Complexity of Flow Analysis

David Van Horn and Harry Mairson
Overview

★ Monovariant flow analysis (0CFA) is complete for $\text{PTIME}$
  — Analysis & evaluation are identical on linear programs
  — 0CFA (and approximations) are inherently sequential

★ For any $k > 0$, $k$CFA is complete for time $\text{EXPTIME}$
  — Exact characterization of complexity of $k$CFA hierarchy
  — Validates observations that $k$CFA is intractable
Plan

★ Proving lower bounds — *programming with analysis*
  — What is $k$CFA?
  — Linearity and precision
  — Non-linearity and an exponential iterator
★ Simulating exponential Turing machines with $k$CFA
★ Conclusions
Programming with analysis

A lower bound establishes the minimum computational requirements it takes to solve a class of problems.

Another view: What kind of work can you do with an analysis?

Two approaches:
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★ Subvert approximation: Analysis ⇒ Evaluation
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Two approaches:

★ Subvert approximation: Analysis $\Rightarrow$ Evaluation
   — For any abstract interpretation, there is a subset of the language on which abstract and concrete coincide.
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Another view: What kind of work can you do with an analysis?

Two approaches:

⋆ Subvert approximation: Analysis ⇒ Evaluation
  — For any abstract interpretation, there is a subset of the language on which abstract and concrete coincide.

⋆ Exploit approximation: Inexactness as combinatorial tool
Proving lower bounds

$k$CFA is provably intractable (EXPTIME-hard)

The *proof* goes by construction:
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Proving lower bounds

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- given the description of a Turing machine and its input,
- produce an instance of the $k$CFA problem,
- whose analysis faithfully simulates the TM on the input
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  \item given the description of a Turing machine and its input,
  \item produce an instance of the $k$CFA problem,
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  \item for an exponential number of steps.
\end{itemize}
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A compiler!
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\end{itemize}

\textbf{A (weird) compiler!}
Strange animals

A compiler:
⭐ Source: exponential TMs with input
⭐ Target: the $\lambda$-calculus
⭐ Interpreter: $k$CFA (as TM simulator)
∴ $k$CFA is complete for EXPTIME.
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★ Target: the $\lambda$-calculus
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Another compiler:
★ Source: circuit with inputs
★ Target: the linear $\lambda$-calculus
★ Interpreter: $0$CFA (as $\lambda$-evaluator)
∴ $0$CFA is complete for $\text{PTIME}$.
More strange animals

Other compilers (ICFP’07):

★ Source: Boolean formulas
★ Target: the $\lambda$-calculus
★ Interpreter: $k$CFA (as SAT solver)

$\therefore k$CFA is NP-hard.
More strange animals

Other compilers (Mairson, POPL’90):

★ Source: exponential TMs with input
★ Target: ML
★ Interpreter: type inference (as ML evaluator)

∴ ML type inference is complete for \textbf{EXPTIME}. 
More strange animals

Other compilers (Henglein and Mairson, POPL’91):

★ Source: elementary TMs with input
★ Target: System $F_i$
★ Interpreter: type inference (as System $F_i$ evaluator)

∴ System $F_i$ type inference is $\text{DTIME}(K(i, n))$-hard.
More strange animals

Other compilers (Mairson, JFP’04):

⋆ Source: circuit with inputs
⋆ Target: the linear $\lambda$-calculus
⋆ Interpreter: type inference (as $\lambda$-evaluator)

∴ Simple type inference is complete for PTIME.
More strange animals

Other examples (Neergaard and Mairson, ICFP’04):

★ Source: elementary TMs with input
★ Target: the $\lambda$-calculus
★ Interpreter: rank-$k$ $\wedge$-type inference (as $\lambda$-evaluator)

$\therefore$ Rank-$k$ $\wedge$-type inference is complete for $\text{DTIME}(K(k, n))$. 
A complexity zoo of static analysis

\[0\text{CFA} \equiv \text{Simple closure analysis} \equiv \text{Sub-0CFA} \equiv \text{Simple type inference} \equiv \text{Linear } \lambda\text{-calculus} \equiv \text{MLL} \ldots \subset \]

\[k\text{CFA} \equiv \text{ML type inference} \ldots \subset \]

\[\text{Rank-}k \text{ intersection type inference} \ldots \subset \]

\[\text{Exact CFA} \equiv \text{Simply typed } \lambda\text{-calculus} \ldots \subset \]

\[\infty\text{CFA} \equiv \text{The } \lambda\text{-calculus} \ldots \]

"Program analysis is still far from being able to precisely relate ingredients of different approaches to one another."

