CS1800

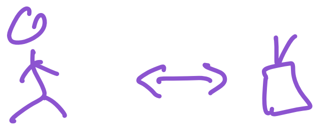
10/31 - Tues.

Admin

- HW5 due Fri
- HW6 out Fri
- back to recitations this week

Agenda

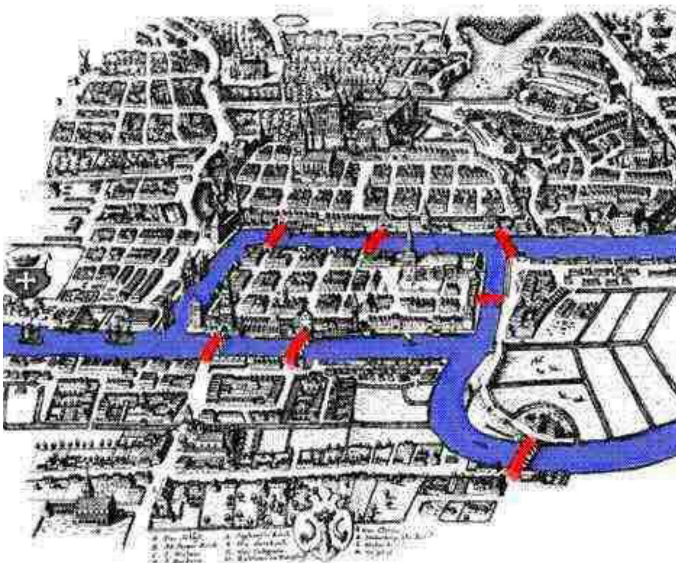
1. Graph Overview
2. Graph representation
3. Graph equality



1. Graph Overview

Graph origin story → Euler - Kronigsberg

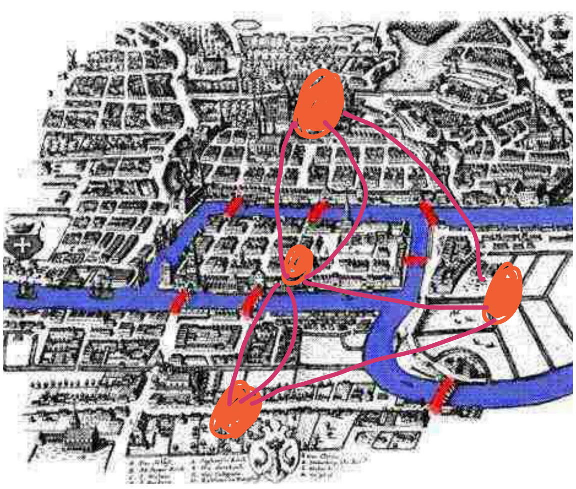
- go to pub in every neighborhood
- cross every bridge ~~only~~ exactly once
- 4 neighborhoods
- 7 bridges

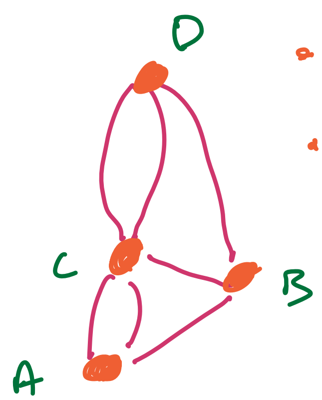


↳ map ... graph (new concept!)

So he could solve the problem
(wants to be mathematicize)

- neighborhood == $\boxed{\text{vertex}}$ 
- bridge == $\boxed{\text{edge}}$ 





- 4 vertices
- 7 edges

goal: visit every vertex
traverse every edge exactly once

- need edge to enter a visit
- need another one to leave

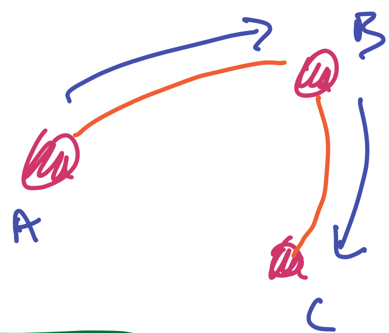
3 edges incident on a vertex
(A, B, D)

A: leave, come, leave

B, D: come, leave, come + $\boxed{\text{now stuck}}$!

