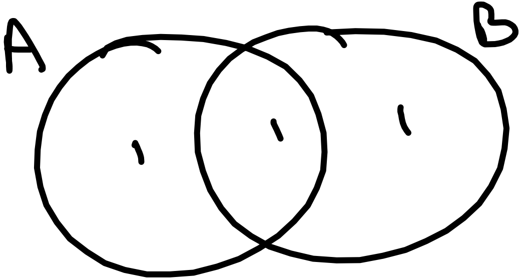
CS1800 Day 9

Admin:
- exam1 is on Oct 17th
- hw3 due today, hw4 released today
- hw4 deadlines are funny (for exam):
    - includes content from day10 (next class)
    - solutions for hw4 released sunday oct 15 @ 12:01am, first thing in the morning
        - good news: allows you study
        - bad news: you may only use up to 1 late day on hw4
- (next tuesday we'll do a "practice" exam together on gradescope so you can see the format, its pretty much a timed HW assignment)

Content:
- review PIE & product rule
- permutations
- count by partition
- count by complement
- count by simplification

PIE

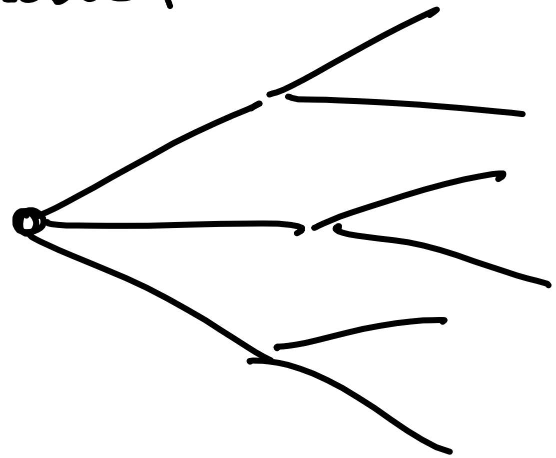


$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$A \cap B = \emptyset$$

Sum Rule $$|A \cup B| = |A| + |B|$$

PRODUCT



$$|A \times B| = |A| \times |B|$$

$A = \{1\ 2\ 3\}$

$B = \{4\ 5\}$

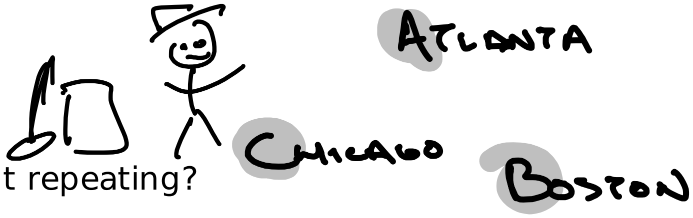
Notation (Reminder)

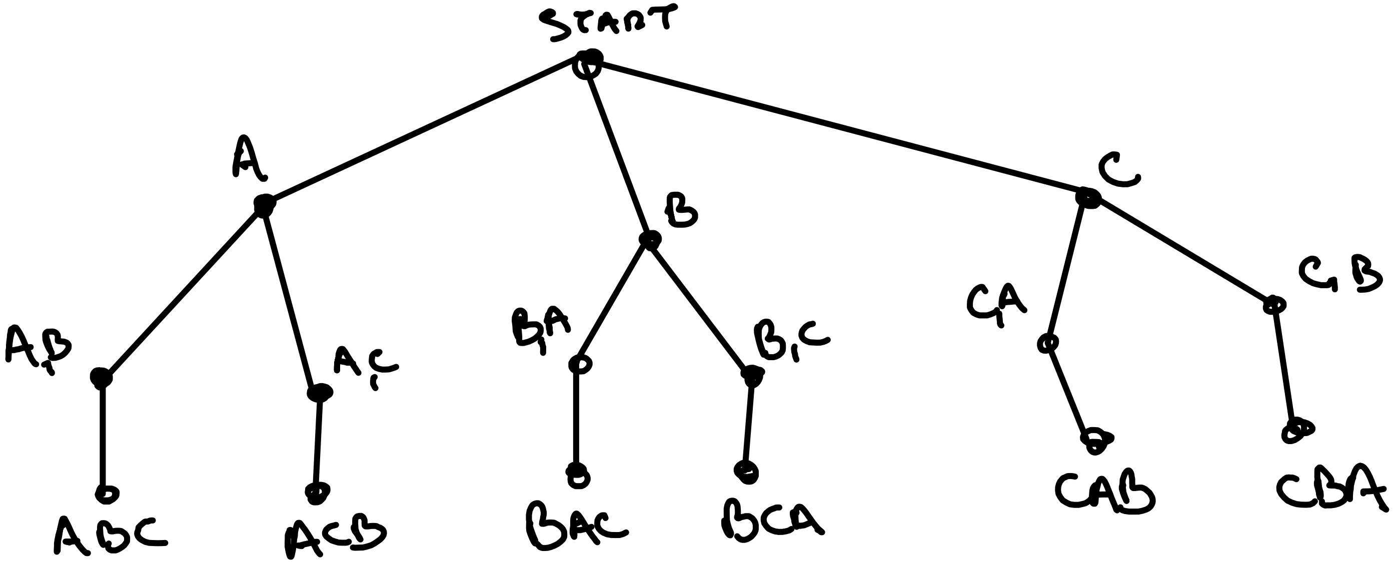
Set	Tuple
$\{a, b, c\}$	(a, b, c, a)
No repeats	May repeat
Unordered	Order matters
	$(a, b) \neq (b, a)$

