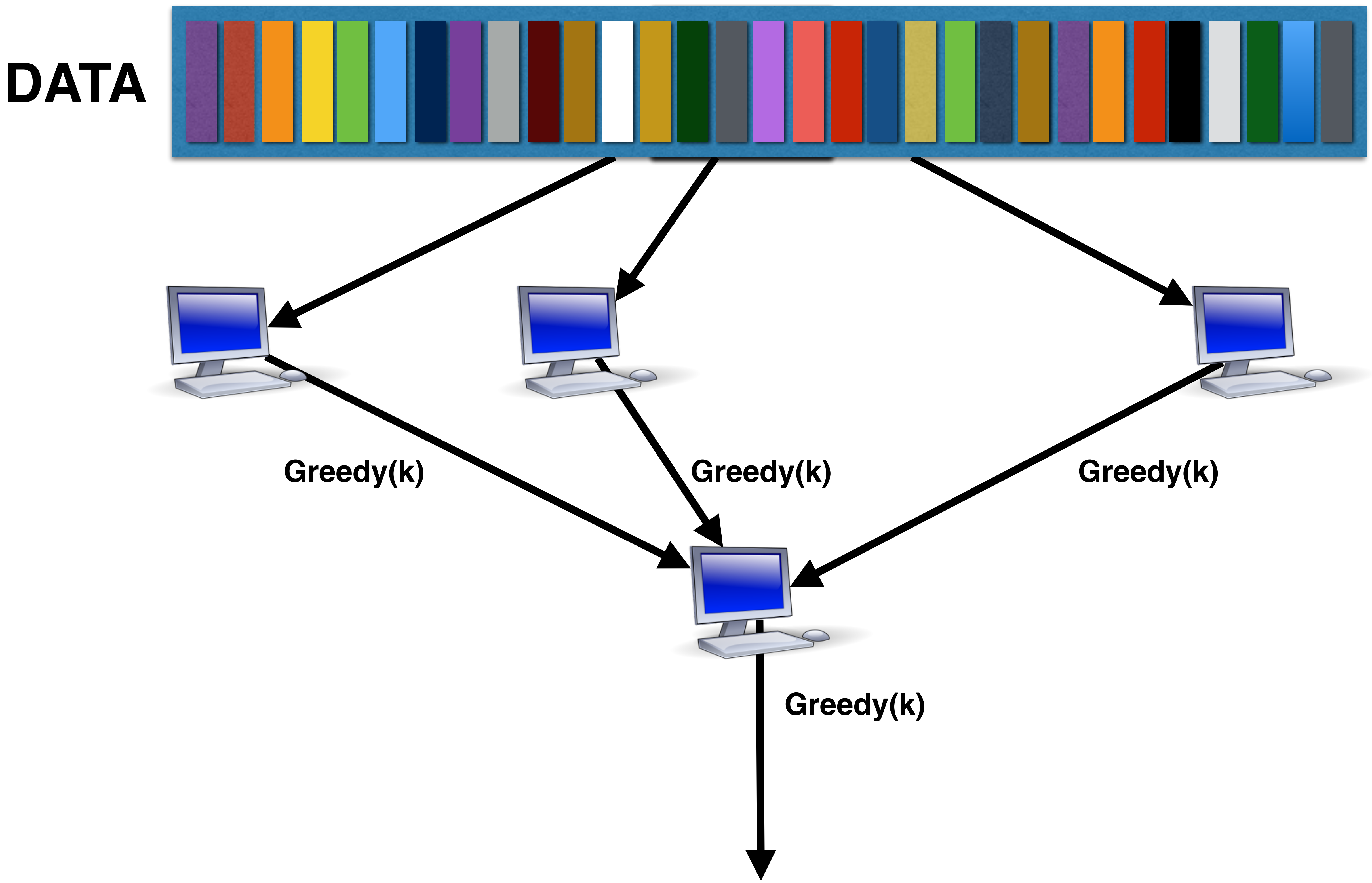
"Distributed Submodular Cover: Succinctly Summarizing Massive Data", NIPS'15

(Mirzasoleiman ,Karbasi,Badanidiyuru, Krause)

"Distributed Submodular Maximization: Identifying Representative Elements in Massive Data", NIPS'13

