CS1800 Day 6

Admin:

- recitation solutions now available Friday (instead of immediately)

Content:

- Sets (subsets, empty set, powerset)
- Set Builder Notation
- Set Operations (Union, Intersection, Complement, Difference)

Sets

A set is a collection of unique objects

{a, b, c} = $\{a, b, c\}$

MY CURLY BRACES ARE NOT GREAT... SORRY!

$$\{1, 2, 3, 4\} = \{1, 2, 3, 4, 4\}$$

POOR FORM

AN ITEM IS IN SET OR NOT, NO ITEM IS IN SET MORE THAN ONCE

Example number sets you should be aware of:

Empty set

$$\emptyset = \{\ \}$$

SET W/ NO ITEMS

Integers

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

Natural Numbers

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

SOMETIMES NOT INCLUDED

Real Numbers

$\mathbb{R}$ CONTAINS

$-2, 0, 1/2, \pi, e$

Set Builder Notation: one way to express a set

SET MEMBERSHIP

$$A = \{x \in \mathbb{N} \mid (3 \leq x) \wedge (x \leq 5)\}$$

A is the set of x in Natural Numbers such that <some condition>

$$\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \ldots\}$$

$$A = \{x \in \mathbb{N} \mid (3 \leq x) \wedge (x \leq 5)\}$$

$$A = \{3, 4, 5\}$$

In Class Activity: Set Builder Practice

$|x|$ ABSOLUTE VALUE (DISTANCE FROM 0)

Express the set A by explicitly listing all items it contains

$$A = \{ x \in \mathbb{Z} \mid |x| < 5 \}$$

$$A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

Express the set B using set builder notation

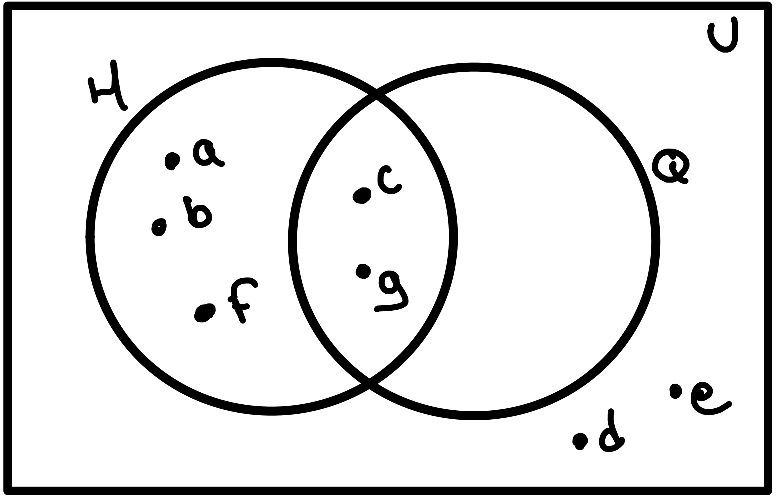
B = set of all natural numbers x which have x mod 3 = 0 and x mod 7 =0 and x < 40

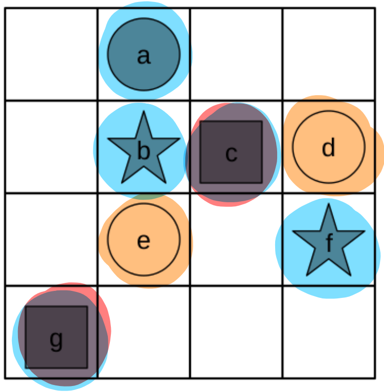
(++ list all of its items)

$$B = \{ x \in \mathbb{N} \mid (x \bmod 3 = 0) \land (x \bmod 7 = 0) \land (x < 40) \}$$

