CS1800 Day 14

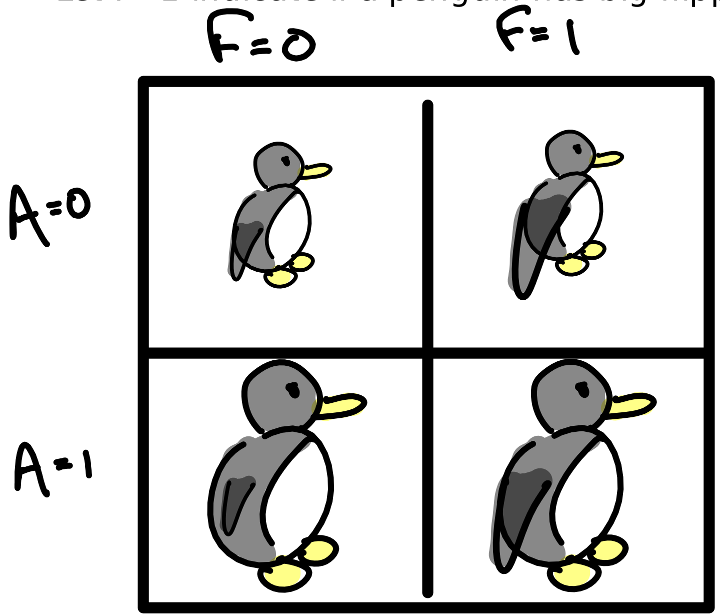
Admin:
- exam results
- tuning up your study process in CS1800
- HW4 results by thursday (hopefully tomorrow)
- grade estimates by Friday (hopefully Thursday)

Content:

Joint Probability Distribution
Marginalization
Conditional Probability
Bayes Rule
Independence

Joint Probability Distribution: A distribution over more than 1 variable at a time

Let A=1 indicate if a penguin is an adult (0 otherwise)
Let F=1 indicate if a penguin has big flippers (0 otherwise)



	F=0	F=1
A=0	3/12	2/12
A=1	1/12	6/12

HALF OF PENGUINS ARE ADULTS w/ BIG FLIPPERS

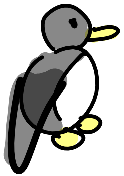
NOTATION

$P(A=0, F=1)$ IS PROB

$A=0$ (NOT ADULT)

$F=1$ (BIG FLIPPER)

HAPPEN AT SAME TIME →

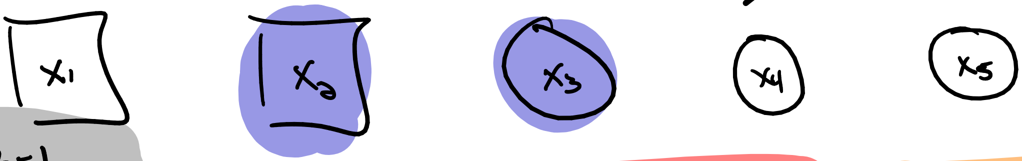


Marginalizing

(Removing a random variable from prob distribution)

B=1 shape is blue

C=1 shape is circle



$$P(B=1)=\frac{2}{5}$$

	B=0	B=1
C=0	1/5	1/5
C=1	2/5	1/5

$$P(B=1) = P(B=1\ C=1) + P(B=1\ C=0)$$

$$= 1/5 + 1/5 = 2/5$$

Remember: To compute P(B) we can sum P(B, A) for all outcomes in sample space of A

$$P(B=b) = \sum_a P(B=b\ A=a)$$

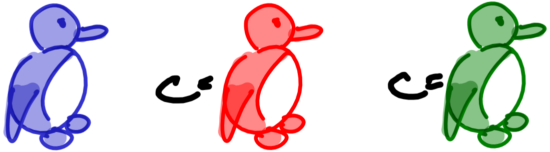
In Class Activity

Let C be a random variable representing penguin color (sample space: blue, red or green)
Let A=1 indicate if a penguin is an adult (0 otherwise)

Given the following distribution of A, C

Compute each of the follow probabilities:

- P(C=blue) $3/12 = 1/12 + 2/12$
- P(C=red) + P(C=green)
  (how is this related to prob above?)
- P(A=1)



	C=	C=	C=
A=0	1/12	3/12	0/12
A=1	2/12	1/12	5/12

$$P(C=\text{Blue}) = P(C=\text{Blue}\ A=0) + P(C=\text{Blue}\ A=1)$$
$$= 1/12 + 2/12 = 3/12$$