(Mirzasoleiman, Karbasi, Sarkar, Krause)

Two-Stage Greedy (Random Splitting)



GreeDi + Random Partition

- Randomization helps! ☺

GreeDi

For monotonic submodular functions, GreeDi gives and random partitioning of data on m machines

$$\mathbb{E}[F(A)] \geq \frac{1 - 1/e}{2} F(A^*)$$

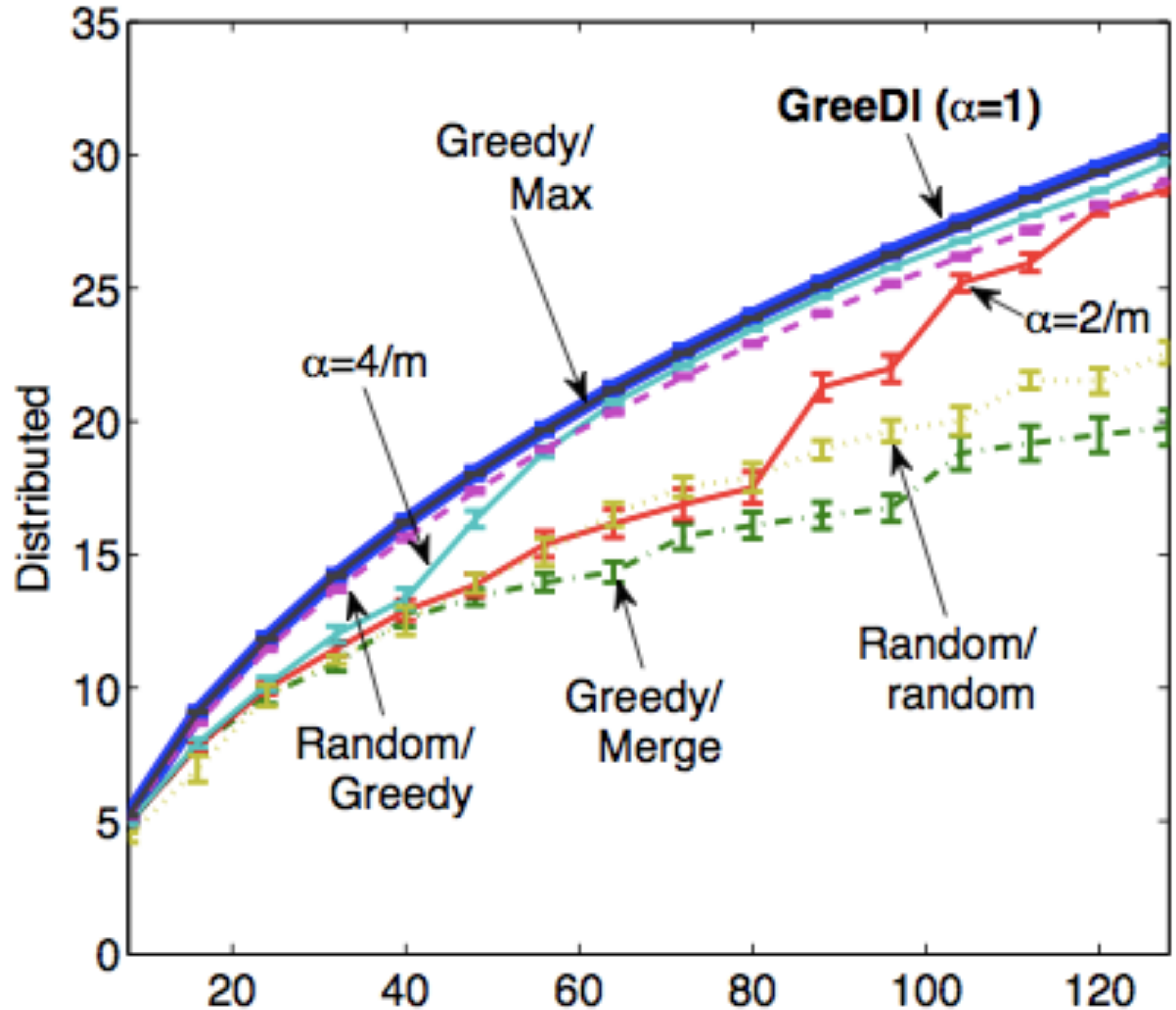
Can do better with **randomization**

"The Power of Randomization: Distributed Submodular Maximization on Massive datasets", ICML'15
Barbosa, Ene, Nguyen, Ward

