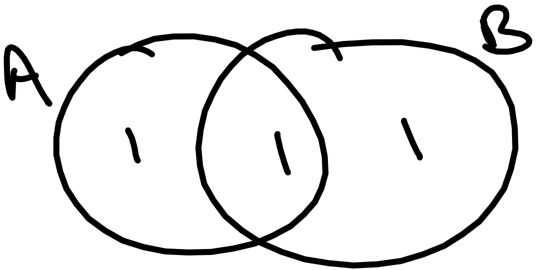
CS1800 Day 9

Admin:
- exam1 is on Oct 17th
- hw3 due today, hw4 released today
- hw4 deadlines are funny (for exam):
    - includes content from day10 (next class)
    - solutions for hw4 released sunday oct 15 @ 12:01am, first thing in the morning
        - good news: allows you study
        - bad news: you may only use up to 1 late day on hw4
- (next tuesday we'll do a "practice" exam together on gradescope so you can see the format, its pretty much a timed HW assignment)

Content:
- review PIE & product rule
- permutations
- count by partition
- count by complement
- count by simplification

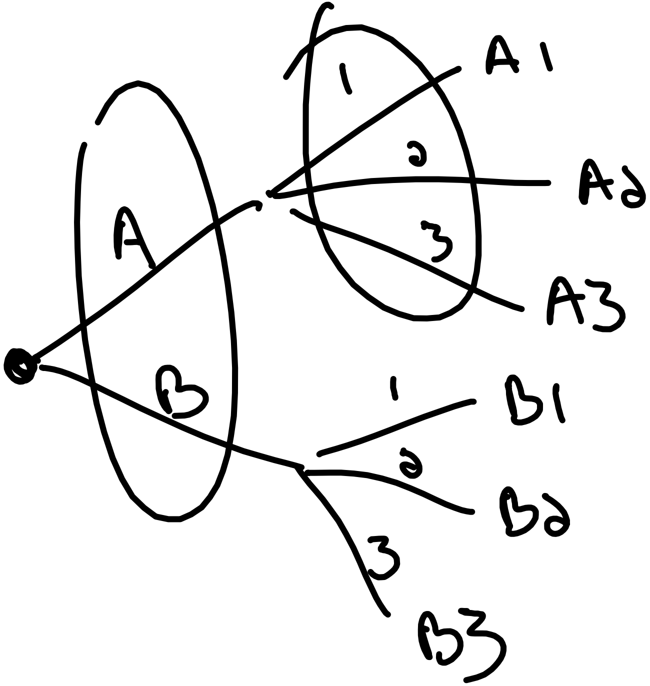
PIE



$$|A \cup B| = |A| + |B| - |A \cap B|$$

SUM RULE (ASSUMES $|A \cap B| = 0$)

$$|A \cup B| = |A| + |B|$$



$2 \times 3 = 6$ TOTAL OPTIONS

PRODUCT

$$|A \times B| = |A| \times |B|$$

NOTATION (REMINDER)

SET

$\{a, b, c\}$

NO REPEATS

UNORDERED

TUPLE

(a, b, c, a)

