CS1800 Day 5

Admin:

- HW1 due today (number representation)
- HW2 released today (logic)

Content:

- conditionals
    - contrapositive, inverse, converse
    - bi-conditionals

- quantifiers (universal & existential)
    - negating each
    - combining them
        - "for every x there exists a y"
        - "there exists a y for every x"

A WALK DOWN MEMORY LANE: BOOLEAN OPERATORS

NOT

X	¬X
0	1
1	0

SWAPS TRUTH VALUE

AND (CONJUNCTIVE)

X	Y	X ∧ Y
0	0	0
0	1	0
1	0	0
1	1	1

TRUE ONLY WHEN ALL INPUTS TRUE

OR (DISJUNCTIVE)

X	Y	X ∨ Y
0	0	0
0	1	1
1	0	1
1	1	1

TRUE ONLY WHEN ANY INPUT TRUE

A WALK DOWN MEMORY LANE: QUANTIFIERS

FOR ALL

$\forall x$ LOVES_CS1800(x)

"EVERY STUDENT X LOVES CS1800"

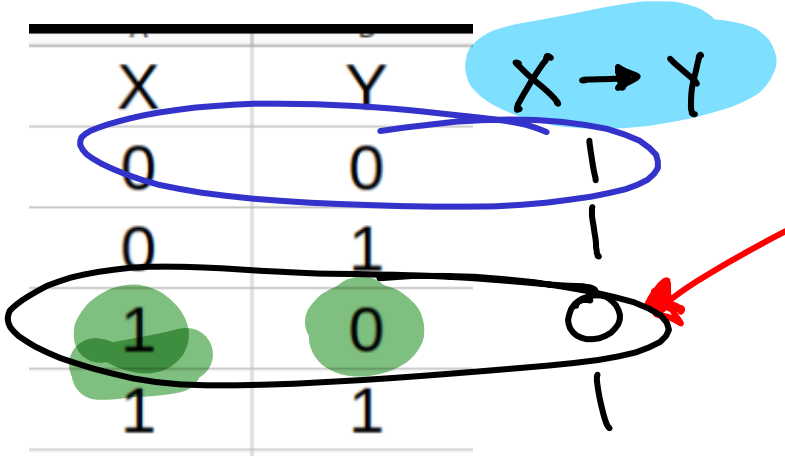
THERE EXISTS

$\exists x$ LOVES_CS1800(x)

"THERE IS A STUDENT X WHO LOVES 1800"

A WALK DOWN MEMORY LANE: CONDITIONALS

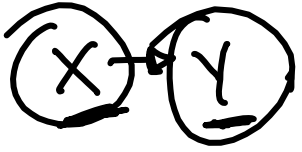
IF X THEN Y



COUNTER-EXAMPLE

- X IS TRUE BUT Y ISN'T
- SHOWS X → Y IS FALSE

Useful Fact



$\rightarrow$ True Everywhere Except

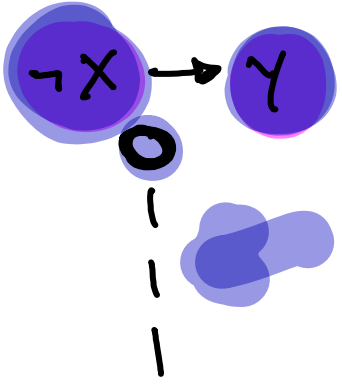
Counter Example

$X \wedge \neg Y$

Quick Example

Complete Truth Table

X	Y
0	0
0	1
1	0
1	1



IN CLASS ACTIVITY

Given the following statements:

G = life you gives you lemons
M = you make lemonade

For each statement below:
- express it using logic symbols
- create a truth table for the statement
  (for every combination of B, K, is it true?)
- identify which of the four statements below are logically
  equivilent to other statements given

Statements:
- If life gives you lemons, then you make lemonade $L \rightarrow M$
- If you are not making lemonade, life hasn't given you lemons $\neg M \rightarrow \neg L$
- If you make lemonade, then life has given you lemons
- If you haven't been given lemons, then you aren't making lemonade

Activity Answers Follow
(No Peeking!)