Conditional Probability (intuition & motivation)

C=1 indicates a person has covid (C=0 otherwise)
T=1 indicates a person has positive test (T=0 otherwise)

Let us discuss (and express) the following probabilities:

- probability person has a positive test $P(T=1)$

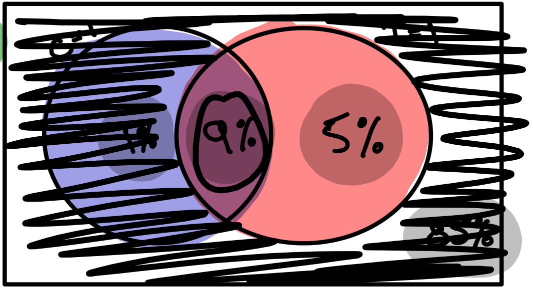
- probability person has positive test given they have covid $P(T=1 \mid C=1)$

"GIVEN C=1"

- probability person has covid given a positive test $P(C=1 \mid T=1)$

Intuition:
Conditional probability P(X=x|Y=y) is the probability of event X=x if we constrain ourselves to a world where Y=y.

Conditional Probability (motivating our formula from intuition)



C=1 indicates a person has covid (C=0 otherwise)
T=1 indicates a person has positive test (T=0 otherwise)

Let us discuss (and express) the following probabilities:

- probability person has a positive test

$$P(T=1) = P(T=1\ C=1) + P(T=1\ C=0) = 9 + 5 = 14\%$$

- probability person has positive test given they have covid

$$P(T=1 \mid C=1) = \frac{9\%}{9\% + 1\%} = 90\%$$

- probability person has covid given a positive test

$$P(C=1 \mid T=1) = \frac{9\%}{9\% + 5\%} = \frac{9}{14} = \frac{P(C=1\ T=1)}{P(T=1)}$$

Intuition:
Conditional probability P(X=x|Y=y) is the probability of event X=x if we constrain ourselves to a world where Y=y.

Conditional Probability (Formula version1: from our intuition)

$$P(a|b) = \frac{P(a\,b)}{P(b)}$$

PROB a HAPPENS GIVEN CONDITION b

PROB a b HAPPEN TOGETHER

PROB b HAPPENS

In Class Activity

Compute each of the probabilitie from the distribution below. For each, write a sentence explaining its meaning which is easily understood by a non-technical reader

Let S be a twitter sentiment score about bitcoin (1=good, 0=neutral, -1=bad)
Let B be the movement of bitcoin price (1=up, -1=down)

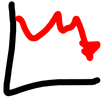
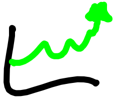
P(S=-1, B=1)
P(S=-1|B=1) ... compare to P(S=-1)
P(B=1|S=-1) ... compare to P(B=1)

	S=-1	S=0	S=1
B=-1	19%	27%	5%
B=1	8%	21%	20%

$$P(S=-1 \mid B=1) = \frac{P(S=-1\ B=1)}{P(B=1)}$$

$$= \frac{.08}{.08+.21+.2}$$

$$= \frac{8}{49} = 16\%$$

	S = -1	S = 0	S = 1
 B = -1	19%	27%	5%
 B = 1	8%	21%	20%

$$P(S=-1) = 19+8$$
$$= 27\%$$

$$P(B=1 \mid S=-1) = \frac{P(B=1\ S=-1)}{P(S=-1)}$$

$$= \frac{.08}{.19+.08}$$

$$= \frac{8}{27} = .3$$

	$S=-1$	$S=0$	$S=1$
$B=-1$	19%	27%	5%
$B=1$	8%	21%	20%

$$P(B=1) = .08+.21+.20$$

$$= .49$$

Conditional Probability (formula version 2: often more useful in our algebraic manipulations)

$$P(a|b) = \frac{P(a\,b)}{P(b)} \quad \longleftrightarrow \quad \underline{P(a|b)}\,\underline{P(b)} = \underline{P(a\,b)}$$

Takeaway above:

Multiplying

- a conditional probability
- the probability of condition

Will yield

- prob both outcomes happen together

BAYES RULE (GLORIFIED CONDITIONAL PROBABILITY)

SEE PREVIOUS SLIDE

$$P(a|b)P(b) = P(ab) = P(b|a)P(a)$$

$$\Rightarrow \quad P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

Notice: this formula "swaps" the order of the conditioning: P(A|B) on left P(B|A) on right
Its typical in a Bayes question to be given variables in one order while question asks for other.

