CS 1800 Day 4

Admin:
- hw1 due Friday
- please read the HW instructions (group members, tagging pages etc)
- tutoring group update (they've been formed, a few missing TAs, we'll be in touch ASAP, if you'd like to join one please see instructions on site)

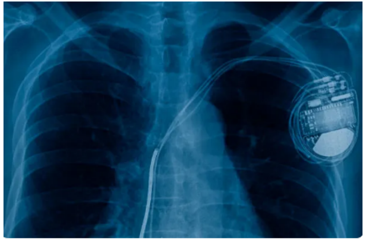
Content:
- logic statements & predicates
- truth tables
- logic operators (AND, NOT, OR)

(just an intro to these topics, we'll do more next lesson too)
- existential / universal quantifier
- conditionals

When should machine:
- give a soda
- return change

When should sunroof:
- open
- close

When should pacemaker:
- send pulse to muscle to pump blood?
- shock to restart heart



Logic gives us an unambiguous language to describe behavior

(spoken languages, like english, can be ambiguous)

STATEMENTS

Statement - a sentence which is either true or false

Which of the following are statements?

1. Today is Sept 19

2. "This big wooden horse definitely doesn't have greek soldiers inside"
    - Greeks who just put soldiers in that horse

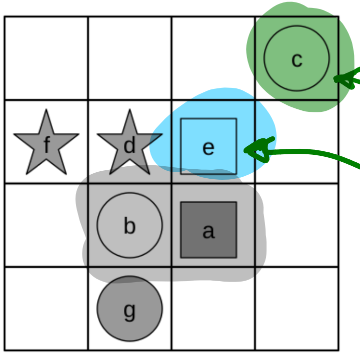
3. What is your favorite color?

4. There is intelligent life on mars

PREDICATES

Predicate - a statement about one or more variables (i.e. mad libs)

TARSKI WORLD



CIRCLE(x) = "THE OBJECT x IS A CIRCLE"

CIRCLE(c) = TRUE

CIRCLE(e) = FALSE

RIGHT_OF(x,y) = "THE OBJECT y IS IMMEDIATELY TO RIGHT OF x"

RIGHT_OF(b,a) = TRUE

RIGHT_OF(a,b) = FALSE

0, 1

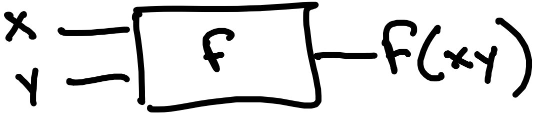
TRUE, FALSE

CONVENTION: BITS AND BOOLEANS

0 = FALSE

1 = TRUE

Truth Tables



We'll often describe a function of one or more inputs (e.g. vending machine operation)

A Truth Table specifies an output associated with every possible combinations of inputs

	X	Y	f(xy)
$(00)_2=0$	0	0	
$(01)_2=1$	0	1	
$(10)_2=2$	1	0	
$(11)_2=3$	1	1	

X	Y	Z	g(xyz)
0	0	0	
0	1	0	
1	0	0	
1	1	0	
0	0	1	
0	1	1	
1	0	1	
1	1	1	

LOGICAL OPERATOR: NOT (NEGATION)

CHANGES TRUTH VALUE

"NOT X"

X	$\neg X$
F	T
T	F

Ex

X = "IT'S RAINING"

$\neg X$ = "IT'S NOT RAINING"

LOGICAL OPERATOR: AND (CONJUNCTIVE)

ONLY TRUE WHEN **ALL** INPUTS ARE TRUE

"X AND Y"

D	P	$D \land P$
F	F	F
F	T	F
T	F	F
T	T	T

D = DRIVER'S LICENSE PRESENTED

P = PASSPORT PRESENTED

$D \land P$ = DRIVER'S LICENSE AND PASSPORT PRESENTED

Logical Operator: OR (Disjunctive Operator)

Only true when **any** input is true

"x **or** y"

X	Y	$X \vee Y$
F	F	F
F	T	T
T	F	T
T	T	T

D = Driver License Presented

P = Passport Presented

$D \vee P$ = Driver's License or Passport Presented

EXCLUSIVE OR: XOR

ONLY TRUE WHEN **EXACTLY ONE** INPUT IS TRUE ←

"WILL YOU HAVE GREENS OR SOUP?"

