

CS1800 Day 6

Admin:

- recitation solutions now available Friday (instead of immediately)

Content:

- Sets (subsets, empty set, powerset)
- Set Builder Notation
- Set Operations (Union, Intersection, Complement, Difference)

Sets

A set is a collection of unique objects

$$\{a, b, c\} = \{a, b, c\}$$

MY CURLY
BRACES ARE
NOT GREAT...

SORRY!



$$\{1, 2, 3, 4\} = \{1, 2, 3, 4, 4\}$$

POOR FORM

AN ITEM IS IN SET OR NOT,
NO ITEM IS IN SET MORE
THAN ONCE

Example number sets you should be aware of:

Empty set

$$\emptyset = \{ \}$$

SET W/ NO
ITEMS

Integers

$$\mathbb{Z} = \{ \dots, -2, -1, 0, 1, 2, \dots \}$$

Natural Numbers

$$\mathbb{N} = \{ 0, 1, 2, 3, \dots \}$$

SOMETIMES NOT
INCLUDED

Real Numbers

\mathbb{R} CONTAINS
 $-2, 0, 1/2, \pi, e$

Set Builder Notation: one way to express a set

$$A = \{ x \in \mathbb{N} \mid (3 \leq x) \wedge (x \leq 5) \}$$

Diagram description: The expression is written in black ink. 'A' is highlighted in yellow. The set symbol '{' is in a grey cloud. 'x ∈ N' is in a blue cloud, with 'SET MEMBERSHIP' written above it and an arrow pointing to 'x'. 'N' is in a green cloud. A vertical purple bar separates the set definition from the condition. The condition '(3 ≤ x) ∧ (x ≤ 5)' is in an orange cloud. A curly brace '}' is on the right.

A is THE SET OF x IN NATURAL NUMBERS SUCH THAT <SOME CONDITION>

$$\mathbb{N} = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \dots \}$$

Diagram description: The natural numbers are listed in black ink. The numbers 0 through 5 are highlighted in a red cloud. The rest of the set notation is in a grey cloud.

$$A = \{ 3, 4, 5 \}$$
$$A = \{ x \in \mathbb{N} \mid (3 \leq x) \wedge (x \leq 5) \}$$

Diagram description: The first equation 'A = {3, 4, 5}' is in black ink. The second equation is a restatement of the set builder notation. 'A' is in black. The set symbol '{' is in a grey cloud. 'x ∈ N' is in black. A vertical purple bar separates the set definition from the condition. The condition '(3 ≤ x) ∧ (x ≤ 5)' is in black. The parts '(3 ≤ x)' and '(x ≤ 5)' are highlighted in grey and red clouds respectively. A curly brace '}' is on the right.

In Class Activity: Set Builder Practice

Express the set A by explicitly listing all items it contains

$$A = \{x \in \mathbb{Z} \mid |x| < 5\}$$

$$|x|$$

ABSOLUTE VALUE
(DISTANCE FROM 0)

$$A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$

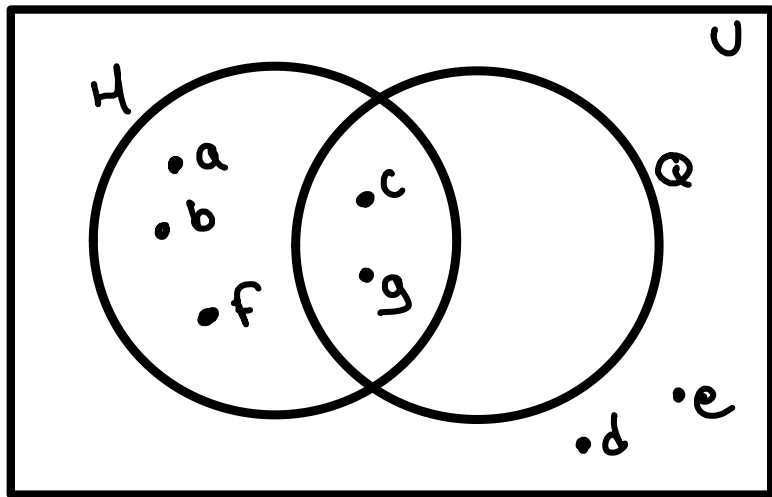
Express the set B using set builder notation

B = set of all natural numbers x which have $x \bmod 3 = 0$ and $x \bmod 7 = 0$ and $x < 40$

