CS1800 Day 6

Admin:
- recitation solutions now available Friday (instead of immediately)

Content:

- Sets (subsets, empty set, powerset)
- Set Builder Notation
- Set Operations (Union, Intersection, Complement, Difference)

Sets

A set is a collection of unique objects

{a, b, c} = $\{a, b, c\}$ MY CURLY BRACES ARE NOT GREAT... SORRY!

$\{1, 2, 3, 4\} = \{1, 2, 3, 4, 4\}$ POOR FORM

AN ITEM IS IN SET OR NOT, NO ITEM IS IN SET MORE THAN ONCE

Example number sets you should be aware of:

Empty set

$\emptyset = \{\ \}$

SET W/ NO ITEMS

Integers

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Z}$

Natural Numbers

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

SOMETIMES NOT INCLUDED

$\mathbb{N}$

Real Numbers

$\mathbb{R}$ CONTAINS

$-2, 0, 1\frac{1}{2}, \pi, e$

Set Builder Notation: one way to express a set

SET INCLUSION

$$A = \{\, x \in \mathbb{N} \mid (3 \le x) \wedge (x \le 5) \,\}$$

A IS THE SET OF x IN NATURAL NUMBERS SUCH THAT <SOME CONDITION>

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, \ldots\}$$

$$A = \{3, 4, 5\}$$

In Class Activity: Set Builder Practice

$|x|$ ABSOLUTE VALUE (DISTANCE FROM 0)

Express the set A by explicitly listing all items it contains

$A = \{ x \in \mathbb{Z} \mid |x| < 5 \}$

$|3| = 3$ $\quad |-123| = 123$

$A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Express the set B using set builder notation

B = set of all natural numbers x which have x mod 3 = 0 and x mod 7 =0 and x < 40

INCLUDE 0

(++ list all of its items)

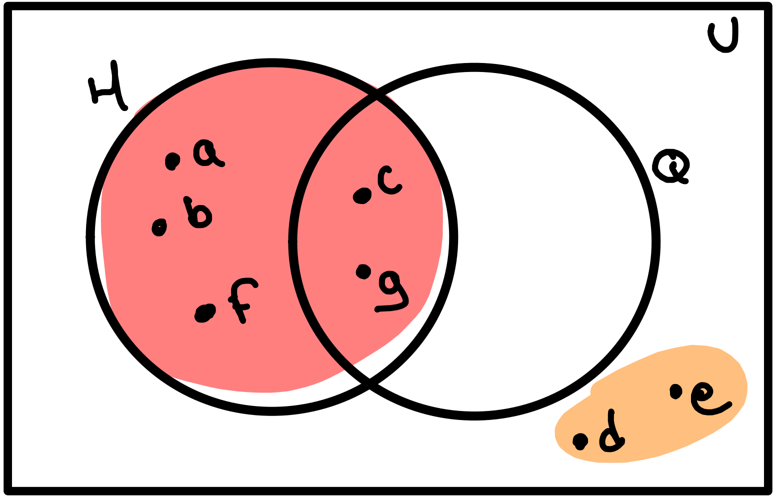
$B = \{ x \in \mathbb{N} \mid (x \bmod 3 = 0) \land (x \bmod 7 = 0) \land x < 40 \}$

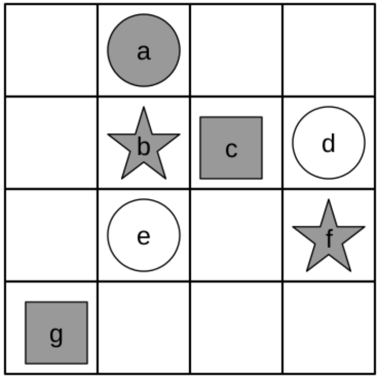
$$B = \{ x \in \mathbb{N} \mid (x \bmod 3 = 0) \land (x \bmod 7 = 0) \land x < 40 \}$$