Permutations: Travelling Salesperson

How many ways can a salesman order 3 city visits?

(How many tuples can we make from items A, B, C without repeating?)





Permutations: Travelling Salesperson

How many ways can a salesman order 3 city visits?

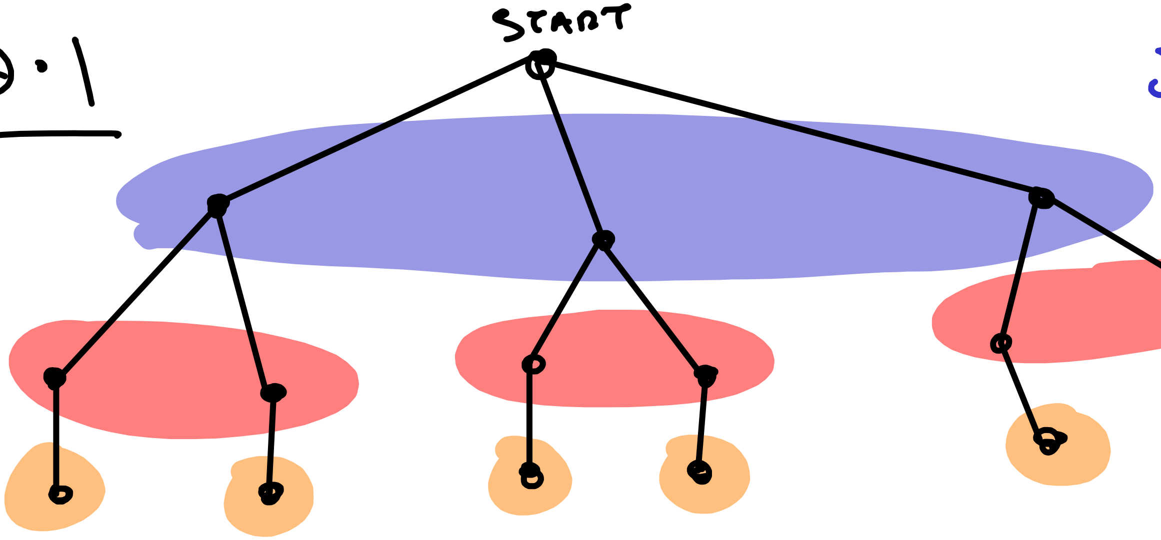
(How many tuples can we make from items A, B, C without repeating?)

ATLANTA

CHICAGO

BOSTON

$3 \cdot 2 \cdot 1$



3 CHOICES FOR 1ST CITY

2 CHOICES FOR 2ND

1 CHOICE FOR LAST CITY

Factorial:

"8 FACTORIAL"

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

CONVENTION $0! = 1$

$1! = 1$

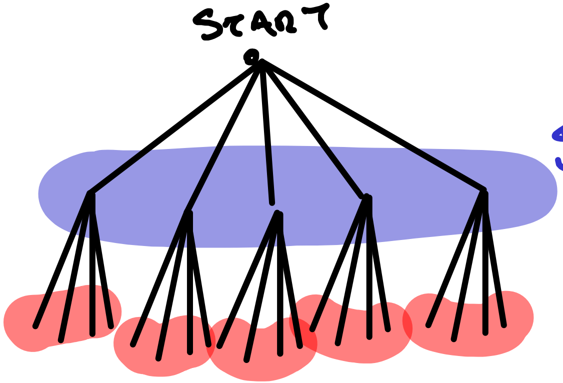
OUR SALESMAN (OF PREVIOUS SLIDE) HAD

$3! = 3 \cdot 2 \cdot 1$ TOTAL ORDERINGS OF 3 CITIES

Permutations: A Travelling (lazy) Salesperson

How many ways can a salesman order 2 of 5 cities?

(How many tuples of length 2 can be made from A, B, C, D, E where no repeats allowed)?



