CS1800
9/26 - Tues.

Admin

- HW2 due Fri 11:59
- Rec 3 ~ Quiz due 10/2 9pm
  Solutions posted Fri (9/29)
- Live Q+A on Piazza (for questions in lec)

Agenda

1. Circuits
2. Sets
3. Set Operations

0. negation of logic statements

$\Rightarrow$ Shortcut

we can do everything with $\wedge$ $\vee$ $\neg$

$P \Rightarrow Q \equiv \neg P \vee Q$ by def.

$\neg(P \Rightarrow Q)$???

$\neg(\neg P \vee Q) \equiv \neg\neg P \wedge \neg Q$

$P \wedge \neg Q$

1. Circuits

- Logic is great on its own! :)

Systems ———————— theory

build computers ~~~~~~~~ predicates
reason
prove things

- Logic: True / False
  ⊘
  1 / 0
  cat / dog

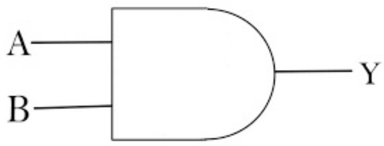
  extension

- Systems
  on / off
  in a transistor

  Solve problems!

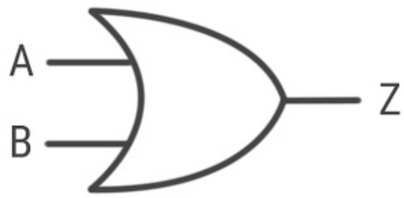
- Operators → gates
- input → 1 / 0
  on / off on a transistor
- Compound logic statement → circuit

and



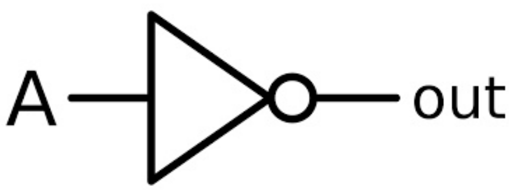
A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

or



A	B	$A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1

not

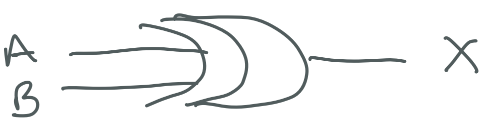


A	$\neg A$
0	1
1	0

all you need!

convenience

xor

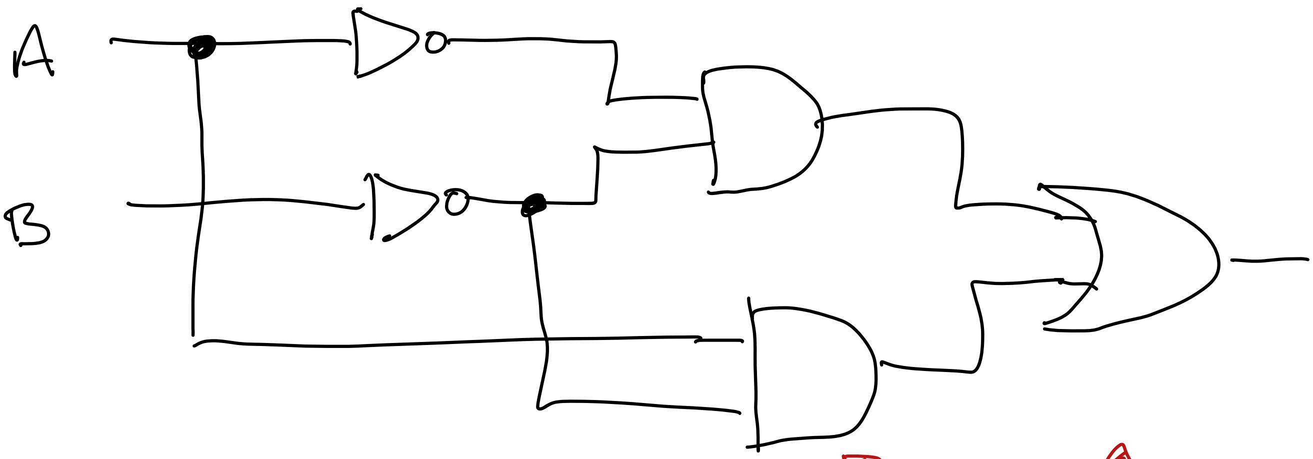


(one or the other, not both)

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

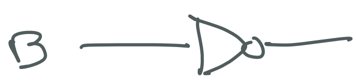
circuit $\Rightarrow$ truth table

(left to right) (one step at a time)



A	B	¬A	¬B	¬A ∧ ¬B ■	A ∧ ¬B ▲	■ ∨ ▲
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

Logic Statement: $(\neg A \wedge \neg B) \vee (A \wedge \neg B) \equiv \neg B$

$\neg B$ 

negate one input

(why we simplify to use as few operators as possible!)

2. Sets

- A set is a discrete structure
- unordered } elements
- distinct

$$S = \{2, 4, 6, 8\}$$

cap → curly braces

$$T = \{2, \text{dog}, \ldots, \text{Snickers}\}$$

(elements are usually connected but don't have to be)

$\in$ "is an element of"

$\boxed{\text{Snickers} \in T}$

- declarative
- truth value

} logic statement!