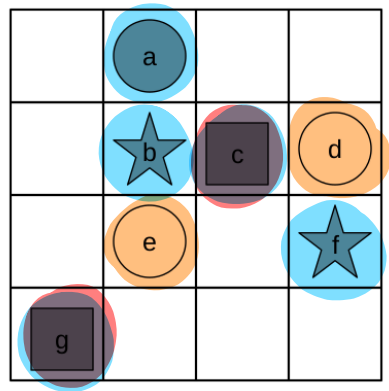
(++ list all of its items)

$$B = \{x \in \mathbb{N} \mid (x \bmod 3 = 0) \wedge (x \bmod 7 = 0) \wedge (x < 40)\}$$

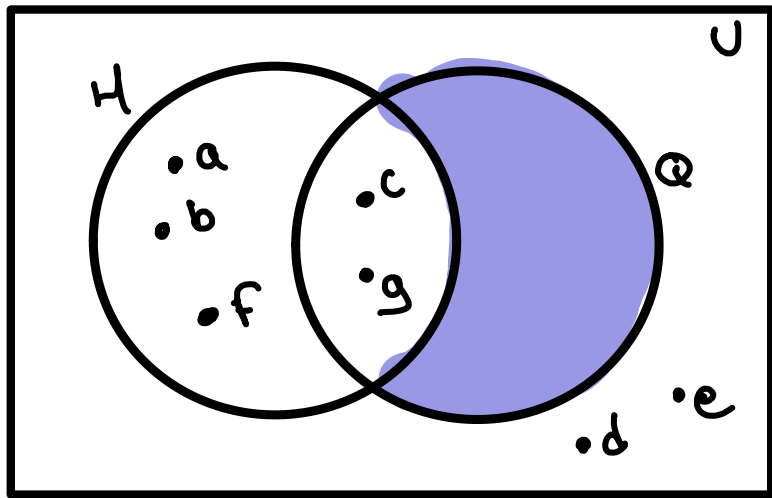
Venn Diagram: a way of visually representing set membership



- H = set of all sHaded shapes
 Q = set of all sQuares
 U = Universal set, contains all shapes

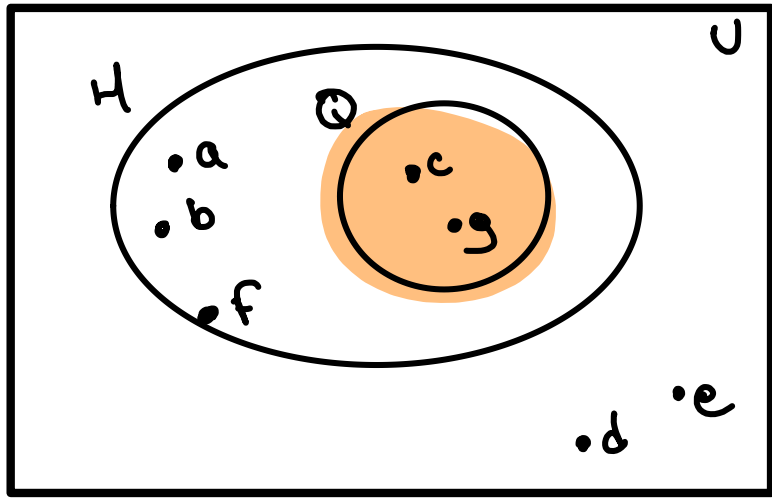


Venn Diagram Gotcha: Just because an area exists, doesn't mean it contains any items (may be empty)



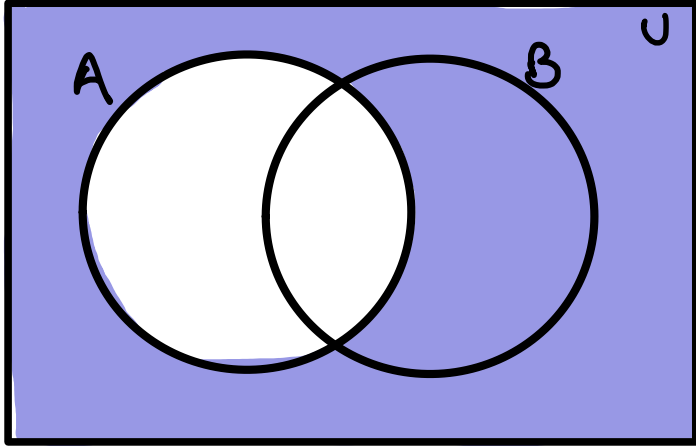
GENERALIZABLE ↗

=



LESS MISLEADING ↗

Set Operation: Complement (all the items NOT in some set)