G = YOU HAVE GREENS

S = YOU HAVE SOUP

G	S	$G \oplus S$
F	F	F
F	T	T
T	F	T
T	T	F

NOTICE DIFFERENCE FROM OR

$G \oplus S$ = "EITHER SOUP OR GREENS, NOT BOTH"

INCLUSIVE OR

X	Y	$X \vee Y$
F	F	F
F	T	T
T	F	T
T	T	T

EXCLUSIVE OR

G	S	$G \oplus S$
F	F	F
F	T	T
T	F	T
T	T	F

"Convention": Most of the time when folks say "or" they intend the inclusive or but not all the time ... good luck! ;)

CONVENTION

$$\neg A \vee B = (\neg A) \vee B$$

Assume the negation operation applies to statement immediately to its right.

If the negation applies to multiple statements, use parenthases as below:

$$\neg(A \vee B)$$

Truth tables allow us to build complex expressions in bite-size steps.

GOAL: TRUTH TABLE FOR $(X \vee Y) \wedge \neg Z$

X	Y	Z	$X \vee Y$	$\neg Z$	$(X \vee Y) \wedge \neg Z$
0	0	0	0	1	0
0	1	0	1	1	1
1	0	0	1	1	1
1	1	0	1	1	1
0	0	1	0	0	0
0	1	1	1	0	0
1	0	1	1	0	0
1	1	1	1	0	0

In Class Assignment:

Build a truth table for each of the two expressions below. Results for both might feel familiar, thats ok :)

$\neg(A \vee B)$

$\neg A \wedge \neg B$

A	B	$A \vee B$	$\neg(A \vee B)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$\neg A$	$\neg B$	$\neg A \wedge \neg B$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Logical (Boolean) Equivilence

Two statements are logically equivalent if their truth table columns are identical.

Statements which are logically equivalent:

- always have the same truth value (True or False)
- may be substituted for each other
    - like one does in our familiar algebra (e.g. x = 3 into 10 = x + y)

Example: logically equivalent statements:
"This shape has exactly four sides of equal length at right angles to each other"

"This shape is a square"

Our previous slide proves logical equivilence of

$$\neg(A \vee B) = \neg A \wedge \neg B$$ (De Morgan's Law)

$$3(2+5) = 3 \cdot 2 + 3 \cdot 5$$

There are other laws too:

- helpful to simplify an expression

- we'll study these alongside set algebra & circuits, which are related topics, more to come later …

Associative Laws

$(P \lor Q) \lor R = P \lor (Q \lor R)$

$(P \land Q) \land R = P \land (Q \land R)$

Double Negation

$\neg \neg P = P$

DeMorgan's Laws

$\neg(P \lor Q) = \neg P \land \neg Q$

$\neg(P \land Q) = \neg P \lor \neg Q$

Distributive Laws

$P \land (Q \lor R) = (P \land Q) \lor (P \land R)$

$P \lor (Q \land R) = (P \lor Q) \land (P \lor R)$

Absorption Laws

$P \land (P \lor Q) = P$

$P \lor (P \land Q) = P$

Complement Laws

$P \lor \neg P = T$

$P \land \neg P = F$

Idempotent Laws

$P \lor P = P$

$P \land P = P$

Identity

$\text{False} \lor P = P$

$\text{True} \land P = P$

Domination:

$\text{True} \lor P = \text{True}$

$\text{False} \land P = \text{False}$

Conditional Statement: (AKA Implication)

If X then Y

X = YOU JOIN TUTORING GROUP

Y = YOU WILL HAVE FUN W/ MATH

$X \to Y$ = IF YOU JOIN TUTOR GROUP THEN YOU'LL HAVE FUN W/ MATH

X	Y	$X \to Y$
0	0	1
0	1	1
1	0	0
1	1	1

X=0, DIDN'T JOIN GROUP $X \to Y$ TRUE BY CONVENTION

STUDENT JOINED GROUP BUT DIDN'T HAVE FUN $X \to Y$ IS FALSE

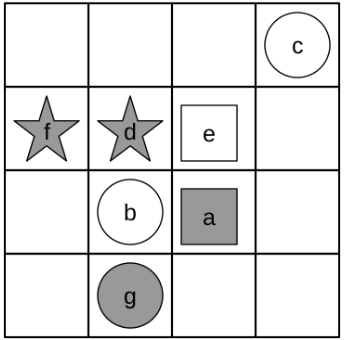
STUDENT JOINED GROUP AND HAD FUN $X \to Y$ IS TRUE

