CS1800
9/29- Fri :)

Admin

- Hw2 due today 11:59pm
- Hw3 out today, due 10/6 11:59pm
- Live Piazza Q+A

Agenda

1. Set equality
2. Computer representation of sets
3. Set Functions

joinpd.com

1. Set Equality

Two sets are equal **if and only if** they have the same elements

$$\forall x \in \mathcal{U} \quad x \in A \iff x \in B$$

Sometimes, by coincidence

(ex) $A = \{1, 5\}$ $B = \{2, 5\}$ $C = \{5\}$

$A \cap B = \{5\}$ $A \cap C = \{5\}$

b/c of elements in the sets

In general... $A \cap B \neq A \cap C$ for arbitrary sets

Mostly we care about equality for arbitrary sets

- A, B sets
- do operations on them
- Same result = Same other result

ex: $(\overline{A \cap B}) - B$ $\overline{B}$

To prove it... **set equality laws/definitions**

↳ all on website!

DeMorgan's $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

Domination $\{\} \cap A = \{\}$

$\mathcal{U} \cup A = \mathcal{U}$

Identity $\{\} \cup A = A$

$\mathcal{U} \cap A = A$

Defs $A - B = A \cap \bar{B}$

$A \triangle B = (A \cup B) - (A \cap B)$

Absorption $A \cap (A \cup B) = A$

Assoc. $A \cup (B \cup C) = (A \cup B) \cup C$

Process to prove 2 sets are equal:

- Start with left-hand side
- Apply one law at a time (& labelled)
- Until it looks like right-hand side

(Venn diagram can help us convince ourselves!)

$\overline{A \cap B} - B = \boxed{\overline{B}}$

$\overline{A \cap B} - B$

$= (\bar{A} \cup B) - B$ # de morgan's

$= (\bar{A} \cup \bar{B}) \cap \overline{B}$ # defn of difference

$= \boxed{\overline{B}}$ # absorption

To show 2 sets are <u>not</u> equal, use a counterexample

10:48

2. Set Representation

- Sets are unordered!

$\{a, e, i, o, u\} = \{i, e, a, u, o\}$

need to express in some kind of order

-  only has 0s and 1s
- Set representation:

1 - element $\in$ set

0 - element $\notin$ set

- Universal set $\rightsquigarrow$ rep with **bitstring**

(ex) $\mathcal{U} = \{0, 1, 2, 3, 4, 5\}$

_ _ _ _ _ _ $\rightsquigarrow$ bitstring

$S = \{1, 2, 5\}$

0	1	1	0	0	1	$= S$
0	1	2	3	4	5	

1 0 0 1 1 0 $= \overline{S}$

$= \{0, 3, 4\}$

$U = \{$Jordan K., Jon K., Donnie W., Danny W., Joey M.$\}$

$\overline{0/1}$	$\overline{0/1}$	$\overline{0/1}$	$\overline{0/1}$	$\overline{0/1}$

$A = \{x \mid$ first initial is J$\}$ 1 1 0 0 1

$B = \{x \mid$ last initial is K$\}$ 1 1 0 0 0

logic

$A \cup B$ 1 1 0 0 1 $\quad (11001 \vee 11000)$

$A \cap B$ 1 1 0 0 0 $\quad (11001 \wedge 11000)$

$\overline{B}$ 0 0 1 1 1 $\quad \neg 11000 = 00111$

$A - B$ 0 0 0 0 1

3. Set Functions

- Powerset
- Cartesian Product

Powerset

↳ collection: set of sets

$\{\{a,e,i,o,u\}, \{a,e\}\}$

every element is itself a set

↳ Powerset: collection

$$\mathcal{P}(S) = \{A \mid A \subseteq S\}$$

- $\{\} \subseteq$ everything
- every set $\subseteq$ of itself

$S = \{a\}$

$\mathcal{P}(S) = \{\{\}, \{a\}\}$

$S = \{a, b\}$

$P(S) = \{\{\}, \{a, b\}, \{a\}, \{b\}\}$

(ex) $A = \{1, 2, 3\}$

$\{1, 2\} \in P(A)$ T

$\{1, 2\} \subseteq P(A)$ F $\{1, 2\} \subseteq A$

$\{\{3\}\} \subseteq P(A)$ T

$|P(A)| = 10$ F

Cartesian Product $\times$

↳ given sets A, B

set of ordered pairs $(x, y) \neq (y, x)$

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

(ex) Strange House

- evening: Laney Show, Tom Show

$(L, T) \rightsquigarrow$ ordered pair

$L = \{gg, murders\}$ $T = \{archer, fg\}$

night at our house: $L \times T$

$= \{(gg, archer), (gg, fg), (murders, archer), (murders, fg)\}$

$|L \times T| = |L| \cdot |T|$

$|P(S)| = 2^{|S|}$

} next week!

set

$A = \{1, 2, 3\}$

$\{1, 2\} \subseteq A$

collection

$B = \{\{1, 2\}, \{3, 4\}\}$

$\{1, 2\} \in B$

$\{\{1, 2\}\} \subseteq B$

(Lanny meets Danny Wood!)