CS1800 day 3

Admin:
- hw1 released today (due the following friday, as nearly all HWs are)
- tutoring groups
- what to do if you can't access piazza (email Kayla & myself please)

Content:
- Two's complement (system to represent negative binary numbers)
- Overflow
- Floating point (system to represent non-whole numbers) (if time)

Whats the difference between operating in base-b and operating in base-b on a computer?

Computers store all values with the same number of bits

why? quicker / easier

Assume: a computer is using a 3-bit representation of values. How does it compute & store the following?

$$(111)_2 + (001)_2 = (1000)_2$$

$$7 + 1 = 8$$

ONLY THESE THREE BITS CAN BE STORED

THIS IS DISCARDED

For today: assume we're working with values on a computer

- all values are N-digits
  (you'll be given this info in problem statement)

- discard the most significant (left-most) digits if needed
  (as shown in green on last slide)

Number Systems:

Currently we're missing:

- negative values (e.g. -43)
- non-whole values (e.g. 321.12358)

Number systems:

- Unsigned Integers:
  - can represent whole, non-negative numbers
  - everything we've done so far are unsigned integers (we just didn't cover name until now)
    - e.g. $(110)_2 = 6$

- Two's Complement:
  - can represent whole (potentialy negative) numbers
  - (will study today)

- Floating Point Values:
  - non whole-numbers
  - (will study today if time)

Number Systems:

Currently we're missing:
- negative values (e.g. -43)
- non-whole values (e.g. 321.12358)

$4+2+1$

Number systems:

- Unsigned Integers:

  can represent whole, non-negative numbers

  everything we've done so far are unsigned integers (we just didn't cover name until now)

  e.g. (110)_2 = 6

$(111)_2 = 7$

- Two's Complement:

  can represent whole (potentialy negative) numbers

  (will study today)

$(111)_2 = ?$

- Floating Point Values:

  non whole-numbers

  (will study today if time)

1.2345

Sign bit*:

A not-so-great number system for negative values

3 BIT "SIGN BIT"

1 IF NEGATIVE

0 OTHERWISE

SIGN BIT

SIGN BIT	2	1
1	0	0

$-(0+0) = -0$

$(001)_2 = 1$

$(000)_2 = 0$

$(111)_2 = -3$

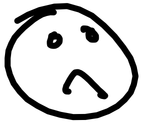
$(101)_2 = -1$

Sign Bit: Problems

No unique zero

$(000)_2 = 0$

$(100)_2 = -0$

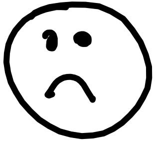


Our operations yield incorrect results

Discard

$(111)_2 + (001)_2 = (\cancel{1}000)_2$

$-3 + 1 = 0$



Two's complement: A better way to store negative numbers

Big idea: the most significant (biggest) place value is negative, all others are positive

-4	2	1
1	1	1

$$-4 + 2 + 1 = -1$$

Example: 3-bit two's complement

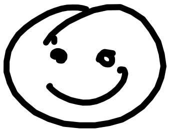
100	-4
101	-3
110	-2
111	-1
000	0
001	1
010	2
011	3

Two's complement is equivilent to unsigned
for non-negative values

Two's complement, problems solved

Unique zero

$$(000)_2 = 0$$

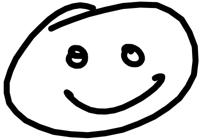


Our operations yield correct* results

Discard

$$(111)_2 + (001)_2 = (\cancel{1}000)_2$$

$$-1 + 1 = 0$$



* Assumes that correct result may be represented (more later)

$(111)_2$

In Class Activity:

If possible, convert each of the following values to the given number system. If not possible, justify why.

(Use guess-and-check as needed, a reliable decimal-to-2's-complement method coming shortly)

0	unsigned 2 bit integer
-2	unsigned 3 bit integer
0	3 bit 2's complement
-4	3 bit 2's complement
-4	4 bit 2's complement
5	4 bit 2's complement
10	4 bit 2's complement
-3	4 bit 2's complement

2	1
0	0

$(000)_2$

-4	2	1
1	0	0

$1 \cdot -4 + 0 \cdot 2 + 0 \cdot 1 = -4$

$(100)_2$

-8	4	2	1
1	1	0	0

$(1100)_2$

(++) What does the 2's complement idea look like in a base which isn't 2? Does it also have the properties we love so much in binary (unique zero, addition operations still work)?

4 Bit Two's Comp

-8	4	2	1
1	1	0	1

5 $(0101)_2$

10

-3 $(1101)_2$

$(0111)_2 = 7$ is biggest 4 bit 2's comp

What values can we represent with N bits?

