CS1800

11/21 - Tues. :)

Admin

- HW8 out, due 12/1
- reg lectures 11/28, 12/1
- exam #3 on 12/5 (short)

Agenda

1. Growth of functions Review
2. Sequences + Series

   2A - value of arbitrary term

   2B - sum the first n terms

1. Growth of Functions

Function $f(n)$ where n = size of input

$f(n)$ = # steps req'd by algorithm

Upper Band - big Oh

$$f(n) = O(g(n))$$

$f(n)$ grows more slowly than $g(n)$

$$\boxed{f(n) \leq c \cdot g(n) \quad \forall n \geq k}$$

Complexity class

$f(n)$ reps ~~an~~ the run-time of algorithm

assign to closest upper band

True or False?

$n^4 = O(n^3)$ F

$4 \lg n + n = O(n \lg n)$ T

$2n = O(n^2)$ T

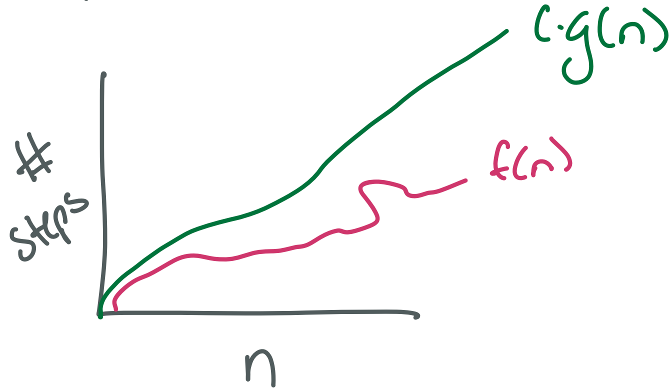
$2n = O(n)$ T $2n \leq c \cdot n \ \forall n \geq k$

$2^{2n} = O(2^{2^n})$ T $2n$ vs. 2^n

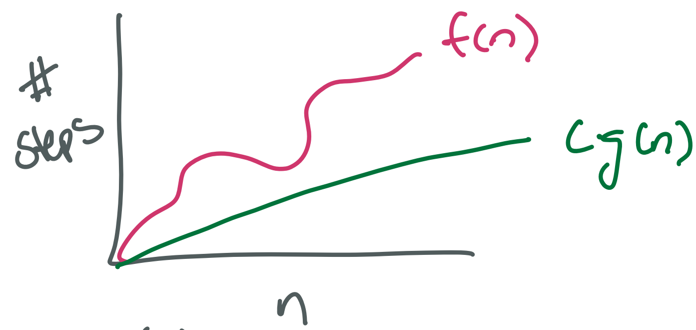
Complexity class?

$14n^3 + 14 + n$	n^3 , $\underline{n^k}$
$4\lg n + 2^{12}$	$\lg n$
2^{2n}	2^n , $\underline{k^n}$
1048231875	$O(1)$, $\underline{k}$
$16n + \lg n + 6n^2$	n^2 , $\underline{n^k}$
$3n + 7$	n

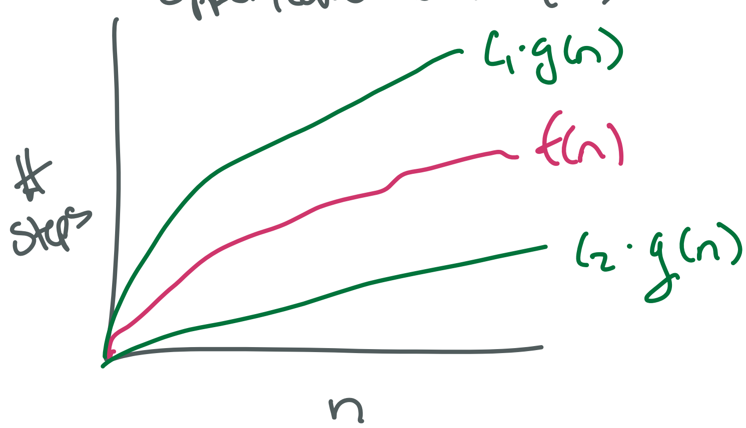
Upper-Band (O)



Lower-Band (Ω)



Upper/Lower band (Θ)



ex: $f(n) = 2n$

$f(n) \leq c_1 \cdot n$

$f(n) \geq c_2 \cdot n$

$c_1 = 3$

$c_2 = 1$

$2n = \Theta(n)$

2. Sequences + Series

Sequence: ordered list of numbers

like a set, except:

- order matters
- dupes are ok
- numbers only

Finite seq: $\{a_1, a_2, a_3, \ldots, a_n\}$

Infinite seq: $\{a_1, a_2, a_3, \ldots\}$

a_k = value of the k^{th} term

We care about:

1. computing the value of a_k
   - sequence has a pattern
   - we can figure out the next term, given the first 3-5
   - want formula for k^{th} term so I don't need to compute everything else
2. Sum of the first n values
   - we could calculate by hand (or in a loop/recursion)
   - want formula for the sum of the first n terms

Goal for both formulas:

- don't need all the previous values to compute
- ~~you~~ given k, a few other things compute a_k in one step $\boxed{O(1)}$
- by hand, or with a loop/recursion, it's k steps $\boxed{\text{linear}}$
- (same with computing sum of first n)

10:37

$\boxed{2A}$ Formula for term $\underline{a_k}$

Depends on what <u>type</u> of sequences

1. arithmetic
2. geometric
3. quadratic

big three!

each has formula for a_k

<u>Arithmetic</u>: always add same value to get to next term

ex $\{\underset{a_1}{4}, \underset{a_2}{7}, \underset{a_3}{10}, \underset{a_4}{13}, \underset{a_5}{16}, \dots\}$