Insight:

- vertex has odd # incident edges
- we can start there, or end there
- <u>but</u>, we need one vertex to start, and one to end
- for problem to be solved, need <u>two</u> vertices with odd # of edges



Every map can be repped as a graph!

- directions
- google maps

Also used for...

- repping relationships (linkedin, insta)
- rec. systems
- dependencies (OS, scheduling)

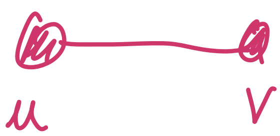
Vocab words

$G = (V, E)$

V → vertices

E → edges

edge (u, v) vertex u and vertex v are connected

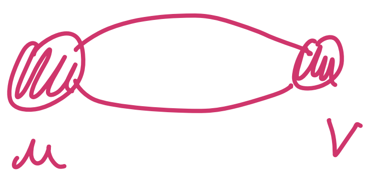


u is adjacent to v

v is adjacent to u

u, v are neighbors

a vertex has incident edges



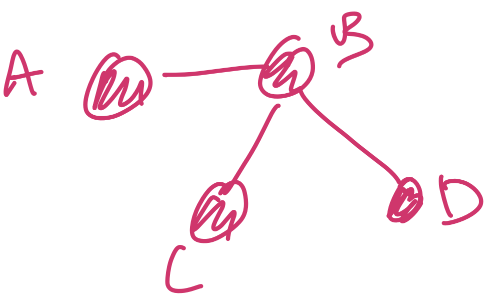
incident edges == "degree" of a vertex

$deg(u) = 2$

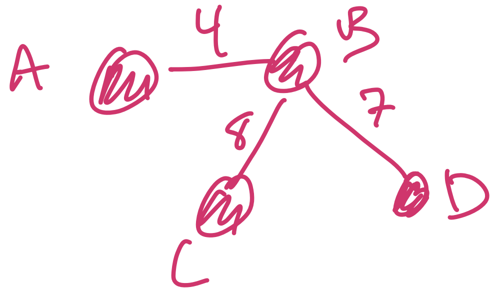
multi edge: two or more edges connecting the same vertices

graph can be: directed, undirected

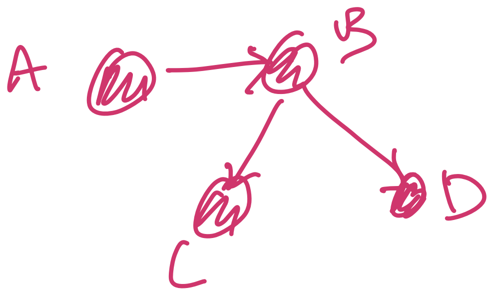
weighted, unweighted



undirected, unweighted



undirected, weighted



(A, B) but not (B, A)

B is adj to A

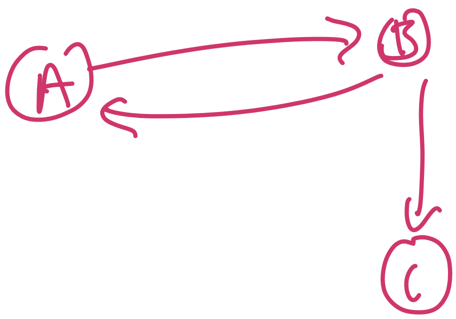
A is not adj to B

directed, unweighted

path: list of vertices, successive edges

A, B, C valid

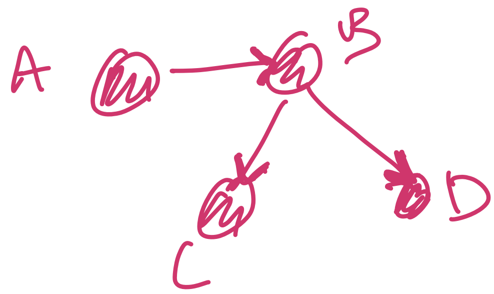
A, C, B invalid



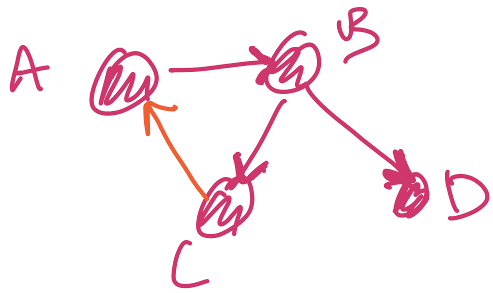
(2 way + one way streets)