IF	GIVEN LEM	NOT MAKE ADE	MAKE ADE	NOT GIVEN LEM
THEN	MAKE ADE	NOT GIVEN LEM	GIVEN LEM	NOT MAKE ADE

G	M	G → M	¬M → ¬G	M → G	¬G → ¬M
0	0	1	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	1	1	1	1	1

COUNTER EXAMPLE: G=1, M=0

M=0, G=1

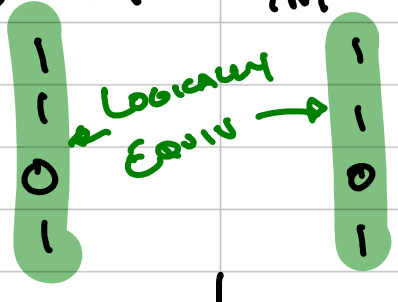
G=0, M=1

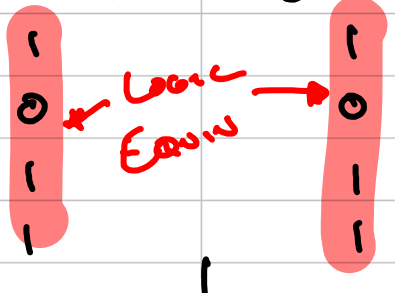
G=0, M=1

if you swap sides and negate both

IF	GIVEN LEM	NOT MAKE ADE	MAKE ADE	NOT GIVEN LEM
THEN	MAKE ADE	NOT GIVEN LEM	GIVEN LEM	NOT MAKE ADE

G	M	$G \rightarrow M$	$\neg M \rightarrow \neg G$	$M \rightarrow G$	$\neg G \rightarrow \neg M$
0	0	1	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	1	1	1	1	1





COUNTER EXAMPLE: $G=1\ M=0$ | $M=0\ G=1$ | $M=1\ G=0$ | $G=0\ M=1$

RELATIVES OF $G \to M$:

G	M	Original Statement $G \to M$	Contrapositive $\neg M \to \neg G$	Converse $M \to G$	Inverse $\neg G \to \neg M$
0	0	1	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	1	1	1	1	1

Takeaways:

- a statement and its contrapositive are logically equivilent
  (tip: it may be easier to work with one or other, use the simpler the one)
- a statement is not logically equivilent to converse or inverse
  "If life gives you lemons, then you make lemonade" does not imply that because you're making lemonade, you must have been given lemons

QUICK NOTATION: BACKWARDS CONDITIONAL

$X \leftarrow Y$ IS SAME AS $Y \rightarrow X$

Bi-conditional: $G \leftrightarrow M = (G \rightarrow M) \wedge (G \leftarrow M)$

G	M	Original Statement $G \rightarrow M$	Converse $M \rightarrow G$	$(G \rightarrow M) \wedge (M \rightarrow G)$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

IF $G \leftrightarrow M$ THEN $(G=M=0)$ OR $(G=M=1)$

EITHER WAY $G = M$

↑ LOGICAL EQUIVALENT

- X AND Y HAVE SAME TRUTH VALUE:
  - WHEN ONE IS TRUE, SO IS OTHER
  - WHEN ONE IS FALSE, SO IS OTHER

NOTATION

X  Y

IFF X THEN Y

New topic:

Quantifiers (negating & combining them)

Negating Quantifiers:

ENGLISH

Statement 1:
Some student, in class, has a birthday today!

Negation of statement 1:

FOR ALL STUDENTS X, ITS NOT THEIR BIRTHDAY!