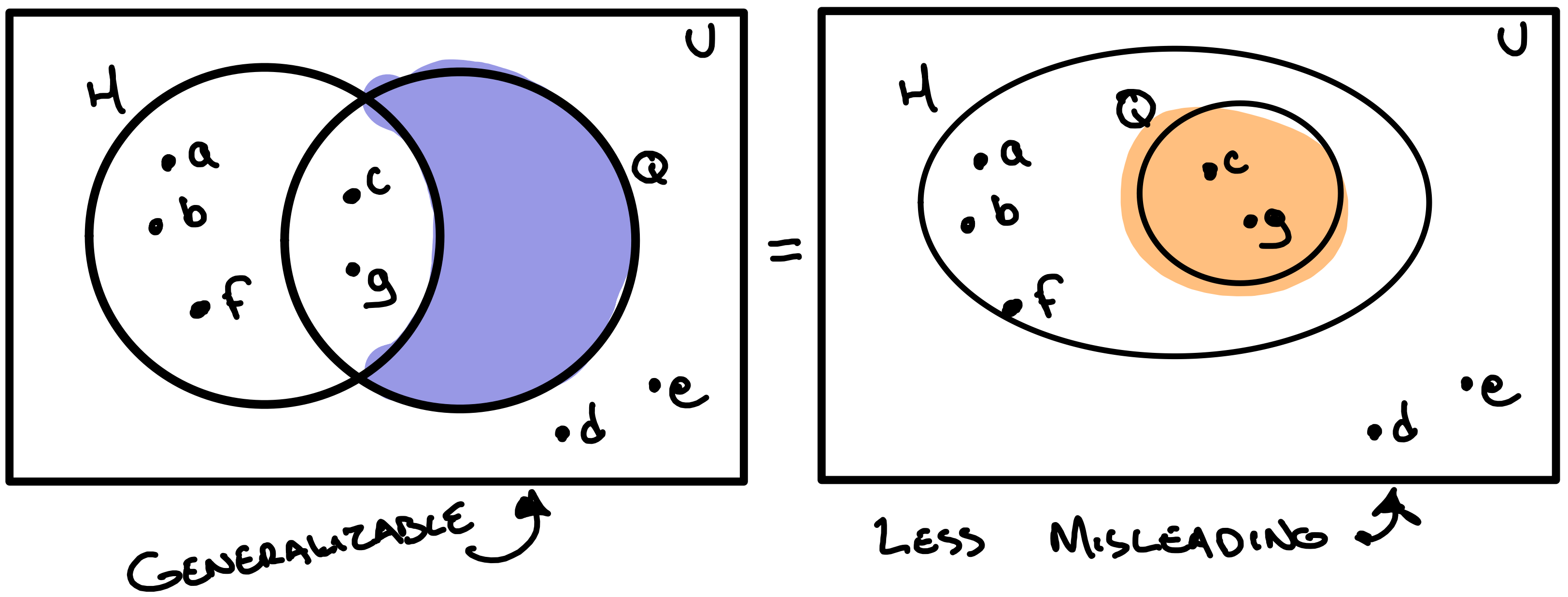
Venn Diagram: a way of visually representing set membership

H = set of all sHaded shapes
Q = set of all sQuares
U = Universal set, contains all shapes

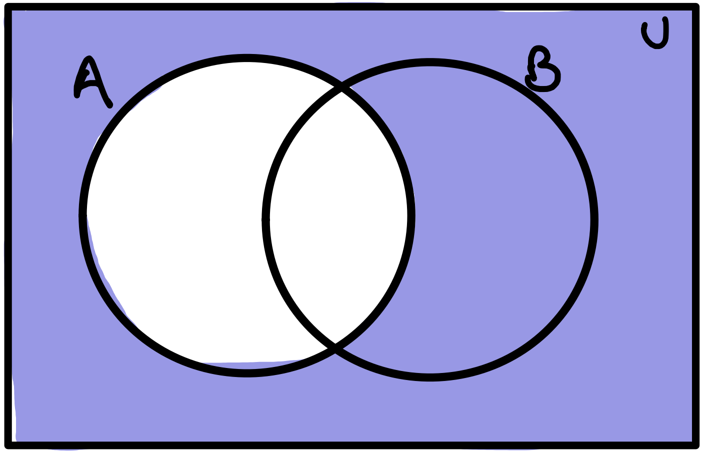




Venn Diagram Gotcha: Just because an area exists, doesn't mean it contains any items (may be empty)



Set Operation: Complement (all the items NOT in some set)



TWO NOTATIONS FOR SAME THING

$$\bar{A} = A^c = \{ x \in U \mid x \notin A \}$$

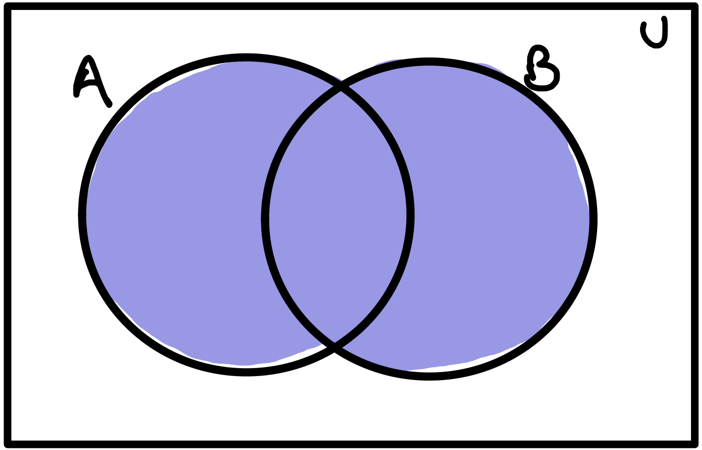
ALL x IN UNIVERSE

SUCH THAT

X IS NOT IN A

Set Operation: Union

(all the items in one set OR another)