<u>not</u> multi-edge

cycle: path that begins/ends at same vertex



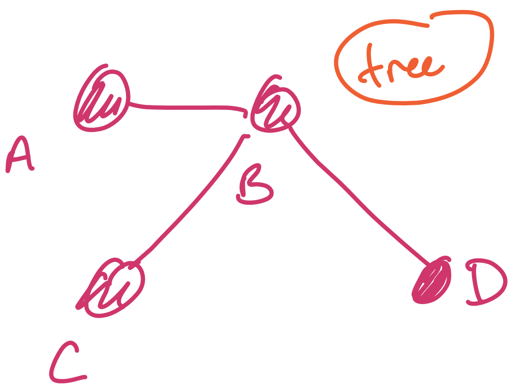
no cycle



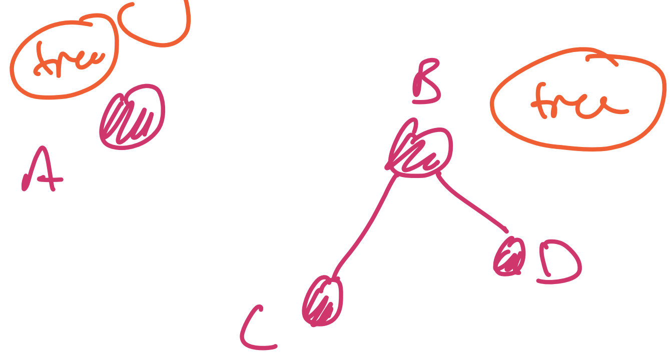
cycle! A, B, C, A

Strongly connected:

path from every vertex to every other vertex



strongly connected



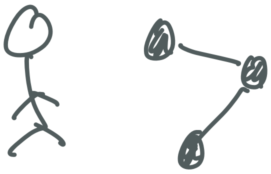
not strongly connected
(A by itself is a valid graph)

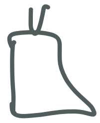
tree: no cycle
↳ type of graph

$\boxed{\text{Simple graph}}$

- undirected
- unweighted
- no multi edges
- no self edges

2. Graph Representation





???

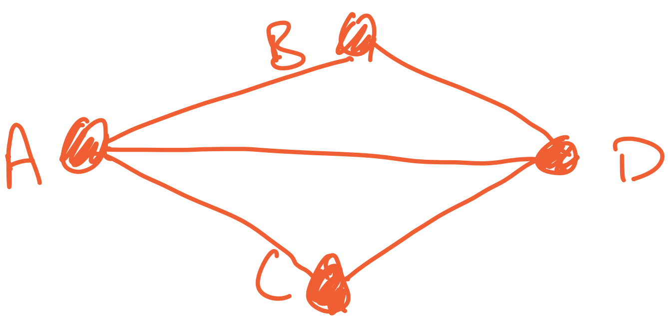
1. Adjacency list
2. Adjacency matrix

1. Adj list

- every vertex
- each has a list of neighbors

ex undirected, unweighted

A: B, C, D ✓
B: A, D ✓
C: A, D ✓
D: A, B, C ✓

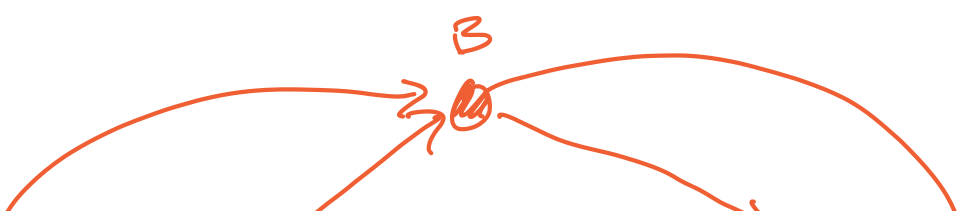


2. Adj matrix

- 2d table
- 1 if edge (u, v)
- 0 otherwise

ex directed, unweighted

	A	B	C	D
A	0	1	1	1
B	0	0	1	1
C	0	0	0	1
D	1	1	0	0



- which do we use?
- Different reps for different problems!

