CS1800 day 3

Admin:
- hw1 released today (due the following friday, as nearly all HWs are)
- tutoring groups
- what to do if you can't access piazza (email Kayla and myself, we'll add you)

Content:
- Two's complement (system to represent negative binary numbers)
- Overflow
- Floating point (system to represent non-whole numbers) (if time)

Whats the difference between operating in base-b and operating in base-b on a computer?

Computers store all values with the same number of bits

why? quicker / easier

Assume: a computer is using a 3-bit representation of values. How does it compute & store the following?

$$(111)_2 + (001)_2 = (1000)_2$$

$$7 + 1 = 8$$

ONLY THESE THREE BITS CAN BE STORED

THIS IS DISCARDED

For today: assume we're working with values on a computer

- all values are N-digits
  (you'll be given this info in problem statement)

- discard the most significant (left-most) digits if needed
  (as shown in green on last slide)

Number Systems:

Currently we're missing:
- negative values (e.g. -43)
- non-whole values (e.g. 321.12358)

Number systems:

- Unsigned Integers:

  $(111)_2 = 7$

  can represent whole, non-negative numbers
  everything we've done so far are unsigned integers (we just didn't cover name until now)
  e.g. (110)_2 = 6

- Two's Complement:

  $(111)_2 = ?$

  can represent whole (potentialy negative) numbers
  (will study today)

- Floating Point Values:
  non whole-numbers
  (will study today if time)

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- Floating Point Values:
  - non whole-numbers
  - (will study today if time)

Sign bit*:

A not-so-great number system for negative values

3 BIT "SIGN BIT"

1 IF NEGATIVE

0 OTHERWISE

SIGN BIT	2	1
1	1	1

$(001)_2 = 1$

$(000)_2 = 0$

$(111)_2 = -3$

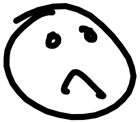
$(101)_2 = -1$

Sign Bit: Problems

No unique zero

$$(000)_2 = 0$$

$$(100)_2 = -0$$



Our operations yield incorrect results

Discard

$$(111)_2 + (001)_2 = (\cancel{1}000)_2$$

$$-3 + 1 = 0$$

Two's complement: A better way to store negative numbers

Big idea: the most significant (biggest) place value is negative, all others are positive

-4	2	1
1	1	1

$-4 + 2 + 1$

Example: 3-bit two's complement

100	-4
101	-3
110	-2
111	-1
000	0
001	1
010	2
011	3

$$\overset{3}{(011)_2} + \overset{1}{(001)_2} = (100)_2$$

$$-4$$

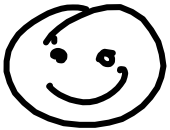
$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 011 \\ 001 \\ \hline (100) \end{array}$$

$(111)_2$

Two's complement, Problems Solved

Unique Zero

$(000)_2 = 0$

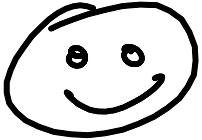


Our operations yield correct* results

Discard

$(111)_2 + (001)_2 = (\cancel{1}000)_2$

$-1 + 1 = 0$



* Assumes that correct result may be represented (more later)

In Class Activity:

If possible, convert each of the following values to the given number system. If not possible, justify why.

(Use guess-and-check as needed, a reliable decimal-to-2's-complement method coming shortly)

0	unsigned 2 bit integer
-2	unsigned 3 bit integer
0	3 bit 2's complement
-4	3 bit 2's complement
-4	4 bit 2's complement
5	4 bit 2's complement
10	4 bit 2's complement
-3	4 bit 2's complement

2	1
0	0

2 BIT USIGNED

-4	2	1
1	0	0

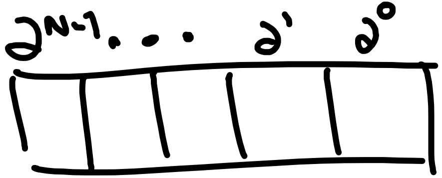
-8	4	2	1
1	1	0	1

$-8+4+0+1 = -3$

(++) What does the 2's complement idea look like in a base which isn't 2? Does it also have the properties we love so much in binary (unique zero, addition operations still work)?

What values can we represent with N bits?

Unsigned Integers



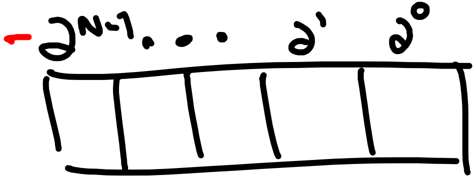
SMALLEST VALUE

0

LARGEST VALUE

$(111111\ldots11111)_2 = 2^N - 1$

Two's Complement



SMALLEST VALUE

$(1000000)_2 = -2^{N-1}$

LARGEST VALUE

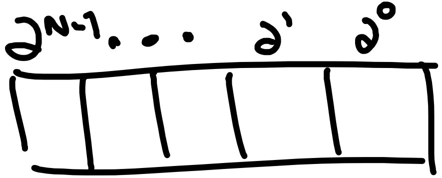
$(0111111)_2 = 2^{N-1} - 1$

$$\cdots + 16 + \underbrace{8 + \underbrace{4 + \underbrace{2 + \underbrace{1 + 1}_{2}}_{4}}_{8}}_{16}$$