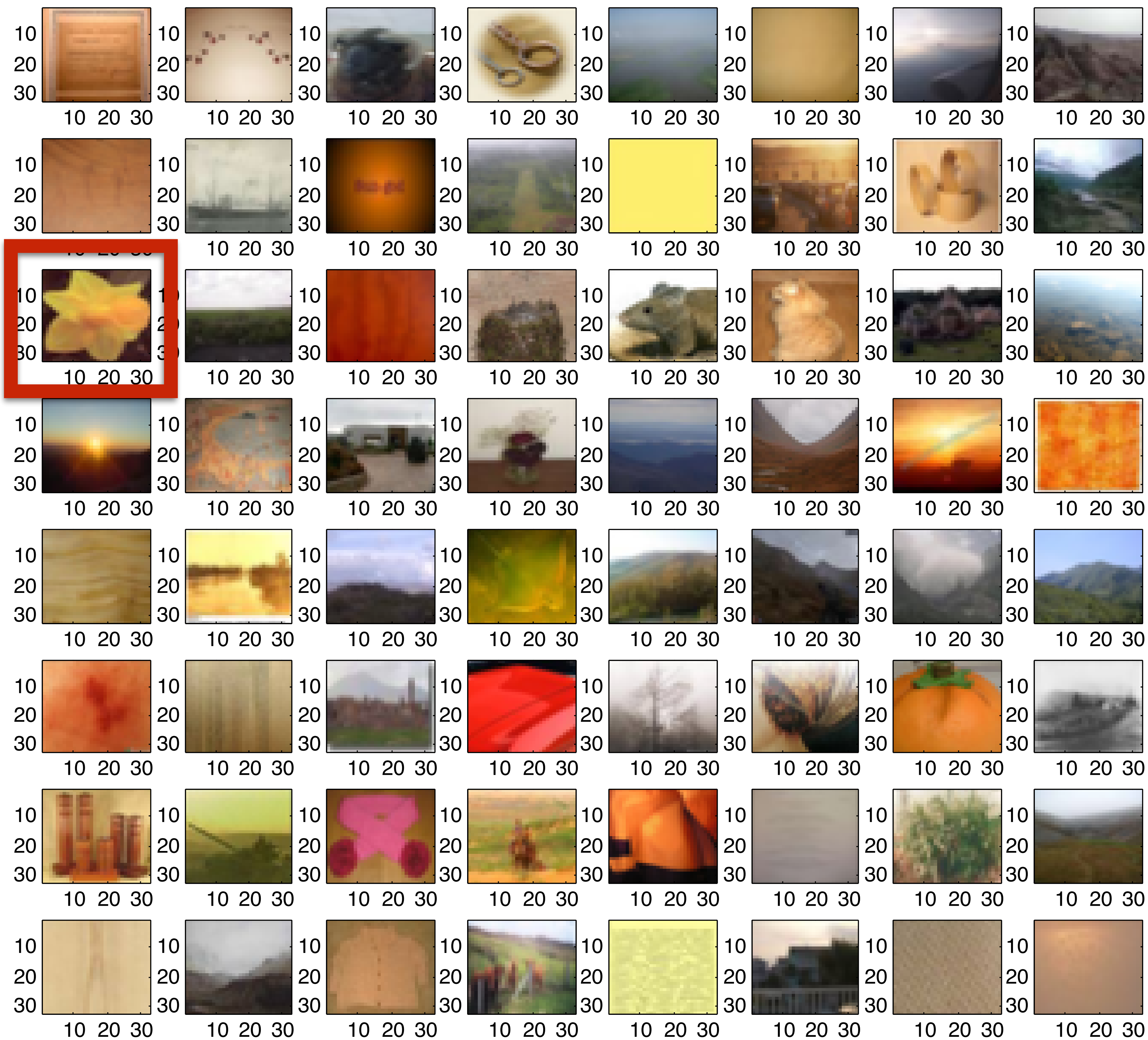
"Randomized Composable Core-sets for Distributed Submodular Maximization", STOC'15
Mirrokni, Zadimoghaddam

