Solving POMDPs Using Quadratically Constrained Linear Programs

Christopher Amato

Daniel S. Bernstein

Shlomo Zilberstein

University of Massachusetts Amherst

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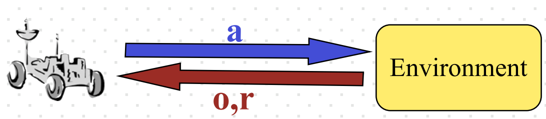
Overview

- POMDPs and their solutions
- Fixing memory with controllers
- Previous approaches
- Representing the optimal controller
- Some experimental results

POMDPs

- Partially observable Markov decision process (POMDP)
- Agent interacts with the environment
  - Sequential decision making under uncertainty
  - At each stage receives:
    - an observation rather than the actual state
    - Receives an immediate reward



POMDP definition

- A POMDP can be defined with the following tuple: M = $\langle S, A, P, R, \Omega, O\rangle$
  - *S*, a finite set of states with designated initial state distribution b_0
  - *A*, a finite set of actions
  - *P*, the state transition model: $P(s'| s, a)$
  - *R*, the reward model: $R(s, a)$
  - Ω, a finite set of observations
  - *O*, the observation model: $O(o|s',a)$

POMDP solutions

- A policy is a mapping $\delta : \Omega^* \rightarrow A$
- Goal is to maximize expected discounted reward over an infinite horizon
- Use a discount factor, γ, to calculate this

Example POMDP: Hallway

Minimize number of steps to the starred square for a given start state distribution

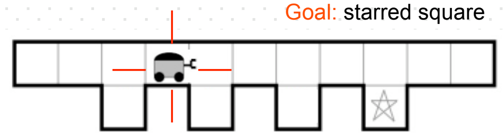
States: grid cells with orientation

Actions: turn , , move forward, stay

Transitions: noisy

Observations: red lines

Goal: starred square



Previous work

- Optimal algorithms
  - Large space requirement
  - Can only solve small problems
- Approximation algorithms
  - provide weak optimality guarantees, if any

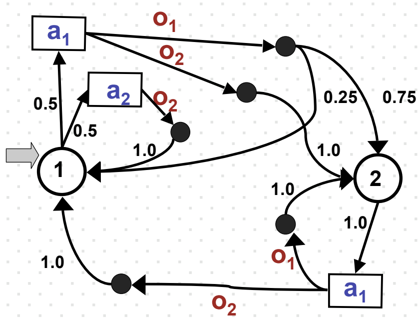
Policies as controllers

- Fixed memory
- Randomness used to offset memory limitations
- Action selection, $\psi : Q \rightarrow \Delta A$
- Transitions, $\eta : Q \times A \times O \rightarrow \Delta Q$
- Value given by Bellman equation:

$$V(q,s) = \sum_a P(a \mid q)\left[R(s,a) + \gamma \sum_{s'} P(s' \mid s,a) \sum_o O(o \mid s',a) \sum_{q'} P(q' \mid q,a,o) V(q',s')\right]$$

Controller example

- Stochastic controller
  - 2 nodes, 2 actions, 2 obs
  - Parameters
    - P(a|q)
    - P(q'|q,a,o)



Optimal controllers

- How do we set the parameters of the controller?
- Deterministic controllers - traditional methods such as branch and bound (Meuleau et al. 99)
- Stochastic controllers - continuous optimization

Gradient ascent

- Gradient ascent (GA)- Meuleau et al. 99
- Create cross-product MDP from POMDP and controller
- Matrix operations then allow a gradient to be calculated

Problems with GA

- Incomplete gradient calculation
- Computationally challenging
- Locally optimal

BPI

- Bounded Policy Iteration (BPI) - Poupart & Boutilier 03
- Alternates between improvement and evaluation until convergence
- Improvement: For each node, find a probability distribution over one-step lookahead values that is greater than the current node's value for all states
- Evaluation: Finds values of all nodes in all states

BPI - Linear program

For a given node, q

Variables: $x(a)$= P(a|q), $x(q',a,o)$=P(q',a|q,o)

Objective: Maximize ε

Improvement Constraints: $\forall s \in S$

$$V(q,s)+\varepsilon \le \sum_a \left[x(a)R(s,a)+\gamma \sum_{s'} P(s'|s,a) \sum_o O(o|s',a) \sum_{q'} x(q',a,o) V(q',s') \right]$$

