

Welcome!

Please don't leave any seats empty between you all
(we might need the room)

thanks!

CS1800 Day 2

Admin:

- Recitation & Recitation Quiz
- Sign up for piazza please (its a great place for student questions :))

Content:

Converting Between Bases:

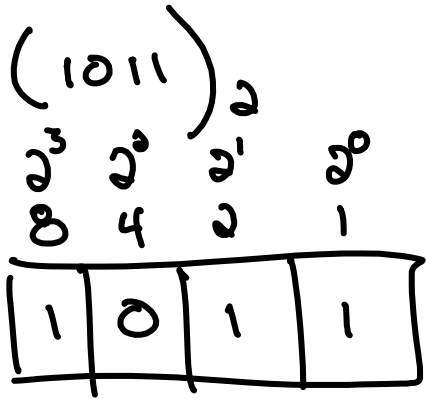
- subtract-largest-power-of-base method (intuitive)
- euclid's division method (easier ... we'll see later they're the same)

Operating (adding & subtracting) in other bases

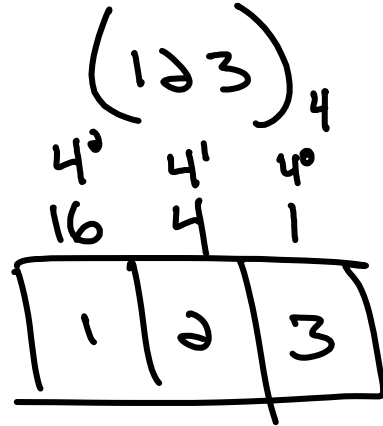
Modular Arithmetic:

In Class Activity (warm-up):

Convert each of the following back to decimal (base-10):

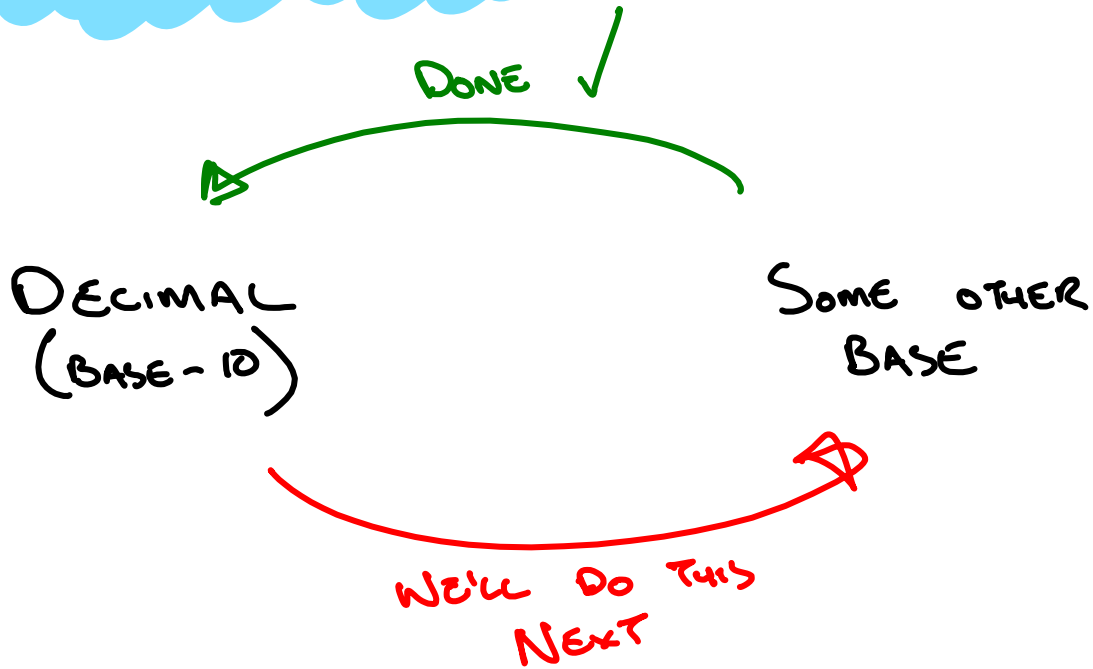


$$8 + 0 + 2 + 1 = 11$$



$$= 1 \cdot 16 + 2 \cdot 4 + 3 \cdot 1$$
$$= 16 + 8 + 3 = 27$$

CONVERTING BETWEEN BASES



DECIMAL TO ANOTHER BASE:

SUBTRACT LARGEST
POWER OF BASE

Solve for x

$$\begin{aligned} 14 &= (x)_2 = 8 + 6 \\ &= 8 + 4 + 2 \\ &= (1110)_2 \end{aligned}$$

8	4	2	1
1	1	1	0

$B=2$

$2^0 = 1$
 $2^1 = 2$
 $2^2 = 4$
 $2^3 = 8$
 $2^4 = 16$
⋮
⋮

DECIMAL TO ANOTHER BASE: EUCLID'S DIVISION METHOD

Solve for x

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$

$$7 = 3 \cdot 2 + 1$$

$$3 = 1 \cdot 2 + 1$$

$$1 = 0 \cdot 2 + 1$$

$$(1110)_2$$

DECIMAL TO ANOTHER BASE: EUCLID'S DIVISION METHOD

Solve for X

$$14 = (x)_2$$

$$14 = 7 \cdot 2 + 0$$

$$7 = 3 \cdot 2 + 1$$

$$3 = 1 \cdot 2 + 1$$

$$1 = 0 \cdot 2 + 1$$

$$(1110)_2$$

1. Given decimal value is first value
2. Divide value by base w/ whole numbers (use a remainder)
3. Set new value as base-multiplier
4. Repeat from step 2 if value is greater or equal to base
5. Glue together all remainders (last-to-first) to produce answer

STOP HERE

$$\text{VALUE} = \text{MULTIPLIER} \cdot \text{BASE} + \text{REMAINDER}$$

$$\begin{aligned} 18 &= (x)_2 \\ &= (10010)_2 \end{aligned}$$

$$\begin{aligned} 18 &= 9 \cdot 2 + 0 \\ 9 &= 4 \cdot 2 + 1 \\ 4 &= 2 \cdot 2 + 0 \\ 2 &= 1 \cdot 2 + 0 \\ 1 &= 0 \cdot 2 + 1 \end{aligned}$$

REMAINDER WILL ALWAYS BE LESS THAN BASE

In Class Activity

Express 23 as a binary value using:

- subtract-largest-power-of-base
- Euclid's division method

(++) How are these methods similar? How are they different? How might you demonstrate that Euclid's division method gives the correct answer?

23 TO BINARY

(LARGEST POWER OF BASE IN REMAINDER)

$$23 = 16 + 7$$

$$= 16 + 4 + 3$$

$$= 16 + 4 + 2 + 1$$

$$= (10111)_2$$

1
2
4
8
16
32
64

EUCLID

$$23 = 11 \cdot 2 + 1$$

$$11 = 5 \cdot 2 + 1$$

$$5 = 2 \cdot 2 + 1$$

$$2 = 1 \cdot 2 + 0$$

$$1 = 0 \cdot 2 + 1$$

$$(10111)_2$$

Operating (adding & multiplying) in another base

(works just like decimal, though it might feels funny at first)

Operating in other bases: addition

Perform each of the following addition operations:

$$\begin{array}{r} 123 + 281 \\ \hline 404 \end{array}$$

$$\left. \begin{array}{r} (3C4)_{16} + (152)_{16} \\ \hline 516 \end{array} \right\}$$

A	10
B	11
C	12
D	13
E	14
F	15

$$17 = 16 + 1 = \begin{pmatrix} 11 \\ 1 \end{pmatrix}_{16}$$

	^{16¹}	^{16⁰}
	F	2

Operating in other bases: multiplication

Perform each of the following multiplication operations:

$$123 \cdot 41$$

$$\begin{array}{r} 123 \\ \times 41 \\ \hline 123 \\ 4920 \\ \hline 5043 \end{array}$$

$$(172)_8 \cdot (21)_8$$

$$\begin{array}{r} 172 \\ \times 21 \\ \hline 172 \\ 3640 \\ \hline 4032 \end{array}$$

$$(4032)_8$$

$$11 = 8 + 3 \\ = (13)_8$$

$$8 = (10)_8$$

$$14 = 1 \cdot 8 + 6 \cdot 1 = (16)_8$$

8^1	8^0
1	6

$$11 = 8 + 3$$

$$= 1 \cdot 8 + 3 \cdot 1$$

1
8
64

Operating in other bases (tips):

- use scratch work on the side (in decimal, to be comfortable)
- don't use base-10 values in original problem (convert to given base!)