$$\{ x \in \mathbb{N} \land (x < 40) \mid x \bmod 3 = 0 \land x \bmod 7 = 0 \}$$

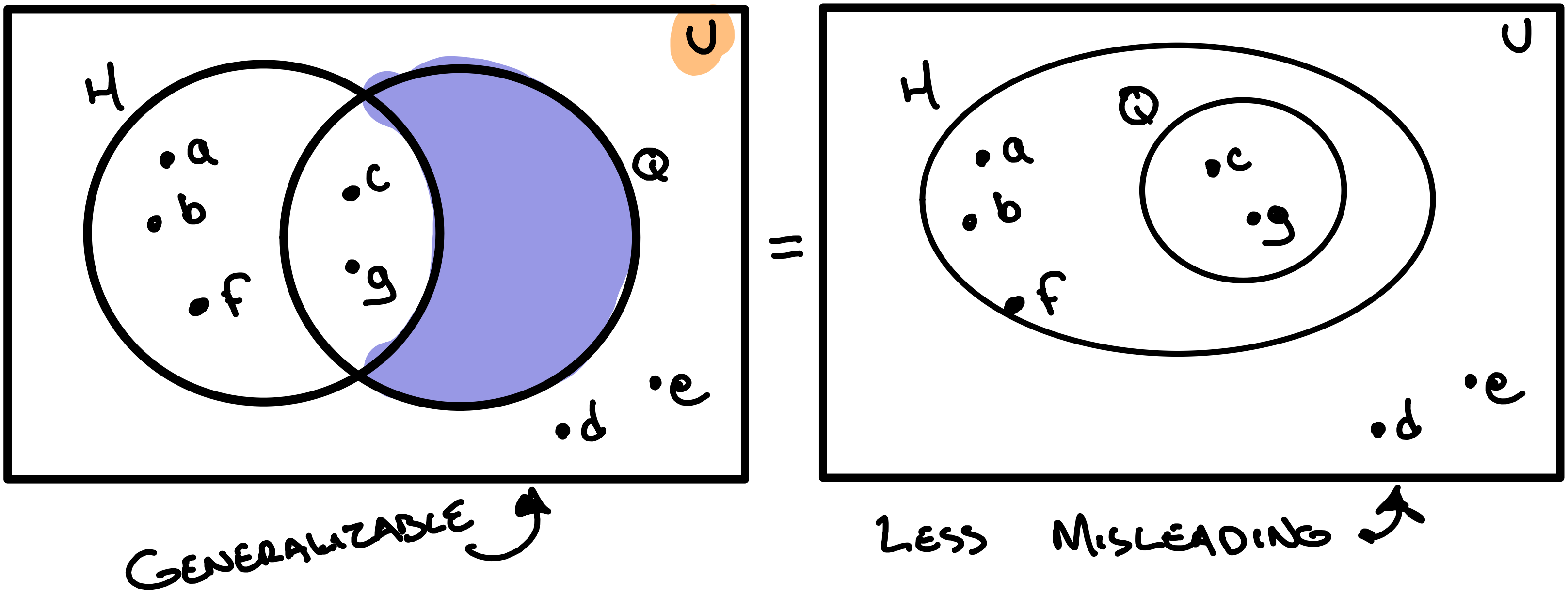
Venn Diagram: a way of visually representing set membership

H = set of all sHaded shapes
Q = set of all sQuares
U = Universal set, contains all shapes

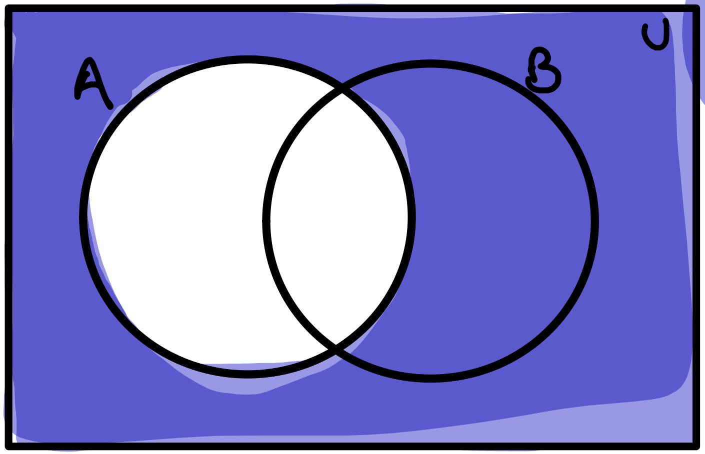




Venn Diagram Gotcha: Just because an area exists, doesn't mean it contains any items (may be empty)