5 CHOICES FOR 1ST CITY

4 CHOICES FOR 2ND CITY

$$5 \cdot 4 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{5!}{3!}$$

Permutations:

The number of ways of ordering k objects, from n total available is:

$10 \cdot 9 \cdot 8$

$$P(n,k) = \frac{n!}{(n-k)!}$$

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \ldots}{7 \cdot 6 \cdot 5}$$

EXAMPLES:

VISIT 3 OF 3 CITIES

$$P(3,3) = \frac{3!}{(3-3)!} = 6$$

VISIT 2 OF 5 CITIES

$$P(5,2) = \frac{5!}{(5-2)!} = 20$$

In Class Activity

$$120 = \frac{5!}{(5-5)!} = P(5,5) = 5!$$

How many ways are there to order 5 people for a family portrait?

How many ways are there to order 6 of 20 people for a family portrait?

(If time): Plug a few of these factorials into calculator or google, how big of a factorial do you need to plug in until you "break" your computer?

$$P(20,6) = \frac{20!}{(20-6)!} = \frac{20!}{14!} = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15$$

Factorials grow really quickly: (more on this when we study "function growth" later)

$10! \approx 3 \cdot 10^{6}$ 3 MILLION

$20! \approx 2 \cdot 10^{18}$

$19! \approx 10^{17}$ SEC SINCE BIG BANG

$50! \approx 10^{80}$ ATOMS IN UNIVERSE

$70! \approx 10^{100}$ "GOOGLE"

Convention (in this class):

FEEL FREE TO LEAVE EXPRESSION AS

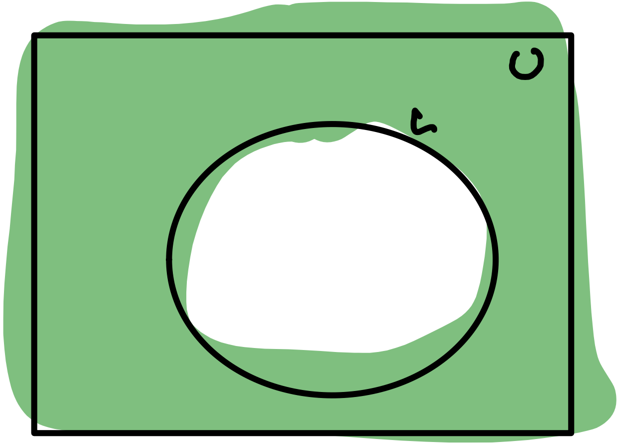
$P(5,3)$ OR $\frac{5!}{2!}$

Counting "moves":

- count by complement
- count by partition
- count by simplify

Count-by-complement

How many ways are there to order 5 people such that person A is not last?



L = SET OF ORDERINGS WHERE A IS LAST

U = ALL ORDERINGS OF 5 PEOPLE

$$|U - L| = |U| - |L|$$

$$= 5! - 4!$$

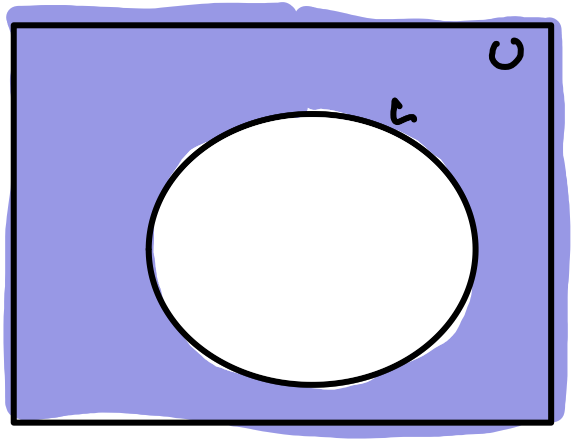
COUNT-BY-SYMMETRY

$$5! \cdot 4/5$$

Count-by-complement

How many ways are there to order 5 people such that person A is not last?

General approach: If we can count everything (U) and all items we're not interested in (L) then we can subtract the two to count the items of interest.



$$|U - L| = |U| - |L|$$

⚠ $|A - B| = |A| - |B|$ NOT TRUE IN GENERAL

Count-by-partition: motivating example

A

How many passwords can be made of lowercase letters which are no longer than 5 characters?