(Nielson et al. 1999)
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Flow analysis

Flow analysis is concerned with the sound approximation of run-time values at compile time.

Analysis answers decision problems such as:

\[ \text{does expression } e \text{ possibly evaluate to value } v? \]

- The most approximate analysis always answers yes.  
  — no resources to compute, but useless
- The most precise analysis answers yes iff \( e \) evaluates to \( v \).  
  — useful, but unbounded resources to compute

For tractability, there is a necessary sacrifice of information in static analysis. (A blessing and a curse.)
Intuition— the more information we compute about contexts, the more precisely we can answer flow questions. But this takes work.
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It did not take long to discover that the basic analysis, for any $k > 0$, was intractably slow for large programs.

Polyvariance

During reduction, a function may copy its argument:

\[
((\lambda f. \cdot \cdot \cdot (f \text{e}_1)^{\ell_1} \cdot \cdot \cdot (f \text{e}_2)^{\ell_2} \cdot \cdot \cdot)(\lambda x. \text{e}))
\]

*Contours* (strings of application labels) let us talk about \text{e} in each of the distinct calling contexts.
Cache-based evaluator

\[
\begin{align*}
\text{Exp} & \quad e ::= t^\ell \\
\text{Term} & \quad t ::= x \mid e \cdot e \mid \lambda x. e
\end{align*}
\]

expressions (or labeled terms)

terms (or unlabeled expressions)

Evaluate the term \( t \), which is closed under environment \( ce \).

Write the result into location \((\ell, \delta)\) of the cache \( C \).

\( C(\ell, \delta) = v \) means \( t^\ell \) evaluates to \( v \) in context \( \delta \).
A cache-based evaluator:

\[ C \in \text{Cache} = (\text{Lab} + \text{Var}) \times \text{Lab}^* \to (\text{Term} \times \text{Env}) \]

\[ \mathcal{E}\llbracket (t^{\ell_1} t^{\ell_2})^{\ell} \rrbracket_{\delta}^{ce} = \mathcal{E}\llbracket t^{\ell_1} \rrbracket_{\delta}^{ce}; \mathcal{E}\llbracket t^{\ell_2} \rrbracket_{\delta}^{ce}; \]

let \( \langle \lambda x. t^{\ell_0}, ce' \rangle = C(\ell_1, \delta) \) in

\[ C(x, \delta^{\ell}) \leftarrow C(\ell_2, \delta); \]

\[ \mathcal{E}\llbracket t^{\ell_0} \rrbracket_{\delta^{\ell}}^{ce'}[x \mapsto \delta^{\ell}]; \]

\[ C(\ell, \delta) \leftarrow C(\ell_0, \delta^{\ell}) \]
An abstraction of the cache-based evaluator:

\[ \hat{C} \in \widehat{\text{Cache}} = (\text{Lab} + \text{Var}) \times \text{Lab} \leq^k \rightarrow \mathcal{P}(\text{Term} \times \text{Env}) \]

\[ \mathcal{A}[\ell_1(\ell_2)t_1 t_2]^{ce}_{\delta} = \mathcal{A}[\ell_1]^{ce}_{\delta} ; \mathcal{A}[\ell_2]^{ce}_{\delta} ; \]

\textbf{foreach} \( \langle \lambda x.t_0^{\ell_0}, ce' \rangle \in \hat{C}(\ell_1, \delta) : \)

\[ \hat{C}(x, \lceil \delta \ell \rceil^k_k) \leftarrow \hat{C}(\ell_2, \delta) ; \]

\[ \mathcal{A}[\ell_0]^{ce'}[x\mapsto \lceil \delta \ell \rceil^k_k] ; \]

\[ \hat{C}(\ell, \delta) \leftarrow \hat{C}(\ell_0, \lceil \delta \ell \rceil^k_k) \]
OCFA

An *abstraction* of the cache-based evaluator:

\[
\hat{C} \in \widehat{\text{Cache}} = (\text{Lab} + \text{Var}) \to \mathcal{P}(\text{Term})
\]

\[
\mathcal{A}[\ell](t_1 t_2) = \mathcal{A}[t_1]; \mathcal{A}[t_2];
\]

foreach \( \lambda x. t_0 \in \hat{C}(\ell_1) \):

\( \hat{C}(x) \leftarrow \hat{C}(\ell_2); \)

\( \mathcal{A}[t_0]; \)

\( \hat{C}(\ell) \leftarrow \hat{C}(\ell_0) \)

(An evaluator for linear \( \lambda \)-calculus).
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Linearity and evaluation