Set Operation: Complement (all the items NOT in some set)



TWO NOTATIONS FOR SAME THING

$$\overline{A} = A^c = \{x \in U \mid x \notin A\}$$

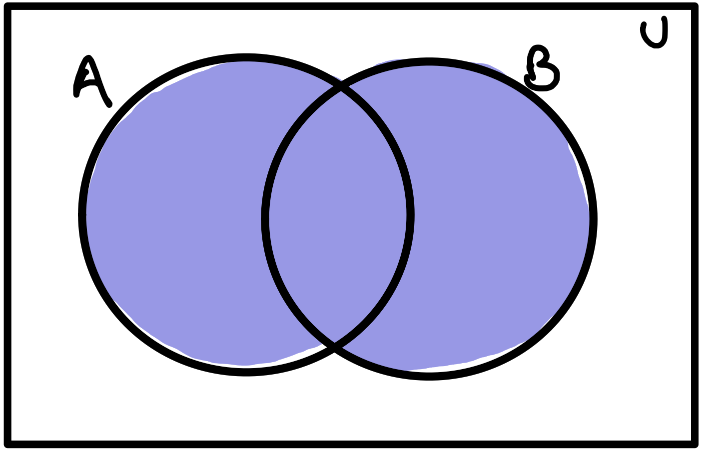
ALL x IN UNIVERSE

SUCH THAT

X IS NOT IN A

Set Operation: Union

(all the items in one set OR another)



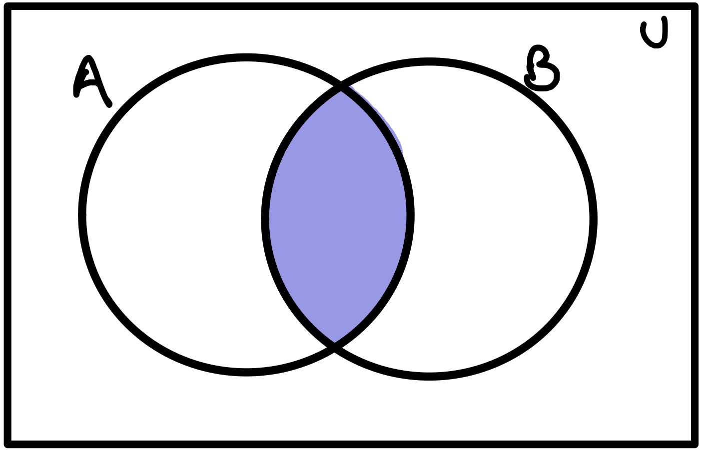
$A \cup B = \{x \in U \mid x \in A \vee x \in B\}$

ALL x IN UNIVERSE SUCH THAT

x IS IN A OR x IS IN B

Set Operation: Intersection

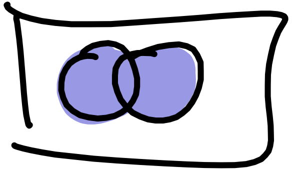
(all the items in one set AND another)



$A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$

ALL x IN UNIVERSE SUCH THAT

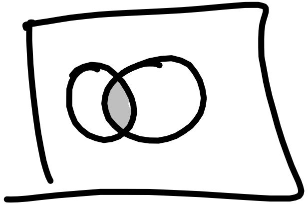
x IS IN A AND x IS IN B



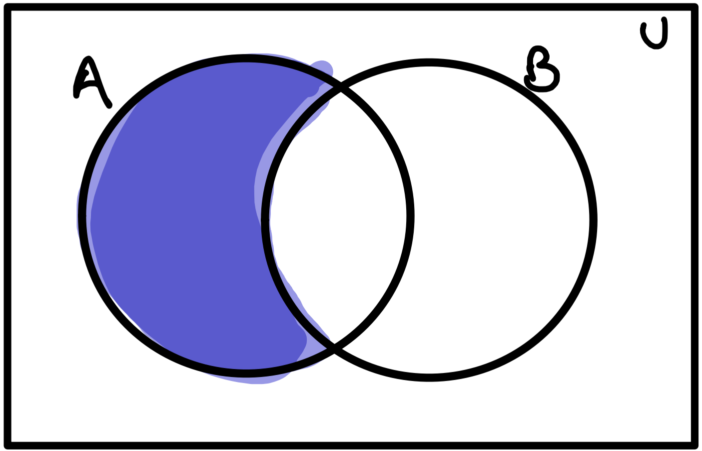
Tip

$\cup$ Union OR Bigger

$\cap$ Intersection AND Smaller



Set Operation: Difference (All items in one set but not another)