List vs. **Matrix**

List

- add new vertex
- (undirected) less space, especially if graph is sparse
- given a vertex, who are neighbors?

Matrix

- does edge (u, v) exist?
- easy to count edges
- easy to add weights

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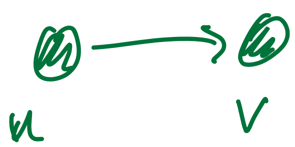


edge u, v

u is not adj to v

v is adj to u

u has neighbor v

v has no neighbors

3. graph equality

- graphs don't need to look the same to be the same
- structural equality

"Isomorphism"

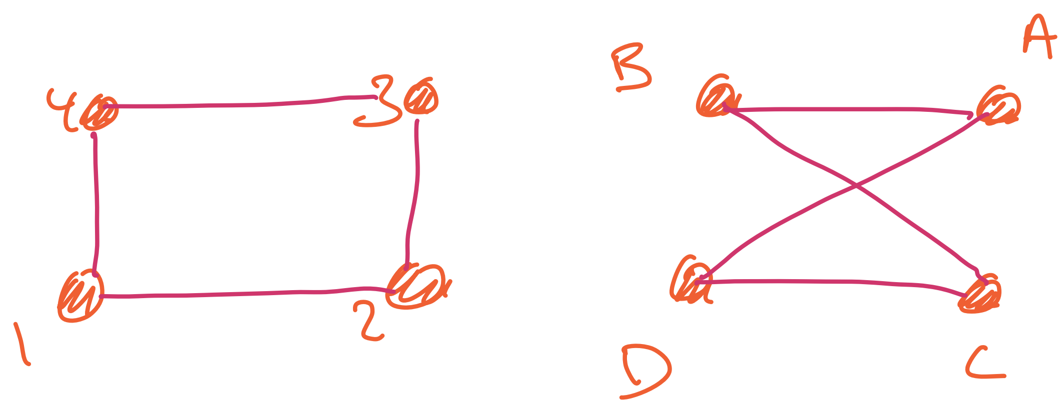
- 2 graphs
- might look diff
- might have diff vertex labels

- they are isomorphic $\boxed{\text{if}}$

$\boxed{\text{1-1 correspondence between vertices that preserves adjacency}}$

Given 2 graphs, are they isomorphic?
↳ if so, what is the mapping?

ex



→ ??

Adj list:

D : C, A
1 : 2, 4

B : 1, A
3 : 2, 4

C : B, D
2 : 3, 1

A : D, B
4 : 1, 3

1 → D
2 → C

1 : 4, 2
D : A, C

4 : 1, 3
A : D, B

3 → B

4 → A

1-1 mapping preserves adjacency

$\overset{3}{B}$: A, C

$\overset{2}{C}$: $\overset{3}{B}$, $\overset{1}{D}$

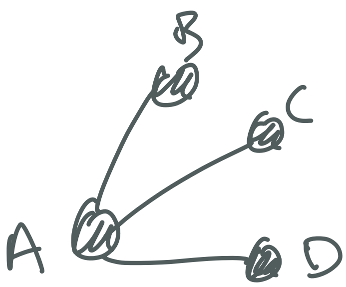
Sometimes helpful... $\boxed{\text{degree sequence}}$

Left: (2, 2, 2, 2) every vertex's degree

If 2 graphs are isomorphic, then they have the same degree sequence

(helpful starting point!)

ex



deg seq: (3, 1, 1, 1)

↳ descending order

A: B C D

B: A

C: A

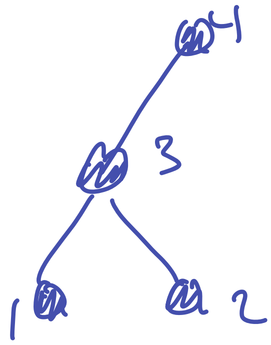
D: A

Any isomorphic graph would have the same degree sequence

Are 2 graphs isomorphic?

- If degree seqs are different, $\boxed{\text{no!}}$

- If degree seqs are the same, $\boxed{\text{maybe!}}$



Start w/ degrees!

(3, 1, 1, 1)

A - 3

B - 1

C - 2

D - 4

Same!

3: 1, 2, 4

4: 3

1: 3

2: 3

A: B C D

D: A

B: A

C: A