$\notin$ "is not an element of"

$\boxed{\text{milky way} \notin T}$ → logic statement!

$\subseteq$ "is a subset of"

$\{2, 6\} \subseteq S$ $\{2\} \subseteq S$ $S \subseteq S$

$\{8, 2\} \subseteq S$ $\{\} \subseteq S$

- every set is a subset of itself

- $\{\}$ is a subset of everything

↑ logic statements!

$\subset$ "is a proper subset of"

$\{4, 6, 8\} \subset S$ True

$\{2, 4, 6, 8\} \subset S$ False

$\{\} \subset S$

Representing Sets

- roster - list out all ~~even~~ elements

  $S = \{2, 4, 6, 8\}$

- roster with pattern

  $S = \{2, 4, 6, 8, \ldots, 100\}$

- set builder

  describe arbitrary element of the set, x

  $A = \{x \mid 1 \leq x \leq 100\}$

  $\mid$ such that

  logic!

Need to specify the ~~uni~~ universe!

logic!

$A = \{x \mid \boxed{x \in \mathbb{N}} \wedge \boxed{1 \leq x \leq 100}\}$

↳ natural numbers

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}^+ = \{1, 2, 3, 4, \ldots\}$

10:56

universal set: $\mathcal{U}$ (another way of defining the universe)

$\mathcal{U} = \{x \in \mathbb{Z} \mid 0 < x < 100\}$

$B = \{x \mid 2x \in \mathcal{U} \wedge x \div 2 \in \mathbb{Z}\}$

↳ no .5

num	in set B?		
18	yes		
7	no	$x \div 2 \in \mathbb{Z}$	F
52	no	$2x \in \mathcal{U}$	F
2	yes		
0	no	$2x \in \mathcal{U}$	F

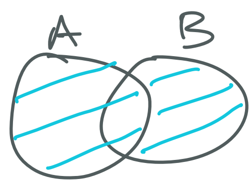
3. Set Operations

↳ just like logical operators

input: set(s)
output: set(s)

union (or) $\cup$

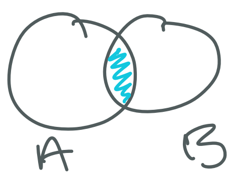
$$A \cup B = \{x \mid x \in A \lor x \in B\}$$



(venn diagram)

intersection (and) $\cap$

$$A \cap B = \{x \mid x \in A \land x \in B\}$$



Complement (not) $\bar{A}$ A^c

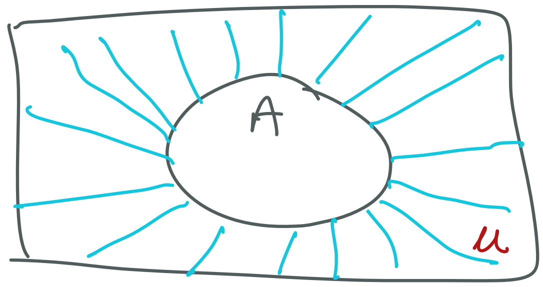
$$\bar{A} = \{x \mid x \notin A\}$$

(need to know universal set)

$A = \{2, 4, 6, 8\}$

w/o universe ... → things in $\bar{A}$

$\mathcal{U} = \{1, 2, 3, \ldots, 10\}$ $\bar{A} = \{1, 3, 5, 7, 9, 10\}$

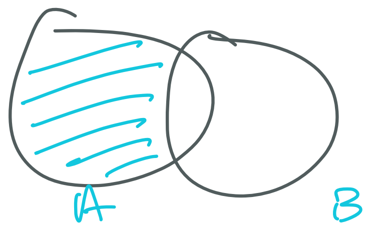


$\overline{A}$ in universal set but not in A

all we need!

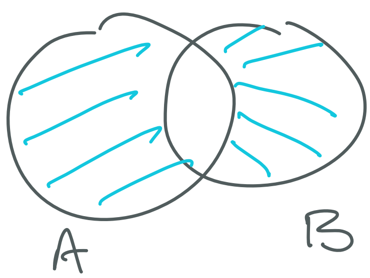
Helpful shortcuts! ↓

Difference: $A - B = \{x \mid x \in A \wedge x \notin B\}$



$A - B \; \ldots \; A \cap \overline{B}$

Symmetric Difference: $A \,\Delta\, B$



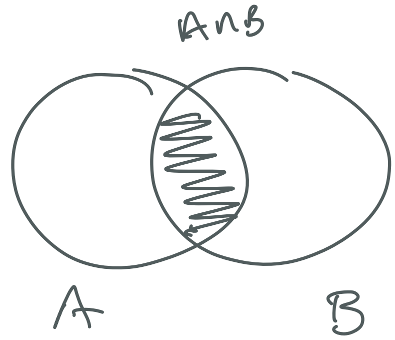
In A or B but not both

(like XOR! $\oplus$

Set operation $(A \cap B) - B$

Draw the Venn Diagram

(can we simplify?)



$(A \cap B) - B = \{\ \}$

$\emptyset$

$\{\ \}$ empty set

$\emptyset$ empty set

$\{\emptyset\}$ a set that contains the empty set

E	S	P	$\neg E$	$E \wedge P$
T	T	T		
T	T	F		
T	F	T		
T	F	F		

⋮