6

$a_6 = 19$

- always add 3 to get to next term

  ↳ d, common difference

- Formula for $\boxed{a_k = a_1 + (k-1)\cdot d}$

According to formula: $a_6 = 4 + (6-1)(3)$

$= 4 + 5\cdot 3$

$= 19$ ✓

$a_{94} = 4 + (94-1)(3)$

$= 4 + 93\cdot 3$

$= 283$

Geometric: always multiply by same value to get next term

ex $\{1, 2, 4, 8, 16, 32, \ldots\}$

$a_1 \; a_2 \; a_3 \; a_4 \; a_5 \; a_6$

6

$a_7 = 64$

- always multiply by 2 to get next value

  ↳ r, common ratio

- Formula for $\boxed{a_k = a_1 \cdot r^{k-1}}$

According to formula

$$a_7 = (1)(2^6) = 64$$

$$a_{13} = (1)(2^{12}) = 4096$$

Quadratic : value of a_k is given by $\boxed{ak^2 + bk + c}$

our job is to figure out a, b, c (k, k^2 are given)

How do we know if a sequence is quadratic?

- its differences are arithmetic

ex $\{-3, 3, 13, 27, 45, \ldots\}$

diffs 6 10 14 18 ⟶ diffs are arithmetic seq (common diff = 4)

4 4 4

Second level difference

this sequence is quadratic, therefore: $a_k = ak^2 + bk + c$

we know

we need: a, b, c

$a_1 = -3 \qquad k=1 \qquad a\cdot 1^2 + b\cdot 1 + c = a+b+c = -3$

$a_2 = 3 \qquad k=2 \qquad a\cdot 2^2 + b\cdot 2 + c = 4a+2b+c = 3$

$a_3 = 13 \qquad k=3 \qquad a\cdot 3^2 + b\cdot 3 + c = 9a+3b+c = 13$

3 equations, need to find 3 variables:

$$\begin{array}{rl} & 4a+2b+c = 3 \\ - & a+b+c = -3 \\ \hline & 3a+b = 6 \end{array}$$

b in terms of a: $b = 6-3a$

$$\begin{array}{rl} & 9a+3b+c = 13 \\ - & a+b+c = -3 \\ \hline & 8a+2b = 16 \end{array}$$

Plug in $b = 6-3a$

Solve for a

$8a+2b=16$

$8a + 2(6-3a) = 16$

$8a + 12 - 6a = 16$

$2a = 4$

$a = 2$

$a_k = \underbrace{2}_{a}\cdot k^2 + bk + c$

Solve for b

$b = 6-3a$

$= 6 - 3\cdot 2 = 0$

$a_k = \underbrace{2}_{a}\cdot k^2 + \underbrace{0}_{b}k + c$

Solve for c

$a + b + c = -3$

$2 + 0 + c = -3$

$2 + c = -3$

$c = -5$

$2x = 2x^2 + -5$

$\boxed{\text{2B - Sum of the first } n \text{ terms}}$ Partial Series

In general: $\displaystyle\sum_{k=1}^{n} a_k = a_1 + a_2 + \dots + a_n$ (by defn)

linear

want: formula to compute the sum

constant

↳ depends on type of sequence!

Arithmetic

↳ ex $\{4, 7, 10, 13, 16, \dots\}$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

6

$\displaystyle\sum_{k=1}^{4} a_k = 4 + 7 + 10 + 13 = 34$

Formula: $\displaystyle\sum_{k=1}^{n} a_k = \frac{(\#\text{terms})(\text{first} + \text{last})}{2}$

According to formula

$\displaystyle\sum_{k=1}^{4} a_k$

\# terms: 4

first : 4

last: $a_4 = 4 + 3 \cdot 3 = 13$

$\dfrac{(4)(4+13)}{2} = 2 \cdot 17$

$= 34$

$$\sum_{k=1}^{100} a_k$$

terms: 100

first : 4

last : $a_{100} = 4 + 99 \cdot 3 = 301$

$$\frac{(100)(4+301)}{2} = 1525$$

~~exa~~ Geometric

↳ ex $\{1, 2, 4, 8, 16, \ldots\}$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \ldots$

$$\sum_{k=1}^{4} a_k = 1 + 2 + 4 + 8 = 15$$

Formula:

$$\boxed{\sum_{k=1}^{n} a_k = a_1 \cdot \frac{1 - r^n}{1 - r}}$$

According to formula:

$$\sum_{k=1}^{4} a_k$$

$a_1 = 1$

$r = 2$

$n = 4$

$$1 \cdot \frac{(1 - 2^4)}{(1-2)} = \frac{1-16}{-1} = \frac{-15}{-1} = 15$$

!!?

$$\sum_{k=1}^{12} a_k$$

$a_1 = 1$

$r = 2$

$n = 12$

$$1 \cdot \frac{((1 - 2^{12}))}{((1-2))} = \frac{1 - 4096}{1 - 2} = 4095$$

!!?

What type of sequence do you have?

- Arithmetic: common difference
- Geometric: common ratio
- Quadratic: differences are arithmetic

Arithmetic

$$\boxed{a_k = a_1 + (k-1)\cdot d}$$

$$\sum_{k=1}^{n} a_k = \boxed{\frac{(\#\text{terms})(\text{first} + \text{last})}{2}}$$

Geometric

$$\boxed{a_k = a_1 \cdot r^{k-1}}$$

$$\boxed{\sum_{k=1}^{n} a_k = a_1 \cdot \frac{1-r^n}{1-r}}$$

Quadratic

$$\boxed{a_k = ak^2 + bk + c}$$

We know

We need: a, b, c