CS1800 Day 14

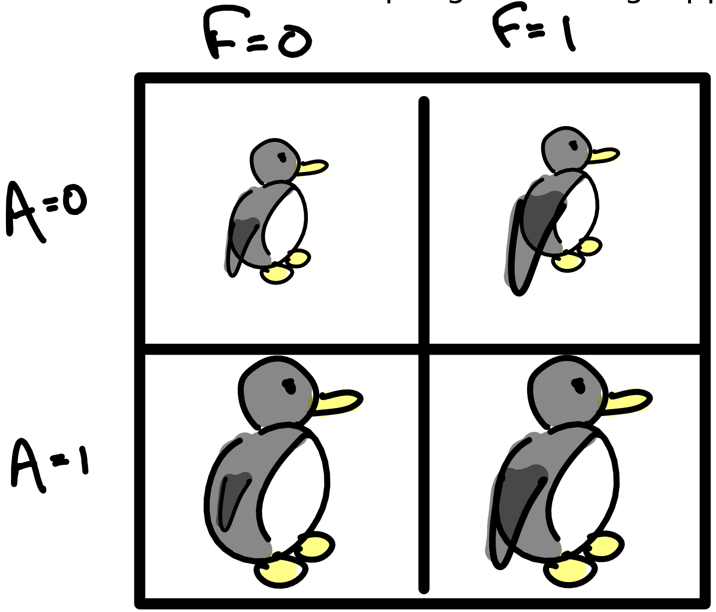
Admin:
- exam results
- tuning up your study process in CS1800
- HW4 results by thursday (hopefully tomorrow)
- grade estimates by Friday (hopefully Thursday)

Content:

Joint Probability Distribution
Marginalization
Conditional Probability
Bayes Rule
Independence

Joint Probability Distribution: A distribution over more than 1 variable at a time

Let A=1 indicate if a penguin is an adult (0 otherwise)
Let F=1 indicate if a penguin has big flippers (0 otherwise)



	F=0	F=1
A=0	3/12	2/12
A=1	1/12	6/12

HALF OF PENGUINS ARE ADULTS w/ BIG FLIPPERS

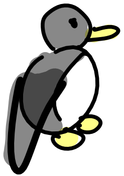
NOTATION

$P(A=0, F=1)$ IS PROB

$A=0$ (NOT ADULT)

$F=1$ (BIG FLIPPER)

HAPPEN AT SAME TIME ⟶

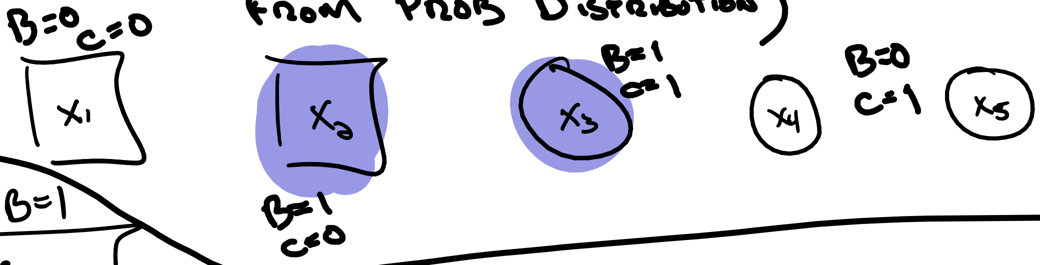


MARGINALIZING

(REMOVING A RANDOM VARIABLE FROM PROB DISTRIBUTION)

B=1 SHAPE IS BLUE

C=1 SHAPE IS CIRCLE



$P(B=1) = 2/5$

	B=0	B=1
C=0	x_1 1/5	x_2 1/5
C=1	x_4 x_5 2/5	x_3 1/5

$$P(B=1) = P(B=1\ C=0) + P(B=1\ C=1)$$

$$= 1/5 + 1/5 = 2/5$$

Remember: To compute P(B) we can sum P(B, A) for all outcomes in sample space of A

$$P(B=b) = \sum_a P(B=b\ A=a)$$

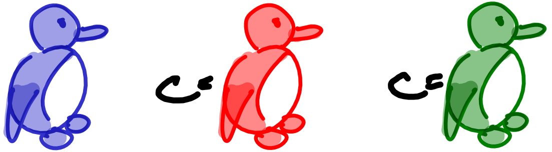
In Class Activity

Let C be a random variable representing penguin color (sample space: blue, red or green)
Let A=1 indicate if a penguin is an adult (0 otherwise)

Given the following distribution of A, C

Compute each of the follow probabilities:

- P(C=blue)
- P(C=red) + P(C=green) $= 9/12$
  (how is this related to prob above?)
- P(A=1)



	C=	C=	C=
A=0	1/12	3/12	0/12
A=1	2/12	1/12	5/12

$$P(C=\text{BLUE}) = P(C=\text{BLUE}\ A=0) + P(C=\text{BLUE}\ A=1)$$
$$= 1/12 + 2/12 = 3/12$$

$$P(A=1) = 2/12 + 1/12 + 5/12$$

Conditional Probability (intuition & motivation)