TWO NOTATIONS FOR SAME THING

$$\overline{A} = A^c = \{x \in U \mid x \notin A\}$$

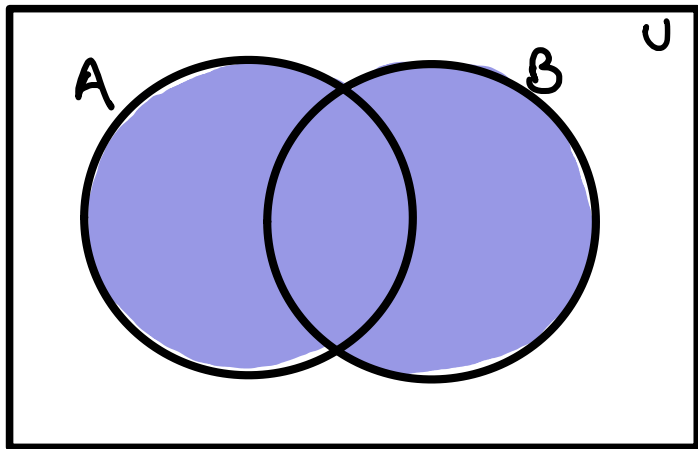
ALL x IN UNIVERSE

SUCH THAT

x IS NOT IN A

Set Operation: Union

(all the items in one set OR another)



$$A \cup B = \{x \in U \mid x \in A \vee x \in B\}$$

ALL x IN UNIVERSE SUCH THAT

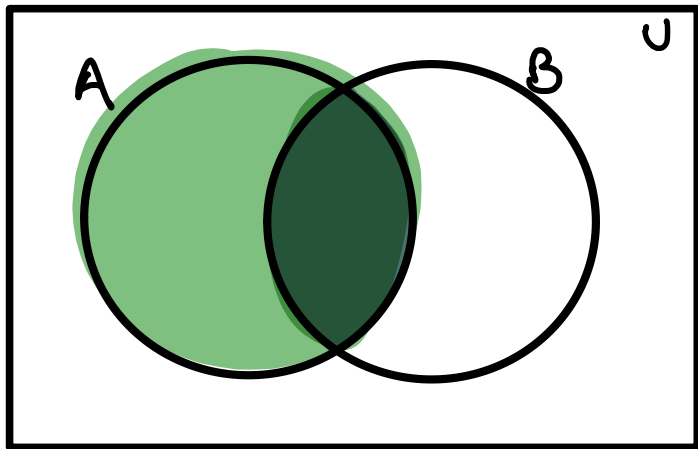
x IS IN A

OR

x IS IN B

Set Operation: Intersection

(all the items in one set AND another)



$$A \cap B = \{x \in U \mid x \in A \wedge x \in B\}$$

ALL x IN UNIVERSE SUCH THAT

x IS IN A

AND

x IS IN B



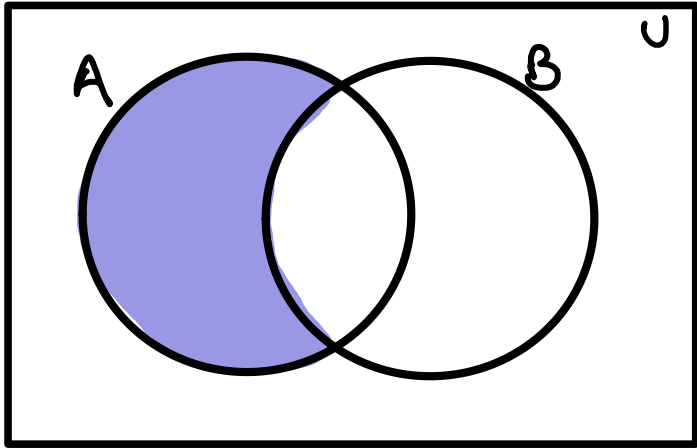
TIP

UNION



INTERSECTION

Set Operation: Difference (All items in one set but not another)



$$A - B = \{x \in U \mid (x \in A) \wedge (x \notin B)\}$$

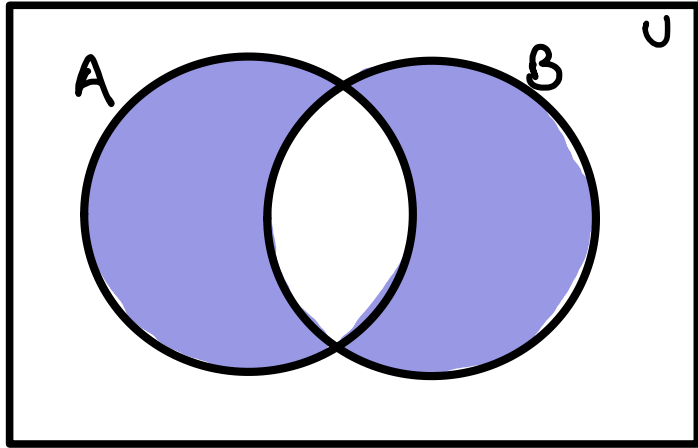
ALL X IN UNIVERSE SUCH THAT

X IS IN A

AND

X IS NOT IN B

Set Operation: Symmetric Difference (All items in one set XOR another)
(All items in one set or the other, but not both)



$$A \Delta B =$$

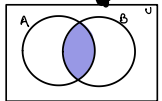
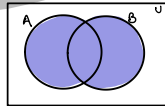
$$\{x \in U \mid x \in (A \cup B) \wedge x \notin (A \cap B)\}$$