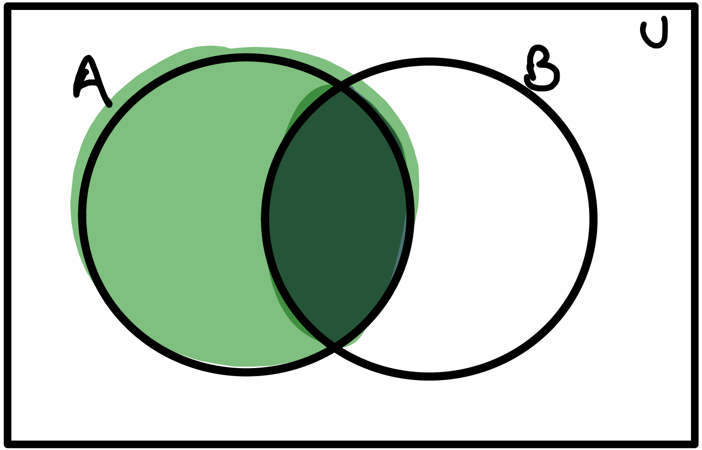
$$A \cup B = \{x \in U \mid x \in A \lor x \in B\}$$

ALL x IN UNIVERSE SUCH THAT

x IS IN A OR x IS IN B

Set Operation: Intersection

(all the items in one set AND another)



$$A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$$

ALL x IN UNIVERSE SUCH THAT

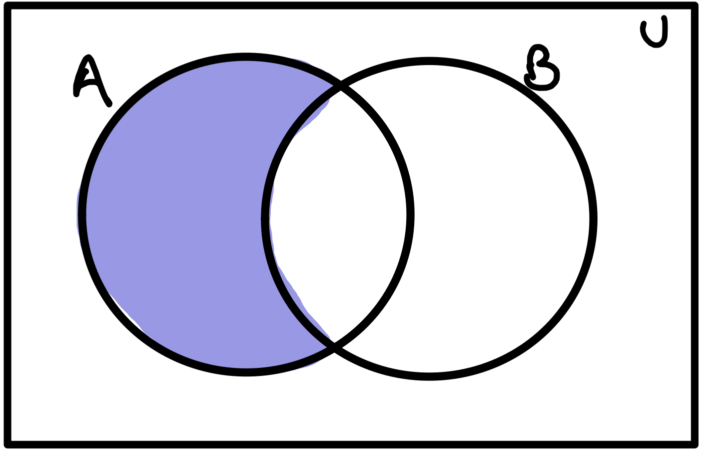
x IS IN A AND x IS IN B

TIP

$\cup$ UNION

$\cap$ INTERSECTION

Set Operation: Difference (All items in one set but not another)

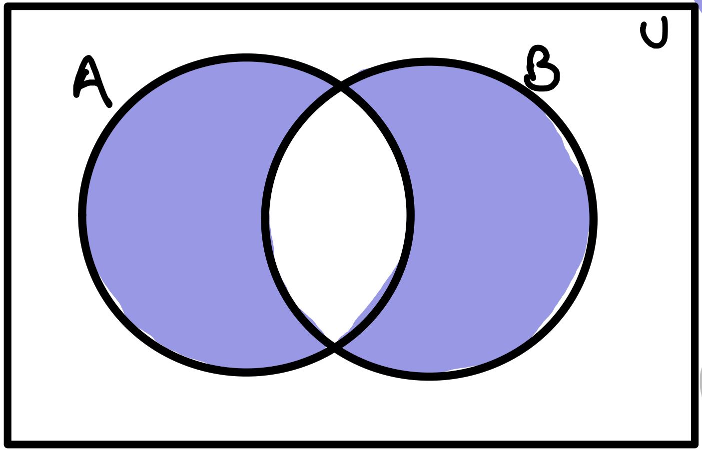


$$A - B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN A AND X IS NOT IN B

