CS1800

9/22 - Fri :)

Admin

- Hw1 (#1 :)) due tonight 11:59
- Hw2 out ~~x~~, due 9/29 11:59
- Rec 2 Quiz due Mon 9/25 @ 9pm

Agenda

1. More implications
2. Predicates
3. Quantifiers

## 0. Review

$\wedge$ and

$\vee$ or

$\neg$ not

$\Rightarrow$ implication & convenience

(and, or, not): can do everything!

$\neg(Q \vee P) \wedge \neg P \vee F$

When...

| | |
|---|---|
| Q is T<br>P is F<br>$\neg(Q \vee P) \wedge \neg P \vee F$<br>T F F<br>$\neg T \wedge T \vee F$<br>$F \wedge T \vee F$<br>F | P is F<br>Q is F<br>$\neg(Q \vee P) \wedge \neg P \vee F$<br>F F F<br>$\neg F$ T F<br>$T \wedge T \vee F$<br>T |

T $\vee$ anything = T

F $\vee$ anything = anything

F $\wedge$ anything = F

T $\wedge$ anything = anything

# 1. More Implications

What conclusions can we make?

If aces win, Larry gets \$ → respect the original

$P$ = aces win

$Q$ = Larry gets \$

$P \Rightarrow Q \equiv \neg P \vee Q$

| $P$ | $Q$ | $P \Rightarrow Q$ | $\neg P$ | $\neg P \vee Q$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

$\neg(P \wedge \neg Q)$

$\neg P \vee Q$

Premise: if aces win, Larry gets \$

- Larry gets \$
- Did the aces win? not for sure

converse

- the Aces didn't win
- Larry got no \$? not for sure

inverse

- Larry didn't get $
- Aces didn't win?, Yes!

contrapositive

ex

P $x = 6$

Q $x^2 = 36$

Premise: $P \Rightarrow Q$

If $x = 6$, then $x^2$ is 36

- Converse / inverse / contrapositive
- write the implication

$x \neq 6$, therefore $x^2 \neq 36$ inverse $\neg P \Rightarrow \neg Q$

$x^2 \neq 36$, therefore $x \neq 6$ contra $\neg Q \Rightarrow \neg P$

$x^2 = 36$, therefore $x = 6$ Converse $Q \Rightarrow P$

- Sometimes, proving the contra is easier, and it is logically equivalent to original

orig converse inverse contra

| $P$ | $Q$ | $P \Rightarrow Q$ | $Q \Rightarrow P$ | $\neg P$ | $\neg Q$ | $\neg P \Rightarrow \neg Q$ | $\neg Q \Rightarrow \neg P$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | F | T | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | T |
| F | F | T | T | T | T | T | T |

$$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$$

## 2. Predicates

- Predicate is a generalization of a logic statement
- one or more variables
- no truth value

not a logic statement, but can be turned into one

$P(x) \ldots \quad x + 1 = 3$

$P(x,y) \ldots \quad 4 + x < y * 2$

predicates
(no truth values)
(not logic statements!)

Turn predicate into logic statement:

1. Plug in specific values
2. Quantifiers

P(x) ... It's x's birthday — predicate

P(Lanny) ... It's Lanny's bday (3/27) logic st. F

P(matt) ... It's matt's bday (7/18) logic st. F

P(Eleanor) ... It's Eleanor's bday logic st. (True!)

$P(x, y) \ldots \quad 4 + x < y * 2$ predicate

$P(10, 6) \ldots \quad 4 + 10 < 6 * 2$ logic st. (False)

$\Leftrightarrow$

10:53

| $P$ | $Q$ | $P \Leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

## 3. Quantifiers

Turn a predicate into a logic statement...

1. Plug in values for variables
2. Quantifiers!

Quantifiers:

$\forall$ for all

$\exists$ there exists

Also need: universe / domain

Last time...

Bostonians love Dunkin Donuts — not logic st.

$P(x)$ $x$ loves Dunkin Donuts

universe / domain: Bostonians

$\boxed{\forall x \; P(x)}$ ... All Bostonians love Dunks

Logic st. — English

False

$\boxed{\exists x \; P(x)}$ .... there exists a Bostonian who loves Dunkin

Logic → True

English

There is at least one Bostonian who loves Dunks

$\boxed{\exists x \; P(x)}$

There are at least 2 diff Bostonians who love Dunks

$\boxed{\exists x, y \;\; P(x) \wedge P(y) \wedge x \neq y}$

$\forall x$ means literally every thing in domain

universe:

Characters in Cobra Kai

Predicates:

johnny(x) ... Johnny fights x

karate(x) ... x studies karate

The only people Johnny fights are people who study karate

$\forall x \; \text{johnn}(x) \Rightarrow \text{karate}(x)$

Johnny doesn't fight everyone who studies karate

$\exists x \; \text{karate}(x) \wedge \neg \text{johnny}(x)$

Universe: integers

$\forall x \exists y \quad x + y = 0 \qquad$ True

For all ints x, there exists an int y such that $x+y=0$

truth value

johnny(x)

character in show

kzrate(johnny(x))

True/False

kzrate(T/F) ????

doesn't work