5 CHARS + 4 CHAR + 3 CHAR + 2 CHAR + 1 CHAR

$26^5 + 26^4 + 26^3 + 26^2 + 26^1 + 26^0$

$A_5, A_4, A_3, A_2, A_1, A_0$

Partition: definition

Intuition: partition of set A divides its items into groups so each item is in exactly one group

$A = \{1, 2, 3, 4\}$

$A_1 = \{2, 3\}$

$A_2 = \{1\}$

$A_3 = \{4\}$

Definition: partition of set A is a set of sets A_1, A_2, ... such that

$i \neq j \rightarrow A_i \cap A_j = \emptyset$

$A_1 \cup A_2 \cup A_3 \ldots \cup A_N = A$

EACH ITEM OF A IS IN

AT MOST ONE A_i

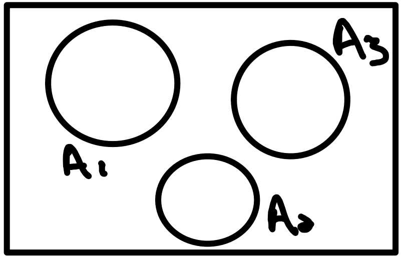
AT LEAST ONE A_i

Count-by-partition:

$$|A| = |A_1 \cup A_2 \cup A_3|$$

$$= |A_1| + |A_2| + |A_3|$$

A_i A_j ARE DISJOINT
(ALL INTERSECTIONS EMPTY)



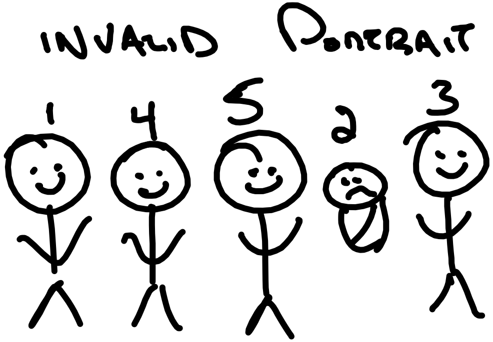
Approach:

Count items by partitioning them into subsets

(common error: ensure that every item is in exactly one subset)

Count-by-simplification:

How many ways can we order 5 family members for a portrait if person 2 is a baby and must be on person 1's immediate right?



Mea Culpa:

"Count-by-simplification" isn't really a particular approach like others …

point is, be on the lookout for equivilent problems more easily counted

In Class Activity

How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

How many ways are there to order 3 people in a wedding photo for romeo and juliet?
Assume:

- there are 10 Montague's (Romeo's family, excluding him) who could be in the photo
- there are 7 Capulets (Juliet's family, excluding her) who could be in the photo
- Romeo and Juliet are too busy dancing to be in any picture
- Montagues and Capulets won't get in the same photo (that whole Tybalt / Mercutio thing...)

(hint: partition, simplify a bit)

How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: partition, complement)

How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

ALL PASSWORD

26^{10}

PASSWORDS START QWERTY

26^{4}

$26^{10} - 26^{4}$

How many ways are there to order 3 people in a wedding photo for romeo and juliet?
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(hint: partition, simplify a bit)

ORDER 3 of 10 M

$$P(10,3) = \frac{10!}{(10-3)!} = 10 \cdot 9 \cdot 8$$

ORDER 3 of 7 C

$$P(7,3) = \frac{7!}{(7-3)!} = 7 \cdot 6 \cdot 5$$

$$P(10,3) + P(7,3)$$

How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: partition, complement)

CASE 1 PERSON 1 NOT IN PICTURE

$$P(6,5) = \frac{6!}{(6-5)!} = 6!$$

PERSON 1 IN PICTURE IN ANY WAY

1 _ _ _ _ $\quad P(6,4)$

_ 1 _ _ _ $\quad P(6,4)$

_ _ 1 _ _ $\quad P(6,4)$

_ _ _ 1 _ $\quad P(6,4)$

_ _ _ _ 1 $\quad P(6,4)$

$$5 \cdot P(6,4)$$

PERSON 1 IN PICTURE IMMEDIATE RIGHT OF 2

2 1 _ _ _ $P(5,3)$

_ 2 1 _ _

_ _ 2 1 _

_ _ _ 2 1

$$4 \cdot P(5,3)$$

$$\frac{6! + 5 \cdot P(6,4) - 4 \cdot P(5,3)}{}$$

A simpler way than what was shown in section3 classtime (thank you thank you thank you :) !)

$$\underbrace{6!}_{\text{Person 1 not included}} + \underbrace{5 \cdot P(6,4)}_{\text{Person 1 included}} = \underbrace{P(7,5)}_{\text{All ways of ordering 5 from 7 (Person 1 included or not)}}$$

matt@matt-yoga-nu: ~

```
matt@matt-yoga-nu:~$ python3
Python 3.10.12 (main, Jun 11 2023, 05:26:28) [GCC 11.4.0] on linux
Type "help", "copyright", "credits" or "license" for more information.
>>> from math import factorial, perm
>>> factorial(6) + 5 * perm(6, 4)
2520
>>> perm(7, 5)
2520
>>>
```