C=1 indicates a person has covid (C=0 otherwise)
T=1 indicates a person has positive test (T=0 otherwise)

Let us discuss (and express) the following probabilities:

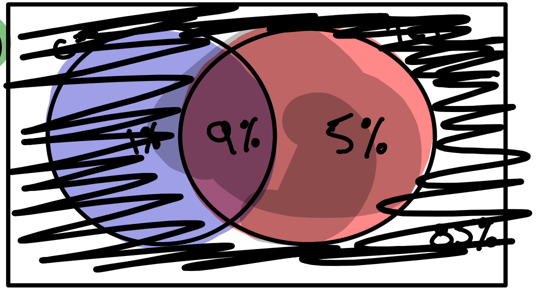
- probability person has a positive test $P(T=1)$

- probability person has positive test given they have covid $P(T=1 \mid C=1)$ "GIVEN C=1"

- probability person has covid given a positive test $P(C=1 \mid T=1)$

Intuition:
Conditional probability P(X=x|Y=y) is the probability of event X=x if we constrain ourselves to a world where Y=y.

Conditional Probability (motivating our formula from intuition)



C=1 indicates a person has covid (C=0 otherwise)
T=1 indicates a person has positive test (T=0 otherwise)

Let us discuss (and express) the following probabilities:

- probability person has a positive test $P(T=1) = 9+5 = 14\%$

- probability person has positive test given they have covid $P(T=1 \mid C=1) = \frac{9}{1+9} = 90\%$

- probability person has covid given a positive test

$$P(C=1 \mid T=1) = \frac{P(C=1, T=1)}{P(T=1)} = \frac{9}{9+5} = \frac{9}{14}$$

Intuition:
Conditional probability $P(X=x|Y=y)$ is the ~~probability~~ of event X=x if we constrain ourselves to a world where Y=y.

Conditional Probability (Formula version1: from our intuition)

PROB a b HAPPEN TOGETHER

$$P(a|b) = \frac{P(a\,b)}{P(b)}$$

PROB a HAPPENS GIVEN CONDITION b

PROB b HAPPENS

In Class Activity

Compute each of the probabilitie from the distribution below. For each, write a sentence explaining its meaning which is easily understood by a non-technical reader

Let S be a twitter sentiment score about bitcoin (1=good, 0=neutral, -1=bad)
Let B be the movement of bitcoin price (1=up, -1=down)

P(S=-1|B=1) ... compare to P(S=-1)
P(B=1|S=-1) ... compare to P(B=1)

	S=-1	S=0	S=1
B=-1	19%	27%	5%
B=1	8%	21%	20%

$$P(S=-1 \mid B=1)$$

$$= \frac{P(S=-1 \quad B=1)}{P(B=1)}$$

$$= \frac{.08}{.08+.21+.20}$$

$$= 16\%$$

	S=-1	S=0	S=1
B=-1	19%	27%	5%
B=1	8%	21%	20%

$$P(S=-1) = .19 + .08$$

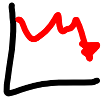
$$= 27\%$$

$$P(B=1 \mid S=-1)$$

"GIVEN S=-1"

$$= \frac{P(B=1 \quad S=-1)}{P(S=-1)}$$

$$= \frac{.08}{.08 + .19} = 30\%$$

		S=-1	S=0	S=1
	B=-1	19%	27%	5%
	B=1	8%	21%	20%

$$P(B=1) = .08 + .21 + .2$$

$$= .49$$

Conditional Probability (formula version 2: often more useful in our algebraic manipulations)

$$P(a|b) = \frac{P(a\,b)}{P(b)} \iff P(a|b)\,P(b) = P(a\,b)$$

Takeaway above:

Multiplying
- a conditional probability
- the probability of condition

Will yield
- prob both outcomes happen together

BAYES RULE (GLORIFIED CONDITIONAL PROBABILITY)

SEE PREVIOUS SLIDE

$$P(a|b)\,P(b) = P(ab) = P(b|a)\,P(a)$$

$$\Rightarrow \quad P(a|b) = \frac{P(b|a)\,P(a)}{P(b)}$$

Notice: this formula "swaps" the order of the conditioning: P(A|B) on left P(B|A) on right
Its typical in a Bayes question to be given variables in one order while question asks for other.

A Helpful Manipulation

Marginalization

$$P(b) = \sum_a P(a\,b)$$

Conditional Prob Definition

$$= \sum_a P(b|a)\,P(a)$$

WHY WAS THAT HELPFUL?

