Welcome!

Please don't leave any seats empty between you all
(we might need the room)

thanks!

CS1800 Day 2

Admin:
- Recitation & Recitation Quiz
- Sign up for piazza please (its a great place for student questions :))

Content:
Converting Between Bases:
	- subtract-largest-power-of-base method (intuitive)
	- euclid's division method (easier ... we'll see later they're the same)

Operating (adding & subtracting) in other bases

Modular Arithmetic:

In Class Activity (warm-up):

Convert each of the following back to decimal (base-10):

$(1011)_2$

2^3	2^2	2^1	2^0
8	4	2	1
1	0	1	1

$8 + 0 + 2 + 1 = 11$

$(123)_4$

4^2	4^1	4^0
16	4	1
1	2	3

$1 \cdot 16 + 2 \cdot 4 + 3 \cdot 1$

$= 16 + 8 + 3 = 27$

Converting Between Bases

Done ✓

Decimal (Base-10)

Some other Base

We'll do this next

Decimal to Another Base:

Subtract largest power of base

$B=2$

$2^0=1$
$2^1=2$
$2^2=4$
$2^3=8$
$2^4=16$
$\vdots$

Solve for x

$$14=(x)_2 = 8+6$$
$$= 8+4+2$$
$$= (1110)_2$$

8	4	2	1
1	1	1	0

Decimal to Another Base: Euclid's Division Method

Solve for x

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$(1110)_2$$

DECIMAL TO ANOTHER BASE: EUCLID'S DIVISION METHOD

SOLVE FOR X

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$
$$7 = 3 \cdot 2 + 1$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$

$$(1110)_2$$

1. Given decimal value is first value

2. Divide value by base w/ whole numbers (use a remainder)

3 Set new value as base-multiplier

4. Repeat from step 2 if value is greater or equal to base

STOP HERE

5. Glue together all remainders (last-to-first) to produce answer

VALUE = MULTIPLIER · BASE + REMAINDER

$$18 = (x)_2$$
$$= (10010)_2$$

$$18 = 9 \cdot 2 + 0$$
$$9 = 4 \cdot 2 + 1$$
$$4 = 2 \cdot 2 + 0$$
$$2 = 1 \cdot 2 + 0$$
$$1 = 0 \cdot 2 + 1$$

REMAINDER WILL ALWAYS BE LESS THAN BASE

In Class Activity

Express 23 as a binary value using:

- subtract-largest-power-of-base
- Euclid's division method

(++) How are these methods similar? How are they different? How might you demonstrate that Euclid's divisoin method gives the correct answer?

23 To Binary

(Largest Power of Base in Remainder)

$$23 = 16 + 7$$
$$= 16 + 4 + 3$$
$$= 16 + 4 + 2 + 1$$
$$= (1\ 0\ 1\ 1\ 1)_2$$

1
2
4
8
16
32
64

Euclid

$$23 = 11 \cdot 2 + 1$$
$$11 = 5 \cdot 2 + 1$$
$$5 = 2 \cdot 2 + 1$$
$$2 = 1 \cdot 2 + 0$$
$$1 = 0 \cdot 2 + 1$$
$$(10111)_2$$

Operating (adding & multiplying) in another base

(works just like decimal, though it might feels funny at first)

Operating in other bases: addition

Perform each of the following addition operations:

$123 + 281$

$$\begin{array}{r} {}^{1}123 \\ +\ 281 \\ \hline 404 \end{array}$$

$(3C4)_{16} + (152)_{16}$

$$\begin{array}{r} {}^{1}3C4 \\ +\ 152 \\ \hline 516 \end{array}$$

A 10
B 11
C 12
D 13
E 14
F 15

$17 = 16 + 1 = (11)_{16}$

$16^1 \quad 16^0$

	F	2

Operating in other bases: multiplication

Perform each of the following multiplication operations:

$123 \cdot 41$

$$\begin{array}{r} 123 \\ \times \quad 41 \\ \hline 123 \\ 4920 \\ \hline 5043 \end{array}$$

