Welcome!

Please be sure not to leave a seat open between you all

(it will ensure we all get a seat)

thank you!

CS1800 Day 2

Admin:
- Recitation & Recitation Quiz
- Sign up for piazza please (its a great place for student questions :))

Content:
Converting Between Bases:
    - subtract-largest-power-of-base method (intuitive)
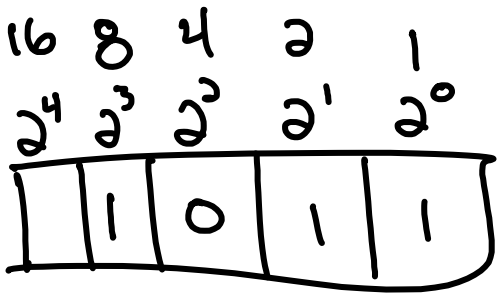   - euclid's division method (easier ... we'll see later they're the same)

Operating (adding & subtracting) in other bases

Modular Arithmetic:

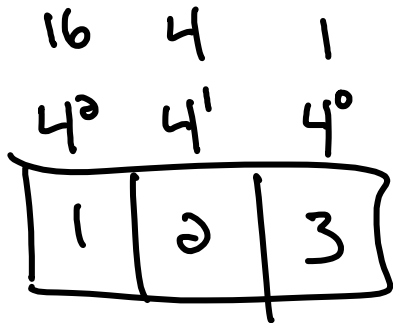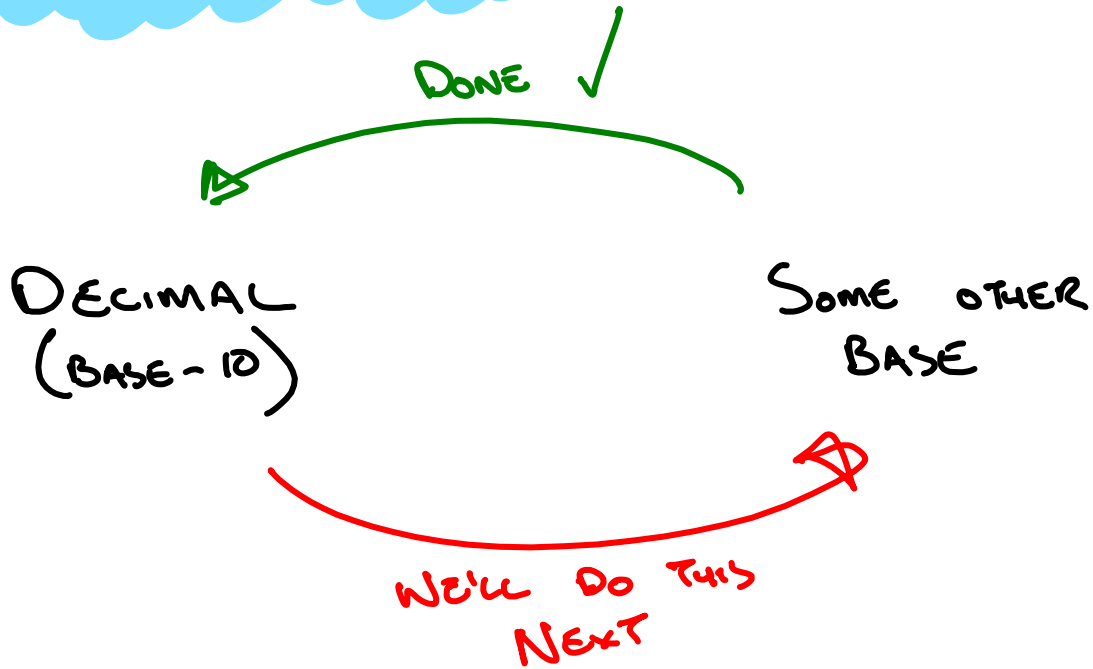Convert each of the following back to decimal (base-10):

$$\left(1011\right)_2$$

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0
1	0	1	1	

$$8 + 0 + 2 + 1 = 11$$

$$\left(123\right)_4$$

16	4	1
4^2	4^1	4^0
1	2	3

$$1 \cdot 16 + 2 \cdot 4 + 3 \cdot 1 = 27$$

Converting Between Bases

Done ✓

Decimal
(Base-10)

Some other
Base

We'll Do This Next

SUBTRACT LARGEST
POWER OF BASE

SOLVE FOR X

$B = 2$

$$14 = (x)_2$$

$$14 = 8 + 4 + 2$$

16	8	4	2	1
0	1	1	1	0

$$(1110)_2$$

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
.
.
.