BAYES RULE 1

$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

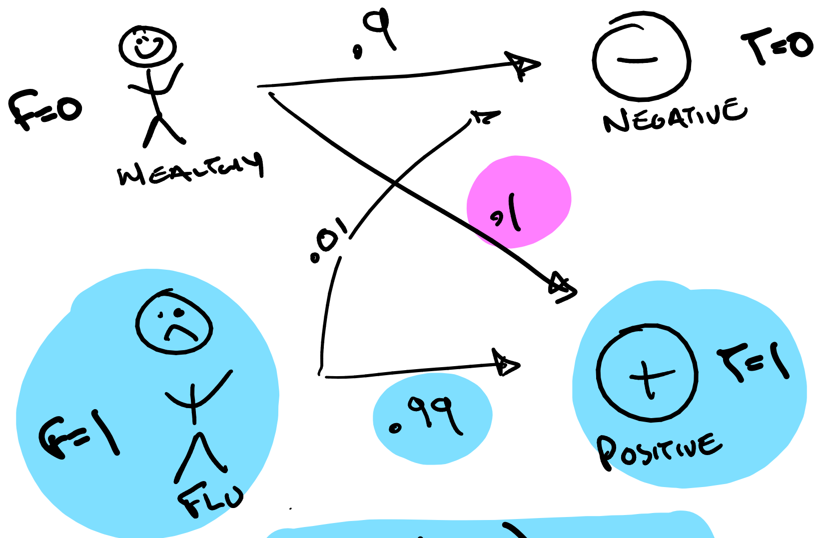
BAYES RULE 2

$$P(a|b) = \frac{P(b|a)P(a)}{\sum_i P(b|a_i)P(a_i)}$$

Notice:
all terms of form P(b|a) and P(a) here

Bayes Rule Ex

Given flu occurs in 4% of population, what is the probability one has flu given they test positive?



$$P(T=1 \mid F=1) = .99$$

$$P(F=1) = .04$$

$$P(F=0) = .96$$

$$P(F=1 \mid T=1) = \frac{P(T=1 \mid F=1)\,P(F=1)}{P(T=1)}$$

$$= \frac{.99 \cdot .04}{.1 \cdot .96 + .99 \cdot .04} \approx .29$$

$$P(T=1) = P(T=1, F=0) + P(T=1, F=1)$$

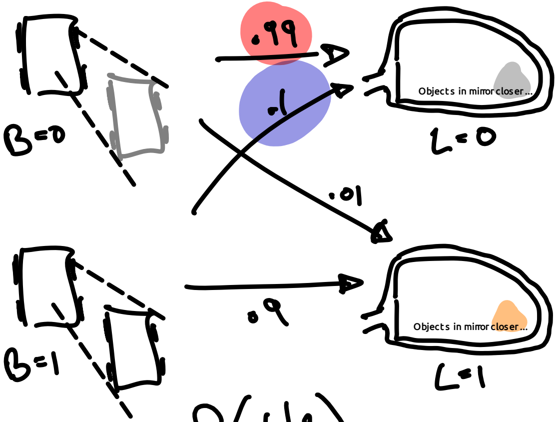
$$= P(T=1 \mid F=0)\,P(F=0) + P(T=1 \mid F=1)\,P(F=1)$$

$$= .1 \cdot .96 + .99 \cdot .04$$

In Class Assignment

A blind spot monitor produces a warning light (L=1) when it estimates that a car is in one's blind spot (B=1). Given that the light is off, whats the probability that a car is one's blind spot? (Assume that a car is in your blindspot 2 percent of the time while driving.)

$P(B=0)=.98$ $P(B=1)=.02$



$P(L|B)$

$$P(B=1|L=0) = \frac{P(L=0|B=1)\cdot P(B=1)}{P(L=0)}$$

$$= \frac{.1\cdot .02}{.99\cdot .98 + .1\cdot .02} \approx .00205$$

$$P(L=0) = P(L=0\ B=0) + P(L=0\ B=1)$$

$$= P(L=0|B=0)P(B=0) + P(L=0|B=1)P(B=1)$$

ALGEBRA + INTUITION

LOVE STORY

AHEAD...

INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x, Y=y) = P(X=x)P(Y=y)$$

Bayes Rule shows the equivilence of the algebraic and intuitive definitions above!

INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x\, Y=y) = P(X=x)\,P(Y=y)$$

$$P(X|Y) = \frac{P(X\,Y)}{P(Y)} = \frac{P(X)\,P(Y)}{P(Y)} = P(X)$$

Notice that P(X|Y) = P(X). Observing Y has no impact on the prob of X!

Joint Distributions w/ Independent Variables

$$P(X\,Y) = \underline{P(X)}\,\underline{P(Y)}$$

$P(X=1) = 1/4$ 0 otherwise

$P(Y=1) = 1/10$ 0 otherwise

	$X=0$	$X=1$
$Y=0$	$\frac{3}{4}\cdot\frac{9}{10}$	$\frac{9}{10}\cdot\frac{1}{4}$
$Y=1$	$\frac{1}{10}\cdot\frac{3}{4}$	$\frac{1}{10}\cdot\frac{1}{4}$