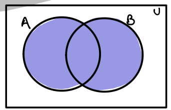
Set Operation: Symmetric Difference (All items in one set XOR another)
(All items in one set or the other, but not both)

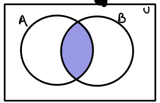


$$A \Delta B = \{x \in U \mid x \in (A \cup B) \land x \notin (A \cap B)\}$$

ALL X IN UNIVERSE SUCH THAT

X IS IN $A \cup B$ AND X NOT IN $A \cap B$

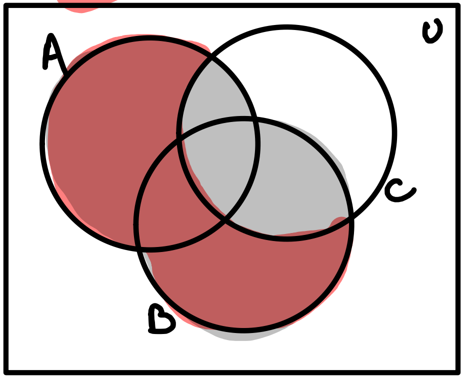




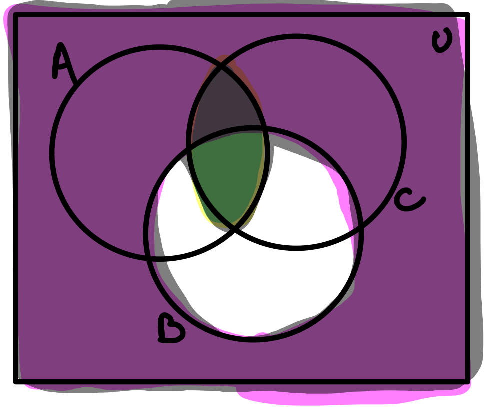
In Class Activity

Shade the indicated areas in each venn diagram

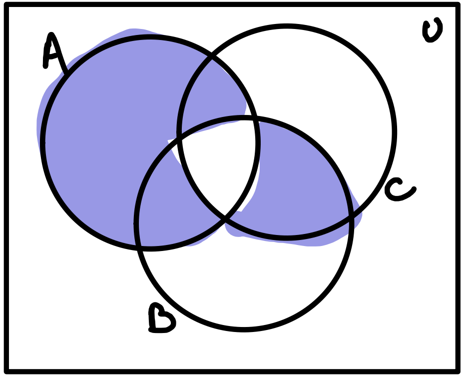
$(A \cup B) - C$



$(A \cap C) \cup B^c$

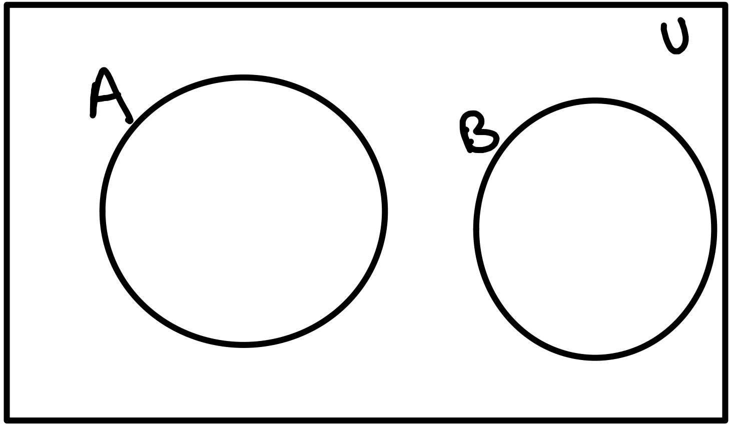


$A \,\Delta\, (B \cap C)$



Set Terminology: Disjoint Sets (two sets are disjoint if no item is in both sets)

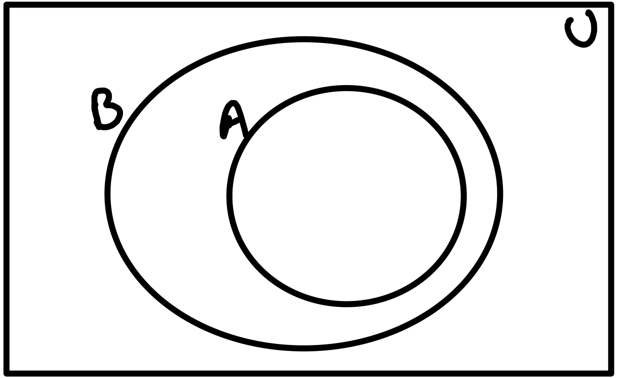
WE SAY A, B ARE DISJOINT IF $A \cap B = \emptyset$



NO ITEM CAN BE IN BOTH A AND B

Set Terminology: subsets

A is subset of B = all items in A are in B



$$A \subseteq B = \forall x \; x \in A \rightarrow x \in B$$

IF X IS IN A THEN X IS IN B

WE ILLUSTRATE LIKE THIS TO SHOW $A - B = \emptyset$

(THERE IS NO ITEM IN A NOT IN B)

Set Terminology: Set Equality

Given sets A, B:

we say that A=B if A is a subset of B and B is a subset of A.

