CS1800

11/7 - tues.

Admin

- HW6 due Fri 11:59pm
- HW7 out Fri
- this week: normal recitation
- next week: exam review in recitation

$\boxed{\text{exam \#2 11/17}}$

Agenda

1. Induction Reasoning
2. Proof structure
3. Example
(4. TA Survey)

1. Induction Reasoning

We live where... the only currency is

$\boxed{\$2}$ $\boxed{\$5}$

we $\boxed{\text{want}}$ to compile any amount of money (whole dollars)

When do we use this proof?

- we could prove ∞ things one at a time
- if we had ∞ time and patience
- technique to short cut doing one at a time

↳ key: Showing we can get from k to $k+1$ ♡ :)

↳ key #2: Showing we can get smallest value we care about ♡

money universe

- goal: show we can compile almost any amount of $

starting point: $P(n)$ states that we can make $\$n$

$\underline{n}$ ↳ $

with $2 \cdot \underline{t} + 5 \cdot \underline{f}$

↳ #2s ↳ #5s

$$P(n) \ldots n = 2t + 5f \text{ with } t, f \in \mathbb{N}$$

Predicate: generalization of logic statement
has no truth value
can't really prove $P(n)$

Turn predicate into logic statement

1. Plug in values for n

$P(15) = 2 \cdot 0 + 5 \cdot 3$
$P(16) = 2 \cdot 3 + 5 \cdot 2$
$P(17) = 2 \cdot 1 + 5 \cdot 3$

key #2

2. Quantifiers

$$\forall n \in \mathbb{Z} \quad n \geq 4 \Rightarrow P(n)$$

key #1

Money Proof (isn)

$\boxed{\text{goal}}$ $\forall n \in \mathbb{Z}\ n \geq 4 \Rightarrow P(n)$

Base Case: plug in smallest n we care about

$P(4) = 2 \cdot 2 + 5 \cdot 0$

$= 4$ ✓

Inductive Step: $P(k) \Rightarrow P(k+1)$

- If I can make \$k, then I can make \$k+1
- Assume $P(k)$ ~ don't need to prove it
- prove $P(k) \Rightarrow P(k+1)$

\$k, then subtract $t-2$ and add $f+1$

Case 1 (no fives)

Assume $P(k)$

$k = 2t + 5f$ with $f = 0$

$= 2t$

goal: show we can make \$k+1

$k+1 = 2t + 1$ ~

$= 2t + (5-4)$

$= 2t - 4 + 5$

$= 2 \cdot \underbrace{(t-2)}_{\#2s} + 5 \cdot \underbrace{1}_{\#5s}$

k, then subtract $f-1$ and add $t+3$

Case 2

(at least one 5)

Assume $P(k) = 2t + 5f$

$$k+1 = 2t + 5f + 1$$
$$= 2t + 5f + (6-5)$$
$$= 2t + 6 + 5f - 5$$
$$= 2\underbrace{(t+3)}_{\#2s} + 5\underbrace{(f-1)}_{\#5s}$$

done!

2. Structure of this proof

Predicate - Logic statement (know what you're proving)

Base Case

- $P(n)$
  ↳ Smallest value we care about
- mini proof (ex: $4 = 2 \cdot 2$)
- very straight-forward
- Showing it works for smallest value

Inductive Step

Prove that $P(k) \Rightarrow P(k+1)$

Assume $P(k)$, don't prove it

Inductive Hypothesis

vocab word!

Use $P(k)$ to show $P(k+1)$

- a little algebra usually
- baby steps!
- keep both sides of equality true

What can we prove with induction?

- Any time you could prove $P(1), P(2), \ldots$ through ∞
- it's a shortcut!
- usually showing $P(n)$ for $n \in \mathbb{Z}^+$, $n \in \mathbb{N}$, or subset
- Inequalities
- number theory (division, primes)
- value of summation $\quad \sum \ldots =$ shortcut

$$\underbrace{\sum_{i=1}^{n} i = 1+2+3+\ldots+n}_{\text{defn}} = \underbrace{\text{shortcut}}_{\text{formula}}$$

- structural proofs (graphs, trees, sets)
- program correctness $\rightarrow$ in real life!

any proof: convince the reader that it's true!

Baby steps!! ☺

10:58

3. Induction Example

- Inequality

$P(n) \qquad 5n + 5 \le n^2$

$P(1) = \text{False} \qquad P(4) = \text{False}$

$P(2) = \text{False} \qquad P(5) = \text{False}$

$P(3) = \text{False} \qquad P(6) = \text{True!}$

$\forall n \in \mathbb{Z}\ n \ge 6\ P(n)$ ~ logic statement we can prove!

Base Case ~ show $P(n)$ for smallest n we care about

$P(n) \ldots P(\overset{}{\underset{n}{6}}) \quad \ldots \quad 5 \cdot \overset{n}{6} + 5 \le \overset{n}{6}^2 \ ?$

$35 \le 36$ ✓

Inductive Step ~ show $P(k) \Rightarrow P(k+1)$

Assume $P(k) \qquad 5k + 5 \le k^2$ Ind. Hypothesis

Goal: use $P(k)$ to show $P(k+1)$

$5(k+1) + 5 \le (k+1)2$

goal

$$5(k+1) + 5 = \underbrace{5k + 5}_{P(k)} + 5$$

$$\le k^2 + 5$$

$\leq k^2 + k$ → things we know to be true!

$k \geq 6$

$\leq k^2 + 2k + 1$

$= (k+1)^2$!! done!

bit.ly/neu-ta-students

Prof Felix's survey !!

thank you!