CS 1800

10/27 - Fri :)

Admin

- HW5 out, due 11/3 11:59pm
- next week: recitation 7

Agenda

1. Binomial Distr.
2. Poisson Distr.
3. Probability Problems

Completing the picture of prob/experiments

$$\Pr(E \mid F) = \frac{\Pr(E \cap F)}{\Pr(F)}$$

Conditional

$$\Pr(E \mid F) = \frac{\Pr(F \mid E) \cdot \Pr(E)}{\Pr(F)}$$

Bayes

$$E[X] = \sum X_i \cdot \Pr(S_i)$$

EV

$$\Pr(F) = \Pr(F \mid E) \cdot \Pr(E) + \Pr(F \mid \neg E) \cdot \Pr(\neg E)$$

$$V[X] = \sum (X_i - \mu)^2 \cdot \Pr(S_i)$$

Variance

X = random var

μ = e.v. (mean)

1. Binomial Distr.

↳ X = random variable

$E[X]$ ~ on avg what happens

$Var[X]$ ~ how far from mean?

Distr ~ model the outcomes

How often will my outcome happen?

Big Picture

Binomial → every chance is independent of others

all experiments have same chance of success

Experiment has <u>two</u> possible outcomes

- success
- failure

even if actual experiment has many outcomes

ex) Flip a coin

Success = heads

failure = tails

ex) Rolling a die

Success: even #

failure: odd #

Bernoulli Trial

- perform experiment w/ two possible outcomes (s/f)
- success = probability $\boxed{p}$
- failure = probability $\boxed{1-p}$

p is fixed, outcome of one trial does not impact outcome of another

Binomial Distr...

- conduct $\underline{n}$ Bernoulli trials
- X = random variable associated with number of successful trials

What's the probability that $X = \underline{k}$?

ex flip a coin

success = heads $\quad p = .5$

failure = tails $\quad 1-p = .5$

$\underline{n}$ trials $\quad \boxed{n=7}$

$Pr(X=k)$ $\underline{k}$ # successes $\quad \boxed{k=4}$

What this looks like?

S/F S/F S/F S/F S/F S/F S/F

total possible outcomes:

$2^n = 2^7$

What outcomes have $X=k$?

4 successes...

- S S S S F F F ✓
- S F F S S S F ✓

How many have $X=k$?

$\binom{7}{4}$ $\quad$ 7 spots

4 succ, 3 failure

SSSS FFF

$p \cdot p \cdot p \cdot p \cdot (1-p)(1-p)(1-p)$

SFFSSS F

$p(1-p)(1-p)ppp(1-p)$

What's prob of one $X=k$ outcome

$p^4 \cdot (1-p)^3 = p^k \cdot (1-p)^{n-k}$

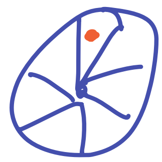
All together...

$$Pr(X=\underset{k}{4}) = \binom{\overset{n}{7}}{4}\cdot(\underset{p}{.5})^4\cdot(\underset{(1-p)}{.5})^3 = .273$$

Binomial Formula

$$\boxed{Pr(X=k) = \binom{n}{k}\cdot(p^k)\cdot(1-p)^{n-k}}$$

ex Spin a roulette wheel



- numbered 1-38
- win = \$10
- lose = -\$1
- Bet on: $\boxed{\text{red}}$ $18/38$

$X = \$$

$$E[X] = 10\cdot 18/38 + -1\cdot 20/38$$
$$= \$4.21$$

$$V[X] = (10-4.21)^2\cdot 18/38 + (-1-4.21)^2\cdot 20/38$$
$$= 2\cancel{1.30}3$$
$$= 30.17$$

$X = \#$ reds

spin 5 times

$Pr(X \geq 4)$?

$n = 5$ $\quad p = 18/38$

$k = 5$ $\quad 1-p = 20/38$

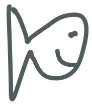
$$Pr(X=5) = \binom{5}{5} \cdot \left(\tfrac{18}{38}\right)^5 \cdot \left(\tfrac{20}{38}\right)^0 = .02$$

$$Pr(X=4) = \binom{5}{4} \cdot \left(\tfrac{18}{38}\right)^4 \cdot \left(\tfrac{20}{38}\right)^1 = .132$$

$$Pr(X \geq 4) = (.02) + (.132) = .152$$

10:55

2. Poisson Distr



Random Variable X = # of occurrences

- avg rate of occurrences
- regular time intervals

} # things / day on avg

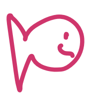
- avg rate
- indv time intervals – sometimes less, sometimes more
- e will in formula (base of ln, ~2.71)

(ex) X = # ppl at Shillman Dunkin in an hour

↳ who walk in

- hour intervals
- on avg, 60 people walk in $\qquad \lambda = 60$
- $Pr(X=5)$ Pr 5 ppl walk in over an hour $\qquad k = 5$

Poisson Formula



$$Pr(X=k) = \boxed{\frac{e^{-\lambda} \cdot \lambda^k}{k!}}$$

$$Pr(X=5) = \frac{e^{-60} \cdot 60^5}{5!} = 5.67 \times 10^{-20}$$

$$Pr(X=70) = \frac{e^{-60} \cdot 60^{70}}{70!} = .021$$

Probability in a given hour of being off from avg

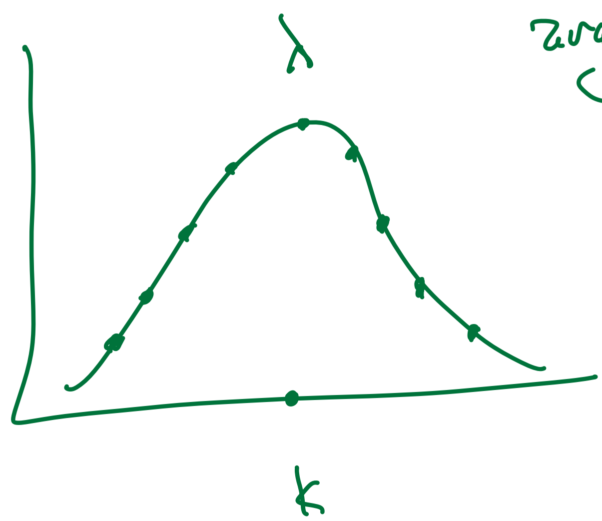
$$Pr(X=60) = \frac{e^{-60} \cdot 60^{60}}{60!} = .051$$

actual avg

(or actual avg)

$$Pr(X=500) = \frac{e^{-60} \cdot 60^{500}}{500!} =$$ google can't :(

$$Pr(X=100) = \frac{e^{-60} \cdot 60^{100}}{100!} = 6.12 \times 10^{-7}$$



avg across all = λ

3. Probability Examples

- Halloween candy 
- 10 choc ones
- 18 fruit ones

Draw 3 candies

Pr(no chocolate)?

order matters

$\frac{18}{28} \cdot \frac{17}{27} \cdot \frac{16}{26} = .249$:)

Pr(at least one) = 1 - .249

order doesn't matter

$|S| = \binom{28}{3}$ all

$|E| = \binom{18}{3}$ no choc :(

$Pr(E) = \frac{\binom{18}{3}}{\binom{28}{3}} \approx .249$

Pr(at least one choc) = 1 - .249