MAY REPEAT

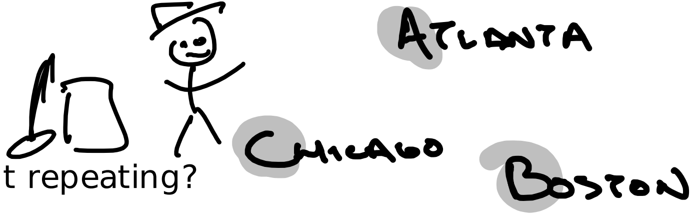
ORDER MATTERS

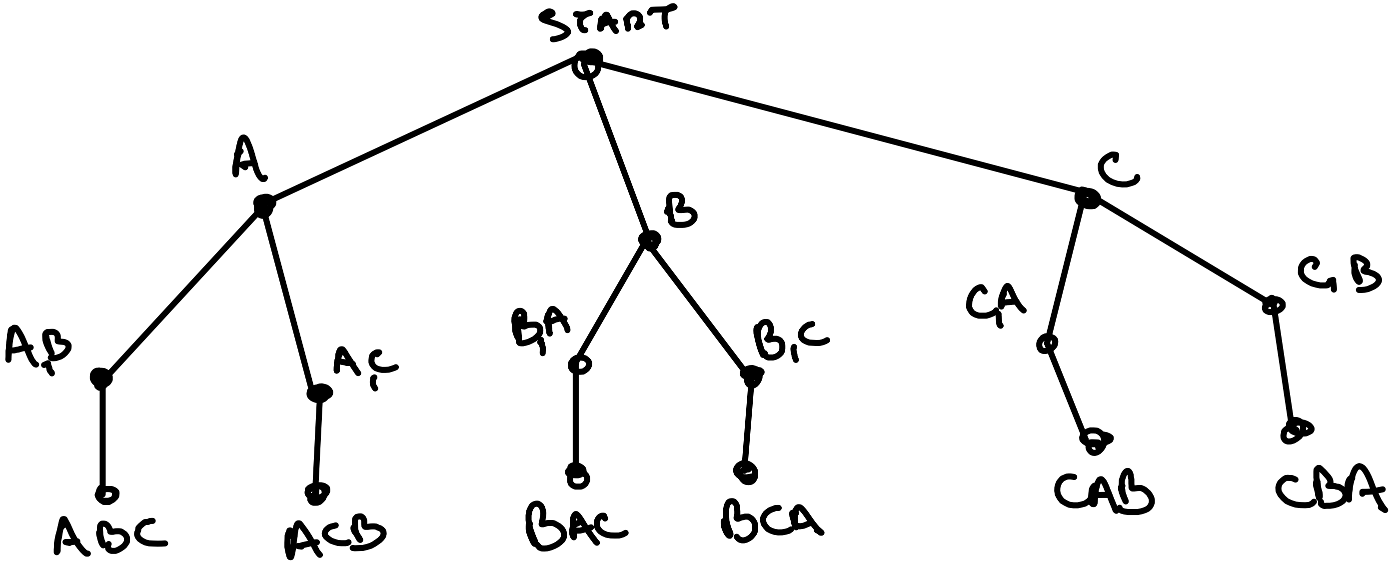
$(a, b) \neq (b, a)$

Permutations: Travelling Salesperson

How many ways can a salesman order 3 city visits?

(How many tuples can we make from items A, B, C without repeating?)





Permutations: Travelling Salesperson

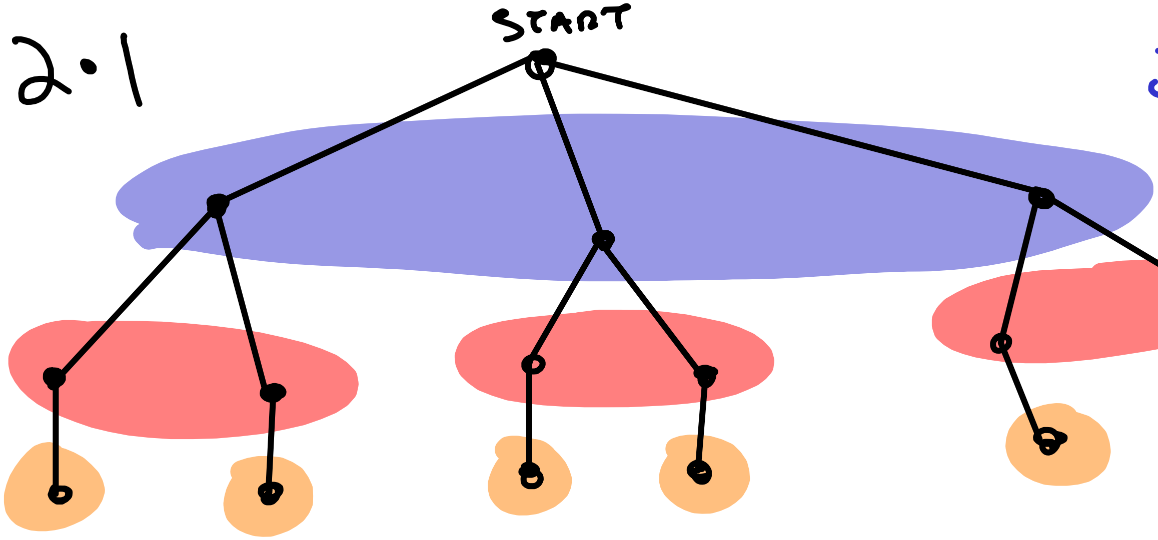
How many ways can a salesman order 3 city visits?

(How many tuples can we make from items A, B, C without repeating?)

ATLANTA

CHICAGO BOSTON

$3 \cdot 2 \cdot 1$



3 CHOICES FOR 1ST CITY

2 CHOICES FOR 2ND

1 CHOICE FOR LAST CITY

Factorial:

"8 FACTORIAL"

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

CONVENTION $0! = 1$

$$1! = 1$$

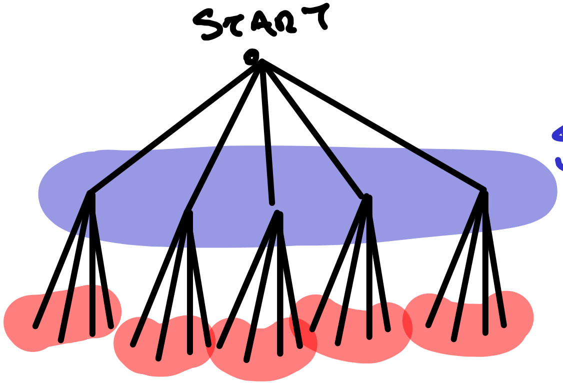
OUR SALESMAN (OF PREVIOUS SLIDE) HAD

$3! = 3 \cdot 2 \cdot 1$ TOTAL ORDERINGS OF 3 CITIES

Permutations: A Travelling (lazy) Salesperson

How many ways can a salesman order 2 of 5 cities?