LOGIC

$\exists x\, BT(x)$

$\forall x \neg BT(x)$

X STUDENT

$BT(x)$ = STUDENT X'S BIRTHDAY IS TODAY

Statement 2:
Every student in the class loves chocolate

Negation of statement 2:

THERE EXIST STUDENT X WHO DOESN'T LOVE CHOC

$\forall x\, C(x)$

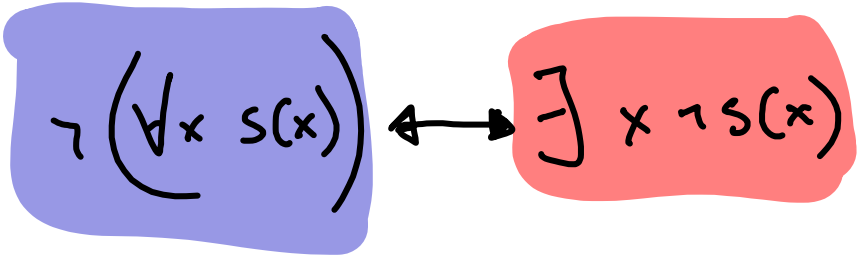
$\exists x \neg C(x)$

X STUDENT

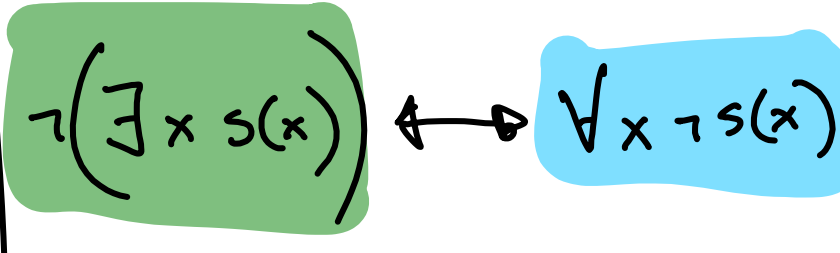
$C(x)$ = STUDENT X LOVES CHOCOLATE

Negating Quantifiers

Iff a statement isn't true for all x,
then there is an x for which it isn't true



Iff there is no x for which a statement is true,
then x is not true for all x



Simultaneous Rock-Paper-Scissors

Combining Quantifiers:

For all x, there exists y

x, y students

Win(x, y) = x beat y @ Rock Paper Scissors

For every student x, there is another student y who they've beaten

$$\forall x \; \exists y \; \text{Win}(x, y)$$

True, everybody x beat somebody else y

Each x may pick its own y

Previously

For all x, there exists y

$$\forall x \exists y$$

Next

There exists y for all x

$$\exists y \forall x$$

COMBINING QUANTIFIERS:

THERE EXISTS Y FOR ALL X

X, Y STUDENTS

$WIN(X,Y)$ = X BEAT Y @ ROCK PAPER SCISSORS

THERE IS A STUDENT Y, WHO FOR EVERY STUDENT X

Y HAS BEATEN X

$$\exists_Y \forall_X \; WIN(Y,X)$$

FALSE, ONE STUDENT DIDN'T BEAT EVERYONE ELSE

EVERY X IS SATISFIED BY SAME Y

FOR ALL x, THERE EXISTS y

$\forall x \exists y$

EACH x PICKS ITS OWN y

THERE EXISTS y FOR ALL x

$\exists y \forall x$

EACH x IS SATISFIED BY SAME y

For each sentence immediately below:
- express it using logical symbols
- express its negation using logical symbols
- translate that negation back to english

Statement:
- There is a good discrete structure textbook

$\exists x\ G(x)$

$\forall x\ \neg G(x)$

- Everybody loves ice cream

$\forall x\ I(x)$

$\exists x\ \neg I(x)$

HAS READ(YX) = STUDENT Y READ BOOK X

$\forall$ STUDENT

X BOOK

G(X) = X IS GOOD DS BOOK

X PEOPLE

I(X) = PERSON X ♡S ICE CREAM

For each sentence immediately below:
- express it using logical symbols

- Everyone has somebody who can make them smile

$\forall x \exists y \ \text{SMILE}(y, x)$

x, y PEOPLE

$\text{SMILE}(y, x)$ = y (PERSON) MAKE x (PERSON) SMILE

- There is someone who ran the race faster than anybody else

$\exists x \forall y \ \text{FASTER}(x, y)$

x, y RUNNERS

$\text{FASTER}(x, y)$ =
x (RUNNER) RAN FASTER THAN y (RUNNER)