If you get stuck, make up and write out a similar decimal example, it will prime your brain to make the same moves in the strange, alien base

In Class Activity

Perform each of the following operations in the given base:

$$(147)_8 + (44)_8$$

$$(32)_4 \cdot (22)_4$$

$$(147)_8 + (44)_8$$

$$7+4=11=8+3=(13)_8$$

$$1+4+4=9=8+1=(11)_8$$

$$1+1$$

$$\begin{array}{r} 147 \\ + 44 \\ \hline 013 \end{array}$$

$$(32)_4 \cdot (22)_4$$

$$2 \cdot 2 = 4 = (10)_4$$

$$1 + 3 \cdot 2 = 7 = 4 + 3 = (13)_4$$

$$\begin{array}{r} \\ \\ \times \\ \hline \\ \\ \hline 2 \end{array}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}_{14}$$

Modular Arithmetic: Motivation via wall-clock time

If the time now is 4 PM:

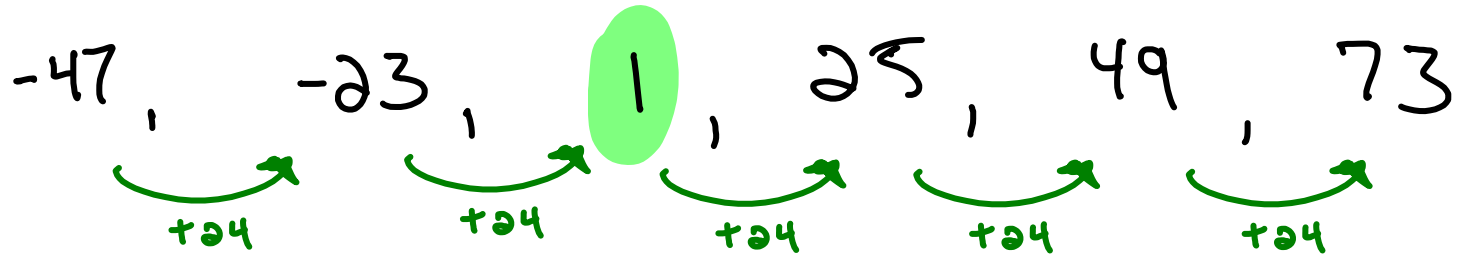
- what time is it in 1 hour?
- what time is it in $25 = 1 + 24 * 1$ hours?
- what time is it in $49 = 1 + 24 * 2$ hours?
- what time is it in $73 = 1 + 24 * 3$ hours?
- what time is it in $1 + 24 * n$ hours (for a whole number n)?

Punchline:

When counting time, values are equivalent if they differ by a factor of 24 (e.g. 24, 48, 72 etc)

Modulo Operator: intuition

What are "all" the values which add 1 hour to the time?



Lets represent this set by its smallest, non-negative value:

$$-47 \bmod 24 = 1$$

$$-23 \bmod 24 = 1$$

$$1 \bmod 24 = 1$$

$$25 \bmod 24 = 1$$

$$49 \bmod 24 = 1$$

$$73 \bmod 24 = 1$$

Modulo Operator: definition

"x mod n" equals the smallest, non-negative value r where $x = c * n + r$ where c is a whole number

Example:

$$14 \bmod 2 = ?$$

Thinking out loud:

- Dividing 14 by 2 gives a remainder zero. $14 = 7 \cdot 2 + 0$

- What are all the values x which also have x divided by 2 gives remainder zero?

... -6, -4, -2, 0, 2, 4, 6, 8, ...

- Which of these is the smallest, non-negative value?

$$14 \bmod 2 = 0$$

$$11 \text{ MOD } 2 = 13 \text{ MOD } 2 = 15 \text{ MOD } 2$$



$$11 = 5 \cdot 2 + 1$$

↑

REMAINDER
NON-NEGATIVE, LESS THAN 2

11 MOD 2 = 1

$$11 \text{ MOD } 3 = 2$$

$$11 = 3 \cdot 3 + 2$$

$$142 \text{ MOD } 10 = 2$$

$$142 = 14 \cdot 10 + 2$$

$$36 \text{ MOD } 6 = 0$$

"ANY VALUE"
ENDING IN
ZERO

$$\text{MOD } 10 = 0$$

$$100 \bmod 3 = 1$$

$$100 = 33 \cdot 3 + 1$$

$$100 = 32 \cdot 3 + 4$$

.