ADD 1, DOUBLED LARGEST POWER

What values can we represent with N bits? (representability)

Unsigned Integers



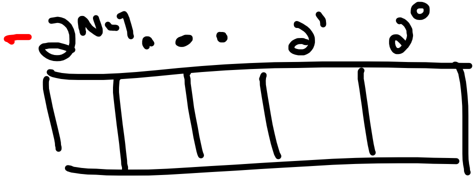
SMALLEST VALUE

0

LARGEST VALUE

$2^N - 1$

Two's Complement



SMALLEST VALUE

-2^{N-1}

$-2^{6-1} = -2^5 = -32$

LARGEST VALUE

$2^{N-1} - 1$

$2^{6-1} - 1 = 2^5 - 1 = 31$

We can represent all whole values from smallest to largest (including smallest & largest)
(we won't justify this)

OVERFLOW

Overflow: the outcome of an operation can't be represented in the given number system

example from earlier in lesson: $(111)_2 + (001)_2 = (\cancel{1}000)_2$

DISCARD

7 + 1 = 8 as 3 bit values

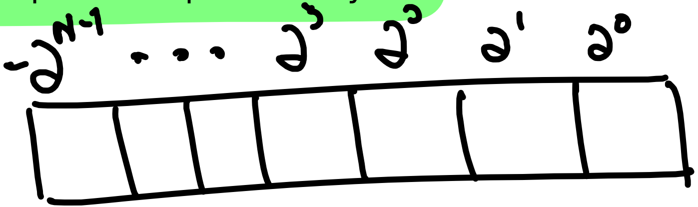
overflow since 8 can't be represented as a 3-bit value

Common misconception:

There are times when we discard a bit but result is correct (no overflow occurs)

punchline: don't conflate discarding the bit with overflow

Decimal to N-bit two's complement: preliminary

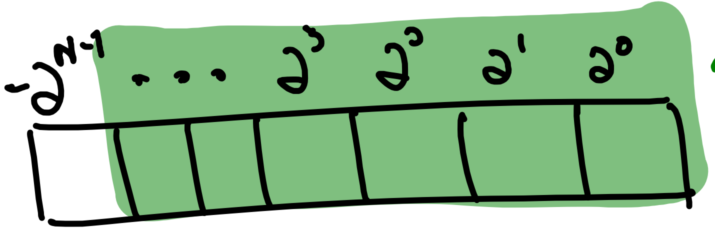


1. Validate that value can be represented as N-bit two's complement (see "representability")

2. If value is positive, its the same as N bit unsigned integer
   methods:
   - subtract largest power of two
   - Euclid's Division Algorithm

3. If value is negative: see "x" method on next slide

Decimal to N-bit two's complement: "x" method for negative representable values



DEFINE X AS VALUE OF LAST N-1 BITS (UNSIGNED)

A. Solve for X
B. Represent X as N-1 bit unsigned int
C. Append a leading 1 to indicate the -2^{N-1}

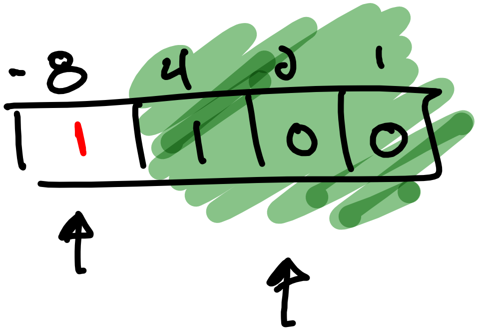
$$\text{VALUE} = -2^{N-1} + X$$

SO THAT

$$X = \text{VALUE} + 2^{N-1}$$

"X" Method Example

Express -4 as 4 bit two's complement



$$-8 + \boxed{x} = -4$$

$$x = 4$$

In Class Activity 2

-32 up to 31

If possible, express each of the following as a 6 bit two's complement value. Use the "x" method where possible.

-5
5
32

-32	16	8	4	2	1
1	1	1	0	1	1

$-32 + x = -5$

$x = 27$

$27 = 13 \cdot 2 + 1$

$13 = 6 \cdot 2 + 1$

$6 = 3 \cdot 2 + 0$

$3 = 1 \cdot 2 + 1$

$1 = 0 \cdot 2 + 1$

(floating point if time)

Floating Point: Representing non-whole values

To express 12.345, rewrite it as:

$$12.345 = \underbrace{12345}_{\text{significand}} \times \underbrace{10}_{\text{base}}^{\overbrace{-3}^{\text{exponent}}}$$

big idea: the signifcand and exponent will always be whole values and we can store those!

A few notes about the "base"

- isn't the same base the number system for significand & exponent number system
  - (you can use base 10, as shown, and still store significand & exponent in binary)
- no need to store floating point base per individual value

img credit: wikipedia