A Helpful Manipulation

Marginalization

$$P(b) = \sum_a P(a\,b)$$

Conditional Prob Definition

$$= \sum_a P(b|a)\,P(a)$$

WHY WAS THAT HELPFUL?

BAYES RULE 1

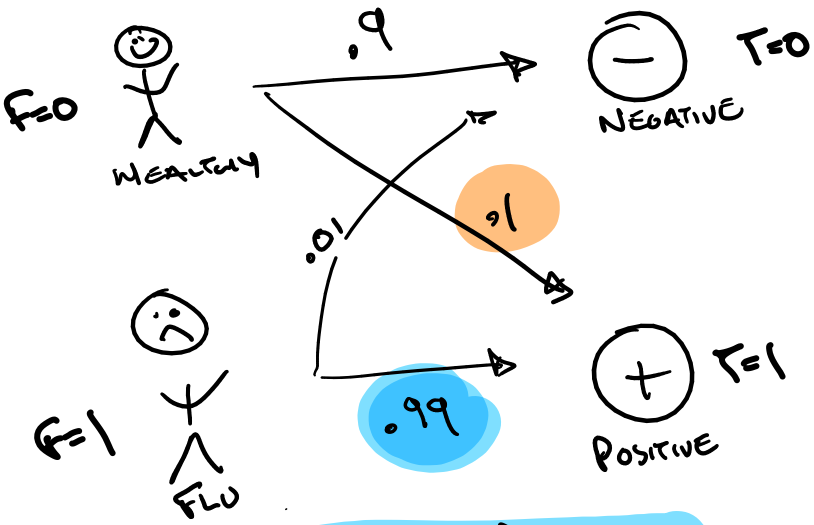
$$P(a|b) = \frac{P(b|a)P(a)}{P(b)}$$

BAYES RULE 2

$$P(a|b) = \frac{P(b|a)P(a)}{\sum_i P(b|a_i)P(a_i)}$$

Notice:
all terms of form P(b|a) and P(a) here

BAYES RULE EX



$$P(T=1 | F=1) = .99$$

Given flu occurs in 4% of population, what is the probability one has flu given they test positive?

$$P(F=1) = .04$$

$$P(F=0) = .96$$

$$P(F=1 | T=1) = \frac{P(T=1 | F=1)\,P(F=1)}{P(T=1)}$$

$$=.29$$

$$P(T=1) = P(T=1\ F=0) + P(T=1\ F=1)$$

$$= P(T=1 | F=0)\,P(F=0) +$$

$$P(T=1 | F=1)\,P(F=1) = .14$$

USEFUL BAYES STUFF

1. BAYES RULE:

$$P(a|b) = \frac{P(b|a)\,P(a)}{P(b)}$$

2. MARGINALIZATION

A

A=0	0	1/4
A=1	1/4	1/2

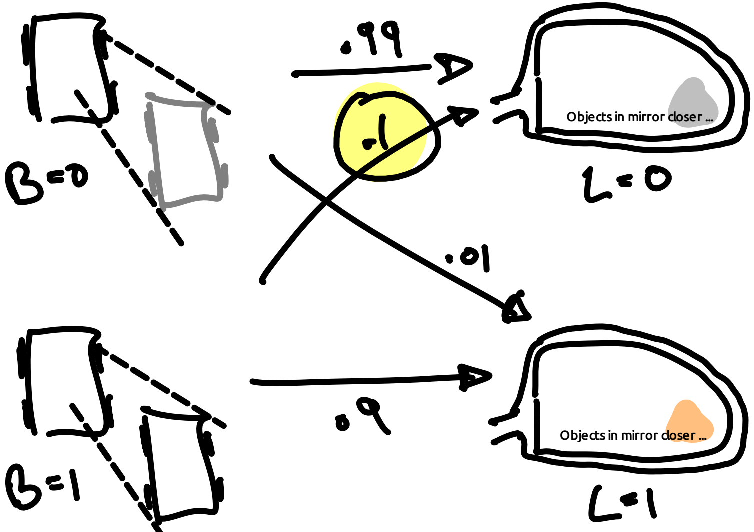
$$P(\;) = P(\;\; A=0) + P(\;\; A=1)$$

3. CONDITIONAL PROB

$$P(a|b)\,P(b) = P(ab)$$

In Class Assignment

A blind spot monitor produces a warning light (L=1) when it estimates that a car is in one's blind spot (B=1). Given that the light is off, whats the probability that a car is one's blind spot? (Assume that a car is in your blindspot 2 percent of the time while driving.)