ALL x IN UNIVERSE SUCH THAT

x IS IN $A \cup B$

AND

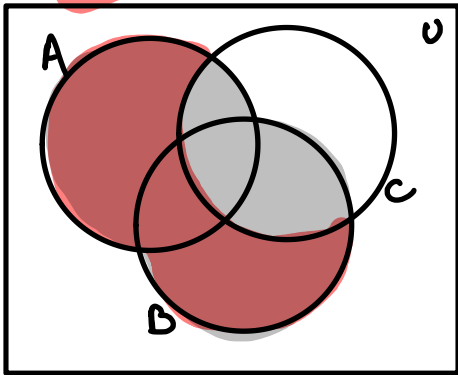
x NOT IN $A \cap B$



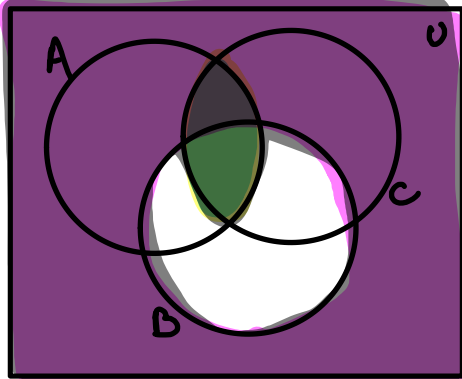
In Class Activity

Shade the indicated areas in each venn diagram

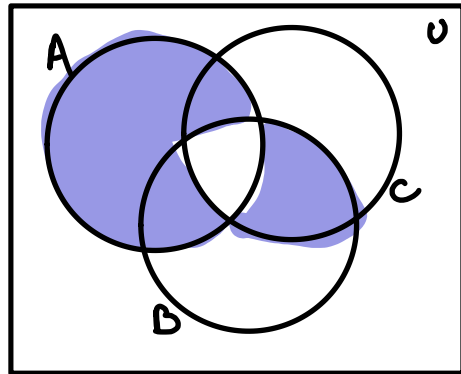
$$(A \cup B) - C$$



$$(A \cap C) \cup B^c$$

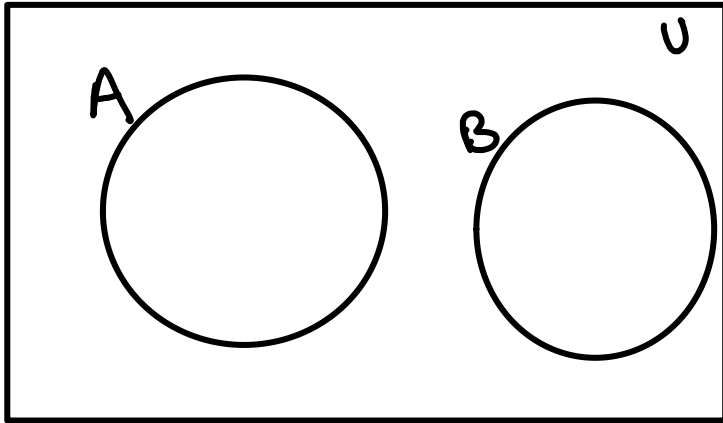


$$A \Delta (B \cap C)$$



Set Terminology: Disjoint Sets (two sets are disjoint if no item is in both sets)

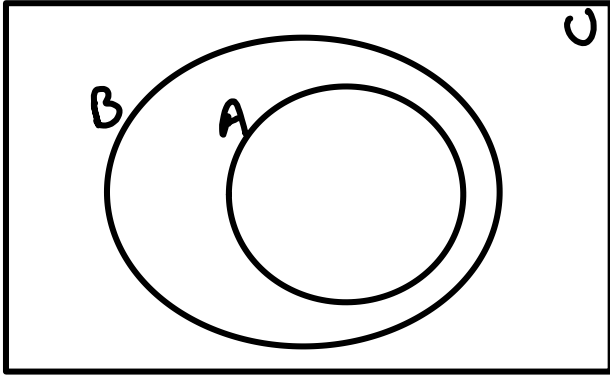
WE SAY A, B ARE DISJOINT IF $A \cap B = \emptyset$