LOGICAL QUANTIFIER: UNIVERSAL (AKA FOR ALL)

$$\forall x \; \text{SHADE}(x)$$

=

"FOR EVERY OBJECT X X IS SHADED"



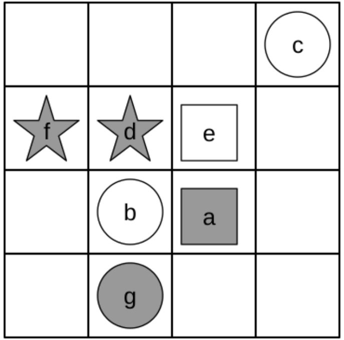
THIS STATEMENT IS FALSE, CONSIDER THAT C IS NOT SHADED

EQUIVALENT

ALL, ANY, EACH, EVERY

QUICK PRACTICE

IS FOLLOWING STATEMENT TRUE?



$\forall x \; \text{STAR}(x) \to \text{SHADE}(x)$

FOR ALL X THAT IS A STAR,
IT IS SHADED

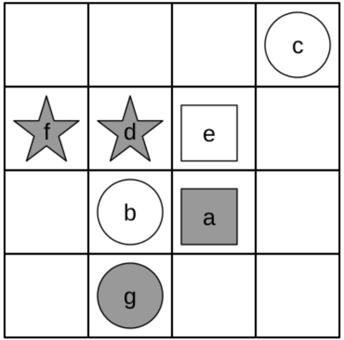
FOR ALL X IF X IS A STAR THEN
X IS SHADED

Logical Quantifier: Existential (aka "There exists")

$\exists x$ Shade(x)

=

"There exists shape x with x is shaded"



This statement is true consider that a is shaded

USEFUL TIP

$\forall$

FOR ALL

UPSIDE DOWN A

$\exists$

THERE EXISTS

BACKWARDS E

In Class Activity:

Using logical operators (AND, OR, NOT) quantifiers (for all, there exists) and conditionals (if-then), translate each statement below:

LET x BE STUDENT

SHOES(x) = STUDENT x WEARING SHOES

DANCE(x) = STUDENT x IS A GREAT DANCER

Logic to english:

$\exists x \; SHOES(x) \lor DANCE(x)$

There exists a student, x, where either x is wearing shoes or is a great dancer

$\forall x \; DANCE(x) \rightarrow \neg SHOES(x)$

for all students, if they're a great dancer then they're not wearing shoes

English to logic (define your own statements & predicates as needed)

- You shall not pass! - Gandalf

P = BALROG PASSES

$\neg P$

$P(x)$ = MONSTER x PASSES

x is a monster

$\forall x \; \neg P(x)$

- "Everybody loves you when you're 6 feet underground" -John Lennon

Y X

$\forall x y \; \neg ALIVE(x) \to LOVE(y, x)$

LOVE$(x\,y)$ = X LOVES Y

ALIVE(x) = X IS ALIVE

- I've got a wallet, keys and a phone in my pocket.

$$W \land K \land P$$

- I never leave the house without my blue shoes or a hat

$$L \rightarrow (B \lor H)$$

$$\neg(B \lor H) \rightarrow \neg L$$

W = I HAVE WALLET IN MY POCKET

K = I HAVE KEYS IN MY POCKET

P = I HAVE PHONE IN MY POCKET

B = I HAVE BLUE SHOES ON

H = I HAVE HAT ON

L = I LEAVE HOUSE

- "There's no place like home" - Dorothy in Wizard of Oz

$$\neg\left(\exists x \ \ \text{HOME}(x)\right)$$

$$= \forall x \ \ \neg\,\text{HOME}(x)$$

HOME(x) = x is LIKE HOME