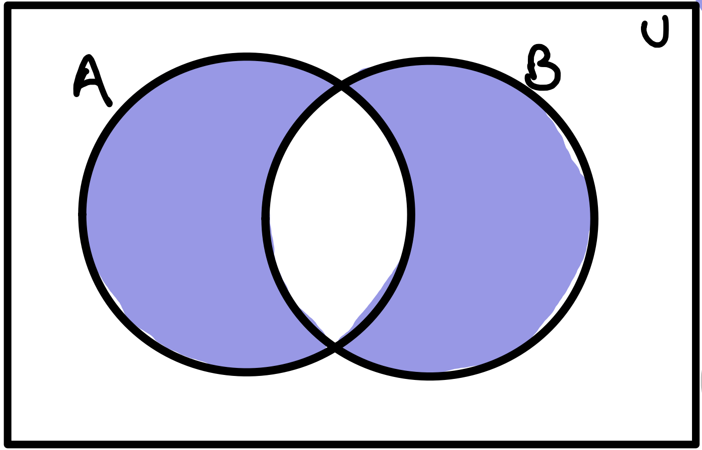


$$A - B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN A AND X IS NOT IN B

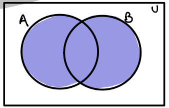
Set Operation: Symmetric Difference (All items in one set XOR another)
(All items in one set or the other, but not both)

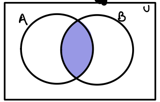


$$A \Delta B = \{x \in U \mid x \in (A \cup B) \land x \notin (A \cap B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN $A \cup B$ AND X NOT IN $A \cap B$

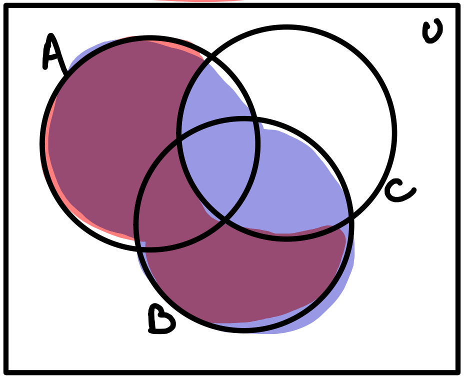




In Class Activity

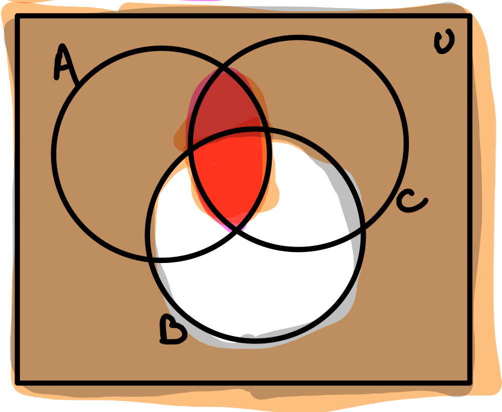
Shade the indicated areas in each venn diagram

$(A \cup B) - C$

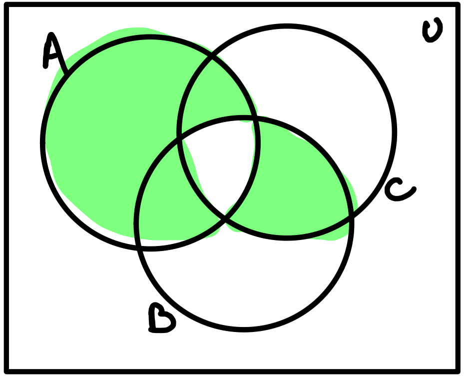


(this one isn't finished, blue area corresponds to blue expression)