Unsigned Integers

2^{N-1}	...	...	2^1	2^0
0	0	0	0	0

SMALLEST VALUE

$$(0000000)_2 = 0$$

LARGEST VALUE

$$(111111111111)_2 = 2^N - 1$$

Two's Complement

-2^{N-1}	...	...	2^1	2^0
0	1	1	1	1

SMALLEST VALUE

$$-2^{N-1}$$

LARGEST VALUE

$$(0111111111)_2 = 2^{N-1} - 1$$

$$2^{N-1} + \dots + 16 + 8 + 4 + 2 + 1 + 1$$

$$= 2 \cdot 2^{N-1} = 2^N$$

What values can we represent with N bits? (representability)

Unsigned Integers

$2^{N-1} \; \ldots \; 2^1 \; 2^0$

SMALLEST VALUE

0

LARGEST VALUE

$2^N - 1$

Two's Complement

$-2^{N-1} \; \ldots \; 2^1 \; 2^0$

4 BIT 2's comp

SMALLEST VALUE

-2^{N-1}

$-2^{4-1} = -8$

LARGEST VALUE

$2^{N-1} - 1$

$2^{4-1} - 1 = 7$

We can represent all whole values from smallest to largest (including smallest & largest)
(we won't justify this)

OVERFLOW

$(111)_2 = 7$

Overflow: the outcome of an operation can't be represented in the given number system

example from earlier in lesson: $(111)_2 + (001)_2 = (\cancel{1}000)_2$

DISCARD

7 + 1 = 8 as 3 bit values

$7 + 1 = 8$ UNSIGNED

overflow since 8 can't be represented as a 3-bit value

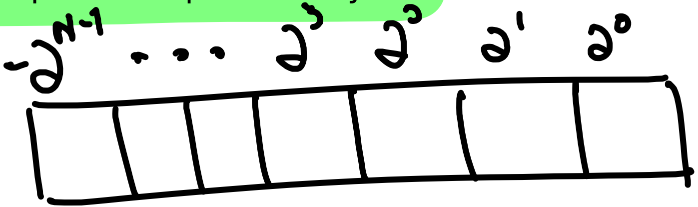
$-1 + 1 = 0$ 2's COMP

Common misconception:

There are times when we discard a bit but result is correct (no overflow occurs)

punchline: don't conflate discarding the bit with overflow

Decimal to N-bit two's complement: preliminary

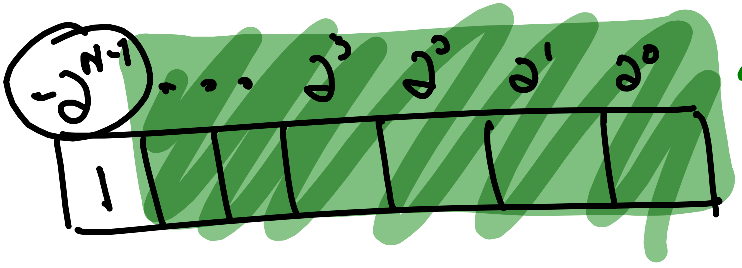


1. Validate that value can be represented as N-bit two's complement (see "representability")

2. If value is non-negative, its the same as N bit unsigned integer
   methods:
   - subtract largest power of two
   - Euclid's Division Algorithm

3. If value is negative: see "x" method on next slide

Decimal to N-bit two's complement: "x" method for negative representable values



DEFINE X AS VALUE OF LAST N-1 BITS (UNSIGNED)

A. Solve for X
B. Represent X as N-1 bit unsigned int
C. Append a leading 1 to indicate the -2^{N-1}

$$\text{VALUE} = -2^{N-1} + X$$

SO THAT

$$X = \text{VALUE} + 2^{N-1}$$

"X" Method Example

Express -4 as 4 bit two's complement

-8	4	2	1
1	1	0	0

$$-4 = -8 + x$$

$$4 = x$$

In Class Activity 2

Smallest $-2^{N-1} = -2^{6-1} = -32$

Largest $2^{N-1} - 1 = 2^{6-1} - 1 = 31$

If possible, express each of the following as a 6 bit two's complement value. Use the "x" method where possible.

-5
5
32

-32	16	8	4	2	1
1	1	1	0	1	1

1. Representable?
2. If non-neg, use unsigned (its same)
3. $-2^{N-1} + X = \text{value}$

$$-32 + X = -5$$

$$X = 27$$

$$= 16 + 8 + 2 + 1$$

$(000101)_2$

$(111011)_2 = -5$

(floating point if time)

Floating Point: Representing non-whole values

To express 12.345, rewrite it as:

$$12.345 = \underbrace{12345}_{\text{significand}} \times \underbrace{10}_{\text{base}}^{\overbrace{-3}^{\text{exponent}}}$$

(AKA MANTISSA)

big idea: the signifcand and exponent will always be whole values and we can store those!

A few notes about the "base"

- isn't the same base the number system for significand & exponent number system
    - (you can use base 10, as shown, and still store significand & exponent in binary)
- no need to store floating point base per individual value

img credit: wikipedia

192.123

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1	9	2	1	2	3

A.BC

16^0	16^{-1}	16^{-2}

$$10 + \frac{11}{16} + \frac{12}{16^2}$$

$$192 = 1 \cdot 100 + 9 \cdot 10 + 2 \cdot 1$$

10^2	10^1	10^0
1	9	2

-8	4	2	1
-2^3	2^2	2^1	2^0
0	1	1	0

$$0 + 4 + 2 + 0 = 6 = (0110)_2$$