← NO ITEM CAN
BE IN BOTH A AND
B

Set Terminology: subsets

A is subset of B = all items in A are in B



$$A \subseteq B = \forall x \quad x \in A \rightarrow x \in B$$

IF x IS IN A THEN x IS IN B

WE ILLUSTRATE LIKE THIS TO SHOW $A - B = \emptyset$
(THERE IS NO ITEM IN A NOT IN B)

Set Terminology: Set Equality

Given sets A, B:

we say that $A=B$ if A is a subset of B and B is a subset of A.

$$A \subseteq B$$

$$x \in A \rightarrow x \in B$$

ALL X IN A ALSO IN B

$$B \subseteq A$$

$$x \in B \rightarrow x \in A$$

ALL X IN B ALSO IN A

INTUITION A, B HAVE SAME ITEMS

KIND OF FUNNY:

$A \subseteq B$ IS TRUE WHEN A, B ARE EQUAL

MIGHT CLARIFY TO ADD SPECIAL LANGUAGE TO DENOTE

- ARE NOT EQUAL

- ONE CONTAINED IN ANOTHER

Set Terminology: Proper Subset (one set is contained in another, larger, set)

$A \subset B$

= ALL ITEMS OF A ARE IN B

AND

B CONTAINS SOME ITEM NOT IN A

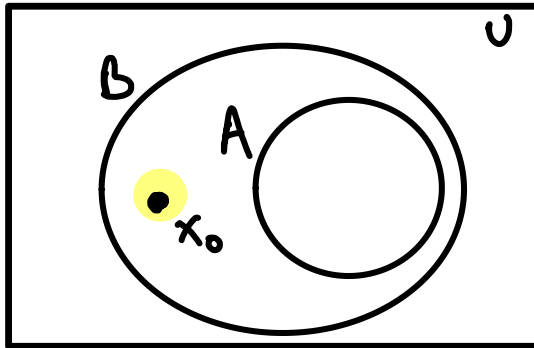
=

$A \subseteq B$

AND

$B - A \neq \emptyset$

"A is PROPER
SUBSET OF B"



SUBSET

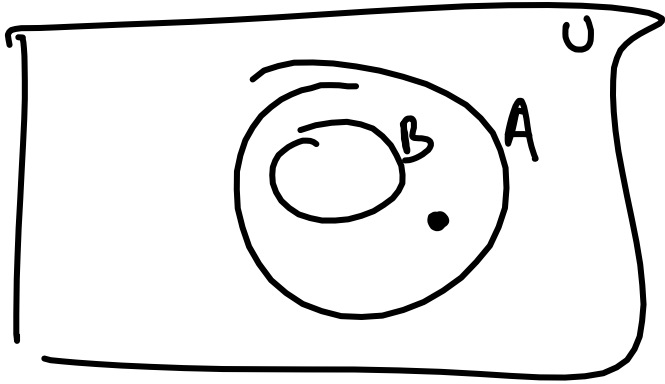
$$A \subseteq B$$

$$7 \leq 8$$

PROPER SUBSET

$$A \subset B$$

$$7 < 8$$



$B \subset A$

Set Terminology: Cardinality (the number of items in a set)

$$A = \{a, b, c, d\}$$

$$|A| = 4$$

)

Set Terminology: Power Set

The power set of set A is the set of all sets which can be made from items in A

$$A = \{1, 0\}$$

$$P(A) = \{ \{1\}, \{0\}, \{1, 0\}, \emptyset \}$$

↓
EMPTY SET

IN CLASS ACTIVITY (IF TIME)

SUPPOSE $A = \{1, 2, 3, 4\}$

COMPUTE

$$|A| = 4$$

$$16 = |P(A)| = \left| \begin{array}{l} \{ \emptyset, \{1\}, \{2\}, \{3\}, \{4\} \\ \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\} \\ \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, A, \{1, 2, 3, 4\} \end{array} \right|$$

$$|A| = n$$

$$\sum_{k=0}^n \binom{n}{k}$$

$$2^n$$

~~A~~

0 ITEM $A = \emptyset$

$$P(A) = \{ \emptyset \}$$

$$2^0 = 1$$

1 ITEM $A = \{i\}$

$$P(A) = \{ \emptyset, i \}$$

$$2^1 = 2$$

2 ITEM $A = \{i, j\}$

$$P(A) = \{ \emptyset, \{i\}, \{j\}, \{i, j\} \}$$

$$2^2 = 4$$