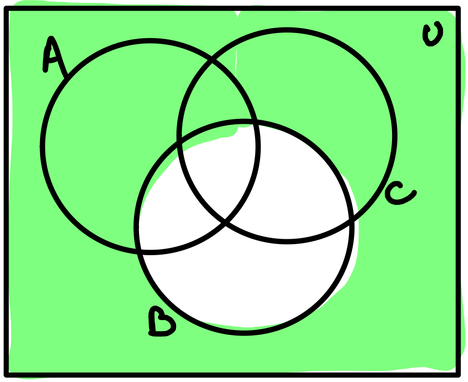
$(A \cap C) \cup B^c$



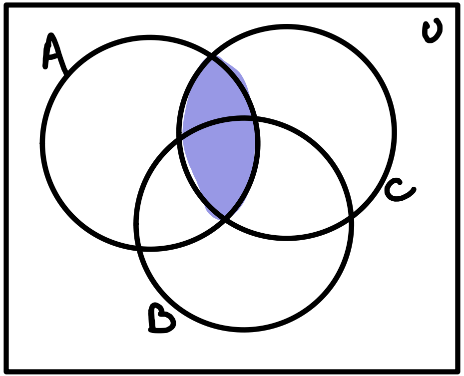
$A \,\Delta\, (B \cap C)$



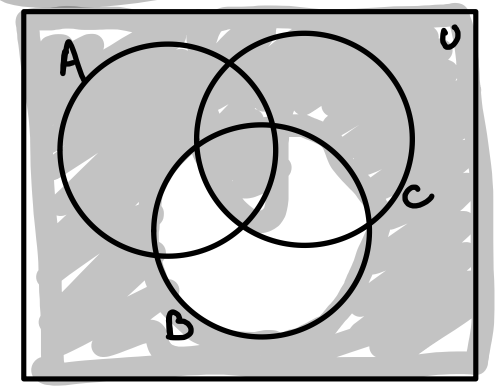
$(A \cap C) \cup B^c$



$(A \cap C) \cup B^c$

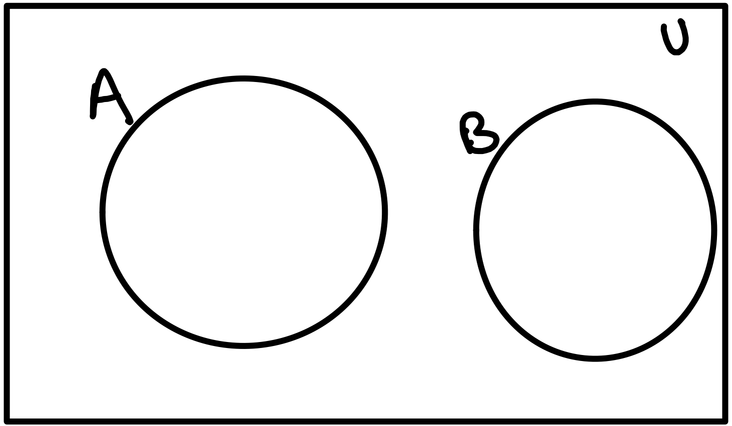


$(A \cap C) \cup B^c$



Set Terminology: Disjoint Sets (two sets are disjoint if no item is in both sets)

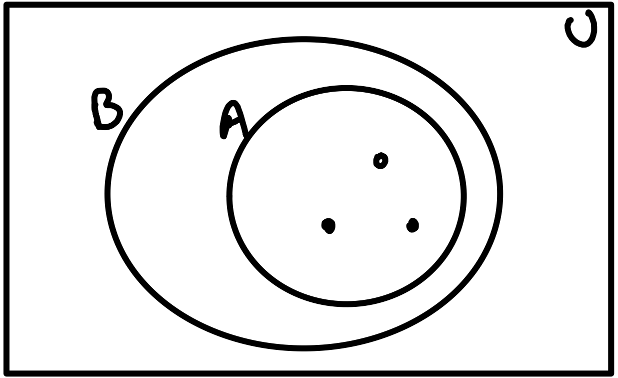
WE SAY A, B ARE DISJOINT IF $A \cap B = \emptyset$



NO ITEM CAN BE IN BOTH A AND B

Set Terminology: subsets

A is subset of B = all items in A are in B



$$A \subseteq B = \forall x \; \underline{x \in A} \rightarrow \underline{x \in B}$$

IF X IS IN A THEN X IS IN B

WE ILLUSTRATE LIKE THIS TO SHOW $A - B = \emptyset$
(THERE IS NO ITEM IN A NOT IN B)

Set Terminology: Set Equality

Given sets A, B:

we say that A=B if A is a subset of B and B is a subset of A.