Since in a *linear* $\lambda$-term,

- each abstraction can be applied to at most one argument
- each variable can be bound to at most one value

Analysis of a linear term coincides exactly with its evaluation.

\[
((\lambda f. \cdots (f e_1) \cdots (f e_2) \cdots))(\lambda x.e))
\]
Boolean logic

Coding Boolean logic in linear $\lambda$-calculus:

\[
\begin{align*}
\text{TT} & \equiv \lambda p. \text{let } \langle x, y \rangle = p \text{ in } \langle x, y \rangle \\
\text{FF} & \equiv \lambda p. \text{let } \langle x, y \rangle = p \text{ in } \langle y, x \rangle
\end{align*}
\]

True $\equiv \langle \text{TT}, \text{FF} \rangle$

False $\equiv \langle \text{FF}, \text{TT} \rangle$

Copy $\equiv \lambda b. \text{let } \langle u, v \rangle = b \text{ in } \langle u \langle \text{TT}, \text{FF} \rangle, v \langle \text{FF}, \text{TT} \rangle \rangle$

Implies $\equiv \lambda b_1. \lambda b_2. \text{ let } \langle p_1, q_1 \rangle = b_1 \text{ in }$

\[
\begin{align*}
\text{let } \langle u_2, v_2 \rangle = b_2 \text{ in } & \\
\text{let } \langle p_1, p_2 \rangle = u_1 \langle u_2, \text{TT} \rangle \text{ in } & \\
\text{let } \langle q_1, q_2 \rangle = v_1 \langle \text{FF}, v_2 \rangle \text{ in } & \\
\langle p_1, q_1 \circ p_2 \circ q_2 \circ \text{FF} \rangle &
\end{align*}
\]
0CFA and \textbf{PTIME}

\[ \lambda e_1.\lambda e_2.\lambda e_3\lambda e_4.\lambda e_5.\lambda e_6. \]
Andgate \( e_2 \) \( e_3 \) (\( \lambda e_7 \).
Andgate \( e_4 \) \( e_5 \) (\( \lambda e_8 \).
Copygate \( f \) (\( \lambda e_9.\lambda e_{10} \).
Orgate \( e_1 \) \( e_9 \) (\( \lambda e_{11} \).
Orgate \( e_{10} \) \( e_6 \) (\( \lambda e_{12} \).
Orgate \( e_{11} \) \( e_{12} \) (\( \lambda o.o \))))))

\textbf{Theorem} 0CFA decision problem is complete for \textbf{PTIME}. 

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Approximation as power tool

Hardness of $k$CFA relies on two insights:

1. Program points are approximated by an exponential number of closures.

2. Inexactness of analysis engenders reevaluation which provides computational power.
Abstract closures

Many closures can flow to a single program point:

\[(\lambda w. w x_1 x_2 \ldots x_n)\]

- \(n\) free variables
- an *exponential* number of possible associated environments mapping these variables to program points (contours of length 1 in 1CFA).
Toy calculation, with insights

Consider the following *non-linear* example

\[(\lambda f. (f \text{ True})(f \text{ False}))
(\lambda x. \\
(\lambda p.p(\lambda u. p(\lambda v. (\text{Implies } u \ v)))))(\lambda w.wx)\]
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A: True: it is equivalent to \text{Implies } x \ x, a tautology.
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A: both \text{True} and \text{False}: \text{Not true evaluation!}

We are computing with the approximation (spurious flows).
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The idea:

★ Break machine ID into an exponential number of pieces

★ Do piecemeal transitions on pairs of puzzle pieces

\[ \langle T, S, H, C, b \rangle \]

“At time \( T \), machine is in state \( S \), the head is at cell \( H \), and cell \( C \) holds symbol \( b \)"
Jigsaw puzzles, Machines

\[ \langle T, S, H, C, b \rangle: \text{“At time } T, \text{ machine is in state } S, \text{ the head is at}\] 
\[ \text{cell } H, \text{ and cell } C \text{ holds symbol } b” \]

1) Compute:
\[ \delta \langle T, S, H, H, b \rangle \langle T, S', H', C', b' \rangle = \]
\[ \langle T + 1, \delta_Q(S, b), \delta_{LR}(S, H, b), H, \delta_{\Sigma}(S, b) \rangle \]
Jigsaw puzzles, Machines

$\langle T, S, H, C, b \rangle$: “At time $T$, machine is in state $S$, the head is at cell $H$, and cell $C$ holds symbol $b$”

1) Compute:
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2) Communicate:
$\delta \langle T + 1, S, H, C, b \rangle \langle T, S', H', C', b' \rangle = \langle T + 1, S, H, C', b' \rangle$