$$P(L=1|B=0) = .01$$

$$P(B=1|L=0) = \frac{P(L=0|B=1)\,P(B=1)}{P(L=0)} = \frac{.1 \; .02}{.9722} \approx .00205$$

$$P(L=0) = P(L=0 \; B=0) + P(L=0 \; B=1)$$

$$= P(L=0|B=0)P(B=0) + P(L=0|B=1)P(B=1)$$

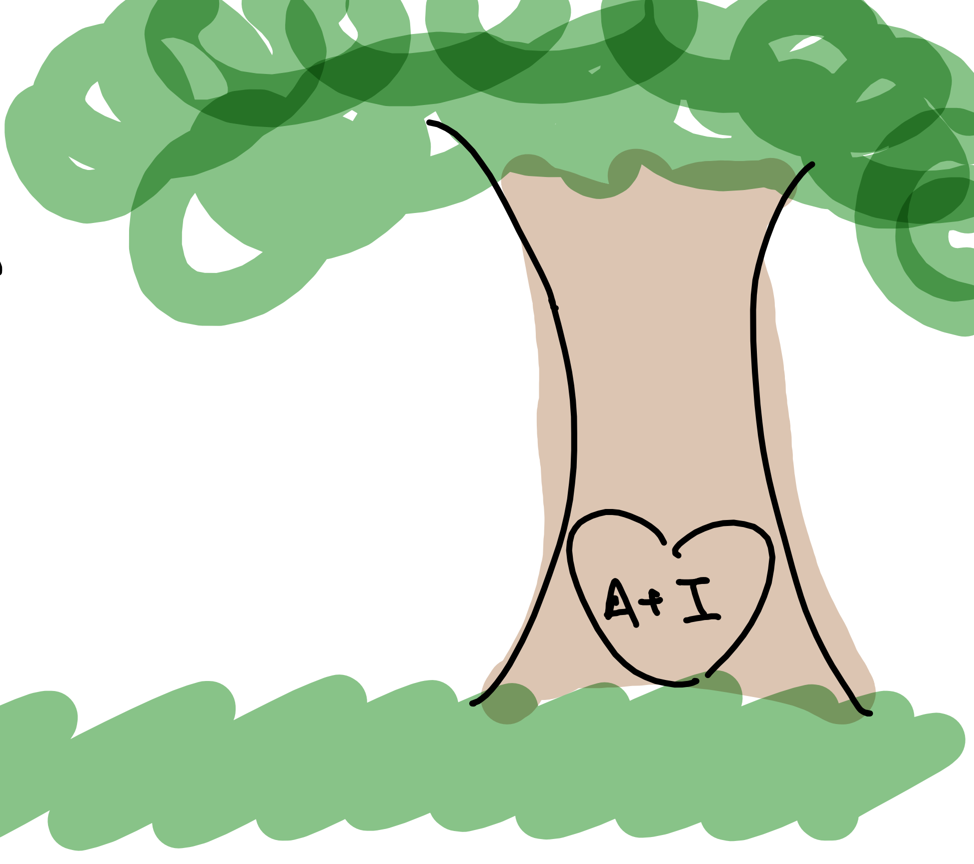
$$= .99 \; .98 + .1 \; .02$$

$$= .9722$$

ALGEBRA + INTUITION

LOVE STORY

AHEAD...



INDEPENDENCE + CONDITIONAL PROB

INDEPENDENCE

INTUITION:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

ALGEBRA:

FOR EACH OUTCOME PAIR x, y

$$P(X=x, Y=y) = P(X=x)\,P(Y=y)$$

Bayes Rule shows the equivilence of the algebraic and intuitive definitions above!

Independence + Conditional Prob

Independence

Intuition:

Random variables x, y are independent if observing any outcome of one doesn't impact our beliefs about the other.

Algebra:

For each outcome pair x, y

$$P(X=x, Y=y) = P(X=x)\,P(Y=y)$$

$$P(X|Y) = \frac{P(X,Y)}{P(Y)} = \frac{P(X)P(Y)}{P(Y)} = P(X)$$

Notice that P(X|Y) = P(X). Observing Y has no impact on the prob of X!