Nonparametric Regression on Hadoop

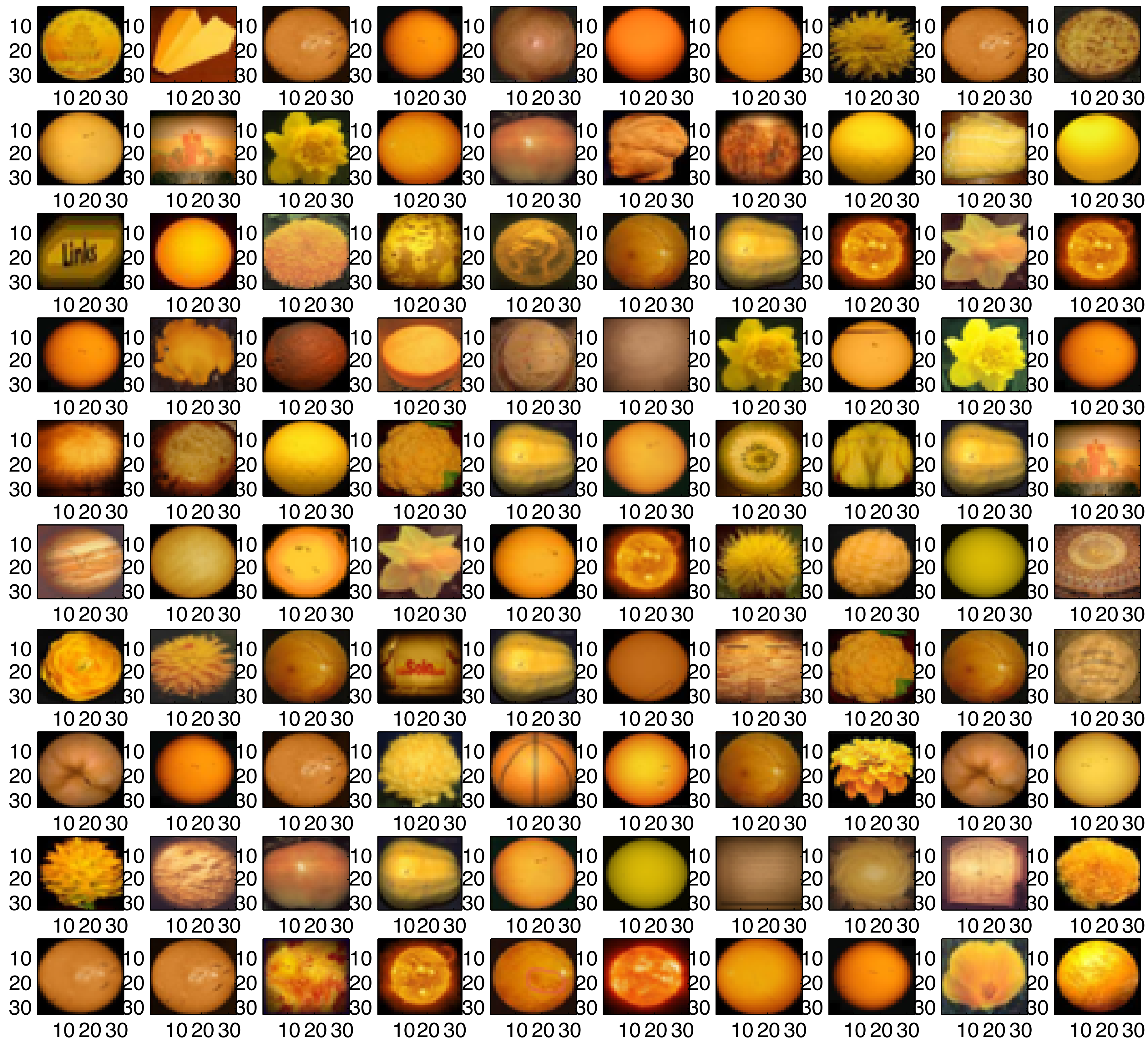
- 45,811,833 user visits on Yahoo! front page