($H' \neq C'$)
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\[ \langle T, S, H, C, b \rangle \text{: “At time } T, \text{ machine is in state } S, \text{ the head is at cell } H, \text{ and cell } C \text{ holds symbol } b \rangle \]

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2) Communicate:
\[
\delta \langle T + 1, S, H, C, b \rangle \langle T, S', H', C', b' \rangle = \langle T + 1, S, H, C', b' \rangle \quad (H' \neq C')
\]

3) Otherwise:
\[
\delta \langle T, S, H, C, b \rangle \langle T', S', H', C', b' \rangle = \langle \text{some goofy null value} \rangle \quad (T \neq T' \text{ and } T \neq T' + 1)
\]

The real deal

Setting up initial ID, iterator, and test:

\[(\lambda f_1.(f_1 \ 0)(f_1 \ 1))\]
\[(\lambda z_1.\]
\[(\lambda f_2.(f_2 \ 0)(f_2 \ 1))\]
\[(\lambda z_2.\]
\[\ldots\]
\[(\lambda f_N.(f_N \ 0)(f_N \ 1))\]
\[(\lambda z_N.\]

(let \( \Phi = \text{coding of transition function of TM in} \)

\[\text{Widget}\left[\text{Extract}(Y \ \Phi (\lambda w.w \ 0 \ldots 0 Q_0 H_0 z_1 z_2 \ldots z_N 0))])\]...)

\[\langle T, S, H, C, b \rangle\]
The real deal

...let $\Phi = \text{coding of transition function of TM in Widget}$

$$\text{Extract}(Y \Phi (\lambda w.w \ 0 \ldots \ 0 \ Q_0 \ H_0 \ z_1z_2 \ldots z_N \ 0)))$$

$$\langle T, S, H, \ C, b \rangle$$

$$\Phi \equiv (\lambda p.p(\lambda x_1.\lambda x_2.\ldots\lambda x_m.p(\lambda y_1.\lambda y_2.\ldots\lambda y_m.($$

$$(\phi x_1x_2 \ldots x_my_1y_2 \ldots y_m))))$$
The real deal

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$$\text{Widget}[\text{Extract}(Y \Phi (\lambda w . w \ 0 \ldots 0 \ Q_0 \ H_0 \ z_1 z_2 \ldots z_N \ 0))] \ldots \langle T, S, H, \ C, b \rangle$$

$$\Phi \equiv (\lambda p . p (\lambda x_1 . \lambda x_2 . \ldots . \lambda x_m . p (\lambda y_1 . \lambda y_2 . \ldots . \lambda y_m . (\phi x_1 x_2 . . . x_m y_1 y_2 . . . y_m))))$$

$\text{Widget}[E] \equiv \ldots f \ldots a \ldots$, where $a$ flows as an argument to $f$ iff a True value flows out of $E$. 
The real deal

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$$\text{Widget}[\text{Extract}(Y\Phi(\lambda w.\lambda w.0\ldots0Q_0H_0z_1z_2\ldots z_N0))\ldots]$$

$$\langle T, S, H, C, b \rangle$$

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**Theorem** In $k\text{CFA}$, $a$ flows to $f$ iff TM accept in $2^n$ steps.
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$$\text{Widget}[\text{Extract}(Y \Phi (\lambda w.w \ 0 \ldots 0 \ 0 Q_0 H_0 z_1z_2 \ldots z_N 0))].$$

$$\langle T, S, H, \ C, b \rangle$$

$$\Phi \equiv (\lambda p.p(\lambda x_1.\lambda x_2.\ldots \lambda x_m.p(\lambda y_1.\lambda y_2.\ldots \lambda y_m.(\phi x_1 x_2 \ldots x_m y_1 y_2 \ldots y_m))))$$

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**Theorem** In $k\text{CFA}$, $a$ flows to $f$ iff TM accept in $2^n$ steps.

**Theorem** $k\text{CFA}$ decision problem is complete for $\text{EXPTIME}$.
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- therefore bounded by a polynomial!
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Might and Shivers’ observation:
improved precision leads to analyzer speedups.
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Analytic understanding: What you pay for in $k$CFA is the junk (spurious flows).
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Analytic understanding:
What you pay for in $k$CFA is the junk (spurious flows).

Work harder to learn less.
Doggie bag

- Linearity is key in understanding static analysis
- 0CFA is inherently sequential
- There is no tractable algorithm for $k$CFA
- The approximation of $k$CFA is what makes it hard
The End

Thank you.