$A \subseteq B$

$x \in A \rightarrow x \in B$

ALL X IN A ALSO IN B

$B \subseteq A$

$x \in B \rightarrow x \in A$

ALL X IN B ALSO IN A

INTUITION A, B HAVE SAME ITEMS

KIND OF FUNNY:

$A \subseteq B$ IS TRUE WHEN A, B ARE EQUAL

MIGHT CLARIFY TO ADD SPECIAL LANGUAGE TO DENOTE

- ARE NOT EQUAL
- ONE CONTAINED IN ANOTHER

Set Terminology: Proper Subset (one set is contained in another, larger, set)

$A \subset B$ = ALL ITEMS OF A ARE IN B = $A \subseteq B$

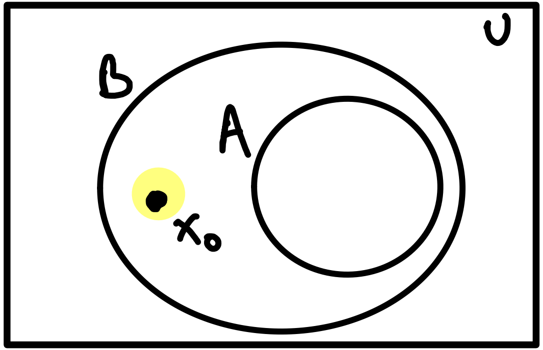
AND

B CONTAINS SOME ITEM NOT IN A

AND

$B - A \neq \emptyset$

"A IS PROPER SUBSET OF B"

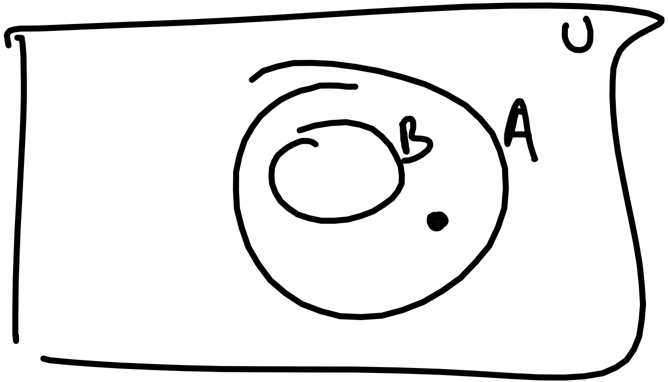


SUBSET

$A \subseteq B$ $\qquad\qquad\qquad$ $7 \leq 8$

PROPER SUBSET

$A \subset B$ $\qquad\qquad\qquad$ $7 < 8$



$B \subset A$

Set Terminology: Cardinality (the number of items in a set)

$$A = \{a, b, c, d\}$$

$$|A| = 4$$

Set Terminology: Power Set

The power set of set A is the set of all sets which can be made from items in A

$$A = \{1, 2\}$$

$$P(A) = \{ \{1\}, \{2\}, \{1, 2\}, \emptyset \}$$

EMPTY SET

IN CLASS ACTIVITY (IF TIME)

SUPPOSE $A = \{1\ 2\ 3\ 4\}$

COMPUTE

$$|A| = 4$$

$$16 = |P(A)| = \big|\{\emptyset, \{1\} \{2\} \{3\} \{4\}$$
$$\{12\} \{13\} \{14\} \{23\} \{24\}$$
$$\{123\} \{124\} \{134\} \{234\}, A\} \{34\}$$

$|A| = n$ $\qquad \sum_{k=0}^{k} \binom{n}{k}$ $\qquad 2^n$

0 ITEM $A = \emptyset$ $\qquad P(A) = \{\emptyset\}$ $\qquad 2^0 = 1$

1 ITEM $A = \{1\}$ $\qquad P(A) = \{\emptyset, 1\}$ $\qquad 2^1 = 2$

2 ITEM $A = \{1, 2\}$ $\qquad P(A) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ $\qquad 2^2 = 4$