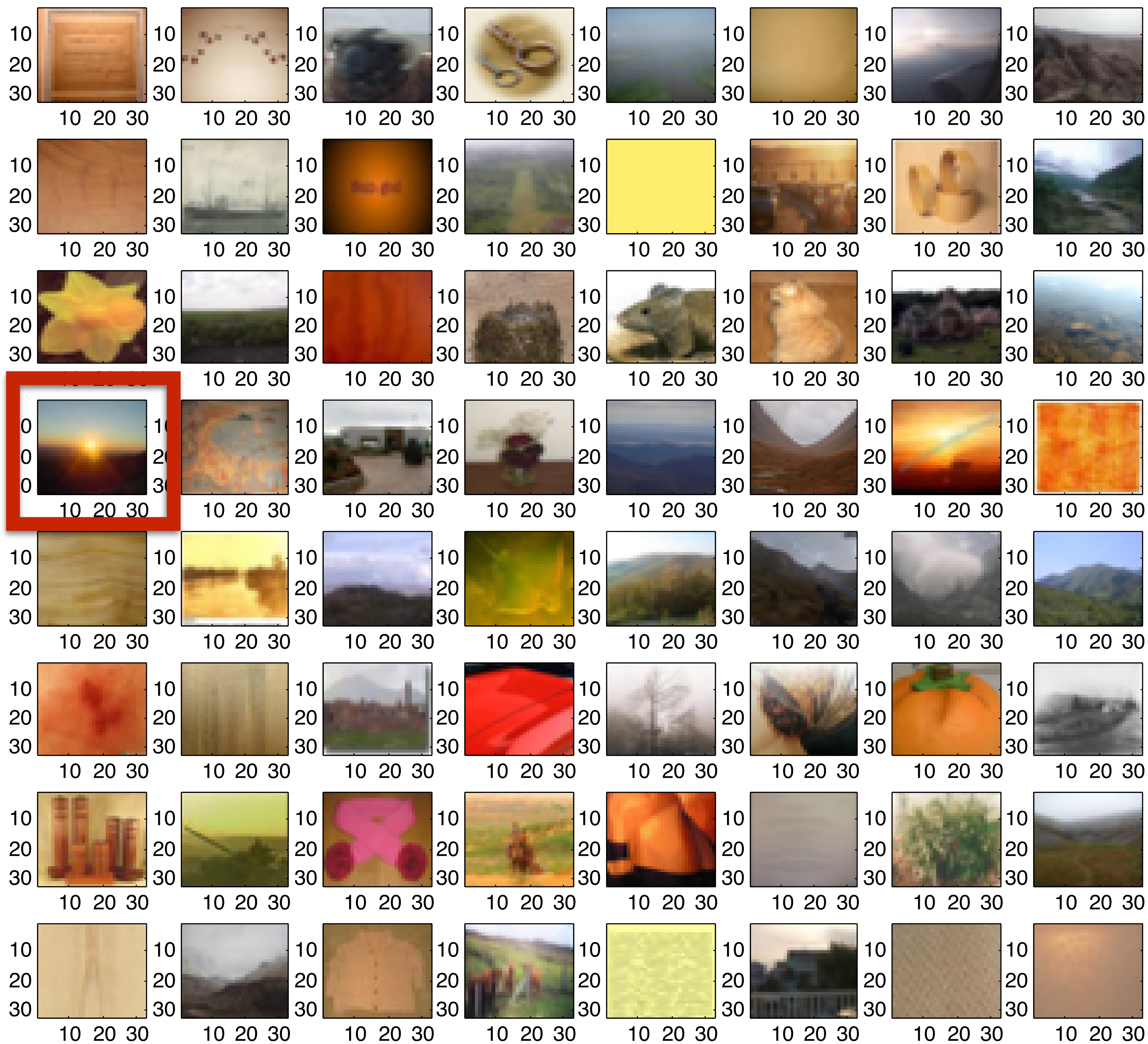
Exemplar-Based Clustering on 80M Images (Hadoop)



Exemplar-Based Clustering on 80M Images (Hadoop)



Exemplar-Based Clustering on 80M Images (Hadoop)



Exemplar-Based Clustering on 80M Images (Hadoop)

Conclusion

- Big data is getting much BIGGER
- Summarization is inevitable
- Submodularity provides a unifying framework
- We have now fast centralised, streaming, and distributed methods to (approximately) optimise submodular functions


Thank You