$$1000 = 01000$$

FLOOR DIVISION

Divide one number by another, round down to nearest whole number

$$17 // 2 = 8$$

$$100 // 1 = 100$$

$$10 // 3 = 3$$

$$-7 // 4 = -2$$

17 DIVIDED BY 8

$$17 = 8 \cdot 2 + 1$$

REMAINDER

$17 // 8$

SOLVE FOR X

$$14 = (x)_2$$

$$14 // 2 = 7$$

$$14 = 7 \cdot 2 + 0$$

$$7 // 2 = 3$$

$$7 = 3 \cdot 2 + 1$$

$$3 // 2 = 1$$

$$3 = 1 \cdot 2 + 1$$

$$1 = 0 \cdot 2 + 1$$

$$(1110)_2$$

$$27 = 13 \cdot 2 + 1$$
$$13 = 6 \cdot 2 + 1$$
$$6 = 3 \cdot 2 + 0$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$27 = (11011)_2$$

DECIMAL TO ANOTHER BASE: EUCLID'S DIVISION METHOD

SOLVE FOR X

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

STOP HERE

$$(1110)_2$$

1. Given decimal value is first value

2. Divide value by base w/ whole numbers (use a remainder)

3 Set new value as base-multiplier

4. Repeat from step 2 if value is greater or equal to base

5. Glue together all remainders (last-to-first) to produce answer

Express 23 as a binary value using:

- subtract-largest-power-of-base
- Euclid's division method

(++) How are these methods similar? How are they different? How might you demonstrate that Euclid's divisoin method gives the correct answer?

Operating (adding & multiplying) in another base

(works just like decimal, though it might feels funny at first)

Perform each of the following addition operations:

$$123 + 281$$

$$(3C4)_{16} + (152)_{16}$$

Perform each of the following multiplication operations:

$$123 \cdot 41$$

$$(172)_8 \cdot (21)_8$$

Operating in other bases (tips):

- use scratch work on the side (in decimal, to be comfortable)

- don't use base-10 values in original problem (convert to given base!)

If you get stuck, make up and write out a similar decimal example, it will prime your brain to make the same moves in the strange, alien base

Perform each of the following operations in the given base:

$$\left(1\,47 \right)_8 + \left(44 \right)_8$$

$$\left(32 \right)_4 \cdot \left(22 \right)_4$$

## Modular Arithmetic: Motivation via wall-clock time

If the time now is 4 PM:
- what time is it in 1 hour?
- what time is it in $25 = 1 + 24 * 1$ hours?
- what time is it in $49 = 1 + 24 * 2$ hours?
- what time is it in $73 = 1 + 24 * 3$ hours?
- what time is it in $1 + 24 * n$ hours (for a whole number n)?

Punchline:
When counting time, values are equivilent if they differ by a factor of 24 (e.g. 24, 48, 72 etc)

What are "all" the values which add 1 hour to the time?

$$-47, \quad -23, \quad 1, \quad 25, \quad 49, \quad 73$$

+24   +24   +24   +24   +24
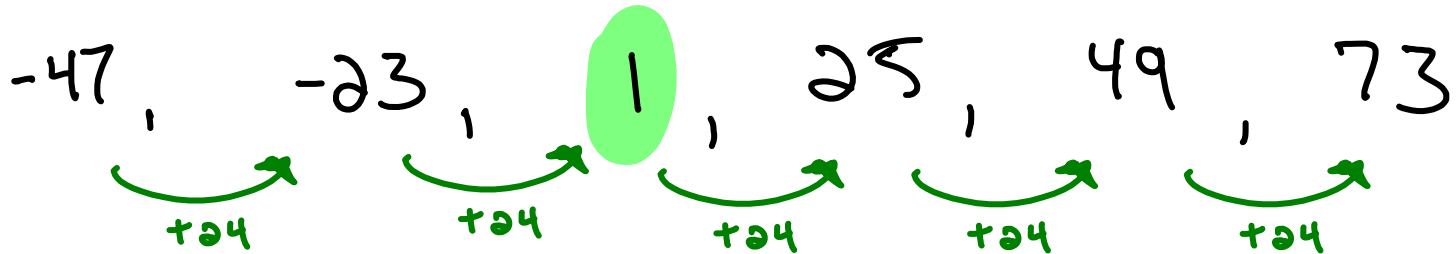
Lets represent this set by its smallest, non-negative value:

-47 mod 24 = 1
-23 mod 24 = 1
1 mod 24 = 1
25 mod 24 = 1
49 mod 24 = 1
73 mod 24 = 1

"x mod n" equals the smallest, non-negative value r where x = c * n + r where c is a whole number

Example:

14 mod 2 = ?

Thinking out loud:
- Dividing 14 by 2 gives a remainder zero. $14 = 7 \cdot 2 + 0$

- What are all the values x which also have x divided by 2 gives remainder zero?

$$\ldots \quad -6, -4, \quad -2, \quad 0, 2, \quad 4, 6, 8, \ldots$$

- Which of these is the smallest, non-negative value?

$$14 \; MOD \; 2 = 0$$

In Class Activity

Compute each of the following:

45 mod 2

11 mod 7

-4 mod 2

"any decimal number ending in zero" mod 0