$(172)_8 \cdot (21)_8$

$$\begin{array}{r} 172 \\ \times \quad 21 \\ \hline 172 \\ + 3640 \\ \hline 4032 \end{array}$$

$(4032)_8$

$11 = 8 + 3$

$= (13)_8$

$8 = (10)_8$

$14 = 1 \cdot 8 + 6 \cdot 1 = (16)_8$

8	1
8^1	8^0
1	6

$$11 = 8 + 3$$
$$= 1 \cdot 8 + 3 \cdot 1$$

1

8

64

Operating in other bases (tips):

- use scratch work on the side (in decimal, to be comfortable)

- don't use base-10 values in original problem (convert to given base!)

If you get stuck, make up and write out a similar decimal example, it will prime your brain to make the same moves in the strange, alien base

In Class Activity

Perform each of the following operations in the given base:

$(147)_8 + (44)_8$

$(32)_4 \cdot (22)_4$

$(147)_8 + (44)_8$

$7+4 = 11 = 8+3 = (13)_8$

$1+4+4 = 9 = 8+1 = (11)_8$

$1+1$

$$\begin{array}{r} {}^{1\,1} \\ 147 \\ +\ \ 44 \\ \hline 213 \end{array}$$

$(32)_4 \cdot (22)_4$

$2 \cdot 2 = 4 = (10)_4$

$1 + 3 \cdot 2 = 7 = 4 + 3 = (13)_4$

$$
\begin{array}{r}
{\scriptstyle 1} \\
32 \\
\times \; 22 \\
\hline
130 \\
{\scriptstyle 1} \; 1300 \\
\hline
2030
\end{array}
$$

$(11)_{14}$

Modular Arithmetic: Motivation via wall-clock time

If the time now is 4 PM:

- what time is it in 1 hour?
- what time is it in 25 = 1 + 24 * 1 hours?
- what time is it in 49 = 1 + 24 * 2 hours?
- what time is it in 73 = 1 + 24 * 3 hours?
- what time is it in 1 + 24 * n hours (for a whole number n)?

Punchline:
When counting time, values are equivilent if they differ by a factor of 24 (e.g. 24, 48, 72 etc)

Modulo Operator: inuition

What are "all" the values which add 1 hour to the time?

$$-47 \xrightarrow{+24} -23 \xrightarrow{+24} 1 \xrightarrow{+24} 25 \xrightarrow{+24} 49 \xrightarrow{+24} 73$$

Lets represent this set by its smallest, non-negative value:

-47 mod 24 = 1
-23 mod 24 = 1
1 mod 24 = 1
25 mod 24 = 1
49 mod 24 = 1
73 mod 24 = 1

Modulo Operator: definition

"x mod n" equals the smallest, non-negative value r where x = c * n + r where c is a whole number

Example:

14 mod 2 = ?

Thinking out loud:

- Dividing 14 by 2 gives a remainder zero. $14 = 7 \cdot 2 + 0$

- What are all the values x which also have x divided by 2 gives remainder zero?

$\ldots, -6, -4, -2, 0, 2, 4, 6, 8, \ldots$

- Which of these is the smallest, non-negative value?

14 MOD 2 = 0

$$11 \bmod 2 = 13 \bmod 2 = 15 \bmod 2$$

$$11 \xrightarrow{+2} 13 \xrightarrow{2} 15$$

$$11 = 5 \cdot 2 + 1$$

REMAINDER

NON-NEGATIVE, LESS THAN 2

$$\boxed{11 \bmod 2 = 1}$$

11 MOD 3 = 2

$11 = 3 \cdot 3 + 2$

142 MOD 10 = 2

$142 = 14 \cdot 10 + 2$

36 MOD 6 = 0

"ANY VALUE ENDING IN ZERO" MOD 10 = 0

$100 \text{ MOD } 3 = 1$

$100 = 33 \cdot 3 + 1$

$100 = 32 \cdot 3 + 4$