Probability constraints: $a \in A$ $\quad \sum_{q'} x(q',a,o) = x(a)$

Also, all probabilities must sum to 1 and be greater than 0

Problems with BPI

- Difficult to improve value for all states
- May require more nodes for a given start state
- Linear program (one step lookahead) results in local optimality
- Must add nodes when stuck

QCLP optimization

- Quadratically constrained linear program (QCLP)
- Consider node value as a variable
- Improvement and evaluation all in one step
- Add constraints to maintain valid values

QCLP intuition

- Value variable allows improvement and evaluation at the same time (infinite lookahead)
- While iterative process of BPI can “get stuck” the QCLP provides the globally optimal solution

QCLP representation

Variables: $x(q',a,q,o) = P(q',a|q,o)$, $y(q,s) = V(q,s)$

Objective: Maximize $\sum_s b_0(s) y(q_0,s)$

Value Constraints: $\forall s \in S,\ q \in Q$

$$y(q,s) = \sum_a \left[\left(\sum_{q'} x(q',a,q,o_k) \right) R(s,a) + \gamma \sum_{s'} P(s'|s,a) \sum_o O(o|s',a) \sum_{q'} x(q',a,q,o) y(q',s') \right]$$

Probability constraints: $\forall q \in Q,\ a \in A,\ o \in \Omega$

$$\sum_{q'} x(q',a,q,o) = \sum_{q'} x(q',a,q,o_k)$$

Also, all probabilities must sum to 1 and be greater than 0

Optimality

Theorem: An optimal solution of the QCLP results in an optimal stochastic controller for the given size and initial state distribution.

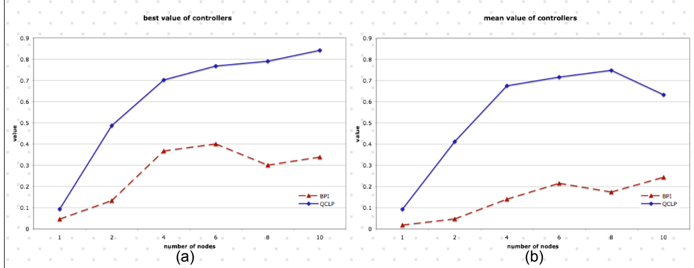
Pros and cons of QCLP

- Pros
  - Retains fixed memory and efficient policy representation
  - Represents optimal policy for given size
  - Takes advantage of known start state
- Cons
  - Difficult to solve optimally

Experiments

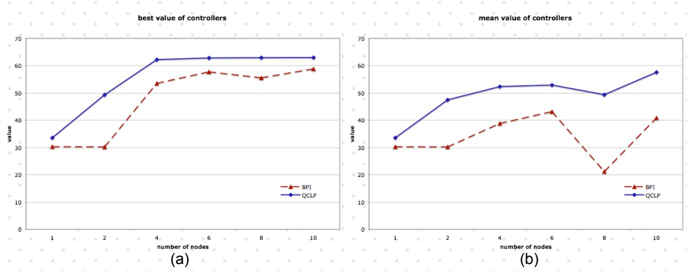
- Nonlinear programming algorithm (snopt) - sequential quadratic programming (SQP)
- Guarantees locally optimal solution
- NEOS server
- 10 random initial controllers for a range of sizes
- Compare the QCLP with BPI

Results



(a) best and (b) mean results of the QCLP and BPI on the hallway domain (57 states, 21 obs, 5 acts)

Results



(a) best and (b) mean results of the QCLP and BPI on the machine maintenance domain (256 states, 16 obs, 4 acts)

Results

- Computation time is comparable to BPI
- Increase as controller size grows offset by better performance

Hallway

# nodes	QCLP	BPI
1	< 1 min	< 1 min
2	< 1min	< 1 min
4	< 1 min	< 1 min
6	1.4 mins	1.6 mins
8	6.9 mins	2.9 mins
10	9.1 mins	4 mins

Machine

# nodes	QCLP	BPI
1	< 1 min	1.3 mins
2	< 1 min	4.6 mins
4	7.9 mins	14.1 mins
6	42.4 mins	25.5 mins
8	57.5 mins	42.9 mins
10	130.8 mins	62.8 mins

Conclusion

- Introduced new fixed-size optimal representation
- Showed consistent improvement over BPI with a locally optimal solver
- In general, the QCLP may allow small optimal controllers to be found
- Also, may provide concise near-optimal approximations of large controllers

Future Work

- Investigate more specialized solution techniques for QCLP formulation
- Greater experimentation and comparison with other methods
- Extension to the multiagent case