$A \subseteq B$

$x \in A \rightarrow x \in B$

ALL X IN A ALSO IN B

$B \subseteq A$

$x \in B \rightarrow x \in A$

ALL X IN B ALSO IN A

INTUITION A, B HAVE SAME ITEMS

KIND OF FUNNY:

$A \subseteq B$ IS TRUE WHEN A, B ARE EQUAL

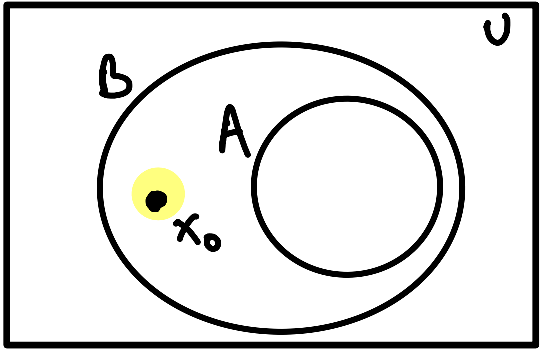
MIGHT CLARIFY TO ADD SPECIAL LANGUAGE TO DENOTE

- ARE NOT EQUAL
- ONE CONTAINED IN ANOTHER

Set Terminology: Proper Subset (one set is contained in another, larger, set)

$A \subset B$ = ALL ITEMS OF A ARE IN B AND B CONTAINS SOME ITEM NOT IN A = $A \subseteq B$ AND $B - A \neq \emptyset$

"A IS PROPER SUBSET OF B"



Subset

$A \subseteq B$

$7 \leq 8$

Proper Subset

$A \subset B$

$7 < 8$

Set Terminology: Cardinality (the number of items in a set)

$$A = \{a, b, c, d\}$$

$$|A| = 4$$

Set Terminology: Power Set

The power set of set A is the set of all sets which can be made from items in A

$$A = \{1, 2\}$$

$$P(A) = \{ \{1\}, \{2\}, \{1, 2\}, \emptyset \}$$

EMPTY SET

$P(B)$ $B = \{\square\ \triangle\}$

$= \{\emptyset, \{\square\ \triangle\}, \{\square\}, \{\triangle\}\}$

IN CLASS ACTIVITY

$\{1\} \in P(A)$

Given:

A = empty set, B = {1}, C = {1, 2}, D = {1, 2, 3}, E = {1, 2, 3, 4}

Compute each of the following:

$|A| = 0$

$|P(A)| = 1$

$|B|$

$|P(B)|$

$|C|$

$|P(C)|$

$|D|$

$|P(D)|$

$P(A) = \{\phi\}$

Can you find a pattern between |P(Z)| and |Z|? Why is it true?

$D = \{1\ 2\ 3\}$

$$
\begin{array}{llll}
P(D) = \{ & \emptyset & & 1 \\
& \{1\}\ \ \{2\}\ \ \{3\} & & 3 \\
& \{1 2\}\ \ \{1 3\}\ \ \{2 3\} & & 3 \\
& \{1 2 3\}\ \} & & 1
\end{array}
$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$|D| = 3 \quad |P(D)| = 8$

$|E| = 4 \quad |P(E)|$

$\lvert S \rvert$	$\lvert P(S) \rvert$
0	1
1	2
2	4
3	8
$\vdots$	$\vdots$

$B = \{1\}$ $\quad P(B) = \{\emptyset \ \{1\}\}$

$C = \{1,2\}$ $\quad P(C) = \{\emptyset, \{1\}, \{2\}$
$\{2,3\}\}$

$D = \{1,2,3\}$

$P(D) = \{\emptyset, \{1\}, \{2\}, \{3\},$
$\{1,2\}, \{1\,3\}, \{2,3\}$
$\{1,2,3\}\}$

CONJECTURE

$$|P(A)| = 2^{|A|}$$

1

1 1

1 2 1

1 3 3 1

$$\{1, 2\} = \{2, 1\}$$

SETS ARE NOT ORDERED