(How many tuples of length 2 can be made from A, B, C, D, E where no repeats allowed)?



5 CHOICES FOR 1ST CITY

4 CHOICES FOR 2ND CITY

$$5 \cdot 4 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{5!}{3!}$$

Permutations:

The number of ways of ordering k objects, from n total available is:

$$P(n,k) = \frac{n!}{(n-k)!}$$

EXAMPLES:

VISIT 3 OF 3 CITIES

$$P(3,3) = \frac{3!}{(3-3)!} = 6$$

VISIT 2 OF 5 CITIES

$$P(5,2) = \frac{5!}{(5-2)!} = 20$$

$$P(n,n) = \frac{n!}{(n-n)!} = n!$$

$$P(n,3) = \frac{n!}{(n-3)!} = \frac{n \cdot n-1 \cdot n-2 \cdot \cancel{n-3} \cdot \cancel{n-4}}{\cancel{n-3} \cdot \cancel{n-4}}$$

$$n \cdot n-1 \cdot n-2$$

In Class Activity

How many ways are there to order 5 people for a family portrait?

$$P(5,5) = \frac{5!}{(5-5)!} = 5! = 120$$

How many ways are there to order 6 of 20 people for a family portrait?

$$P(20,6) = \frac{20!}{(20-6)!} = \frac{20!}{14!} \approx 28 \text{ MILLION}$$

(If time): Plug a few of these factorials into calculator or google, how big of a factorial do you need to plug in until you "break" your computer?

Factorials grow really quickly: (more on this when we study "function growth" later)

$10! \simeq 3 \cdot 10^6$ 3 MILLION

$20! \simeq 2 \cdot 10^{18}$

$19! \simeq 10^{17}$ SEC SINCE BIG BANG

$50! \simeq 10^{80}$ ATOMS IN UNIVERSE

$70! \simeq 10^{100}$ "GOOGLE"

Convention (in this class):

FEEL FREE TO LEAVE EXPRESSION AS

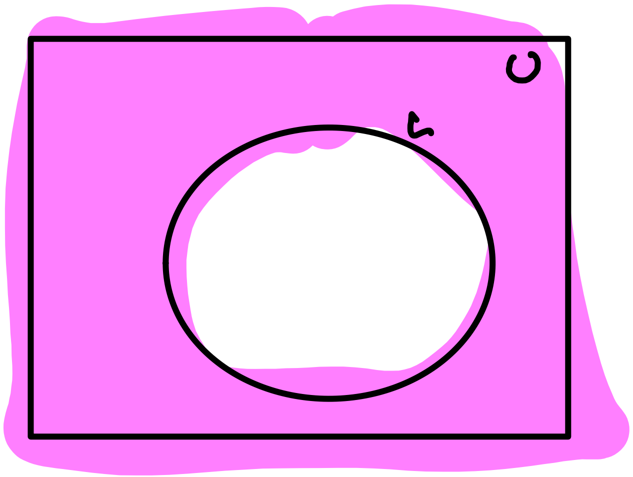
$P(5,3)$ OR $\dfrac{5!}{2!}$

Counting "moves":

- count by complement
- count by partition
- count by simplify

Count-by-complement

How many ways are there to order 5 people such that person A is not last?

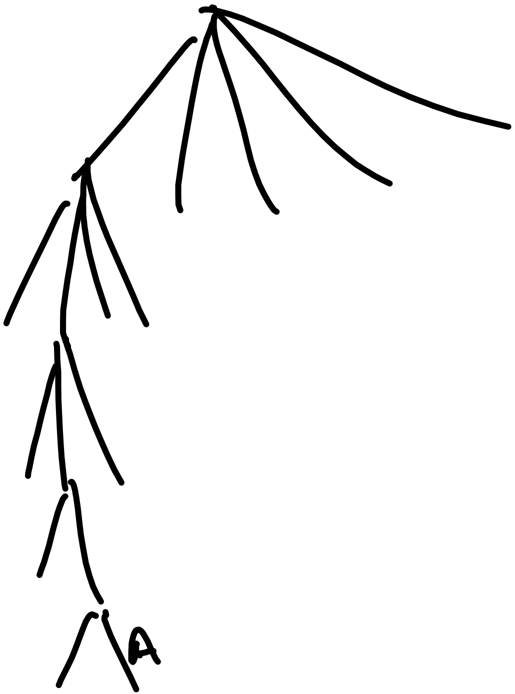


L = SET OF ORDERINGS WHERE A IS LAST

U = ALL ORDERINGS OF 5 PEOPLE

$|U| = 5!$ $|L| = 4!$

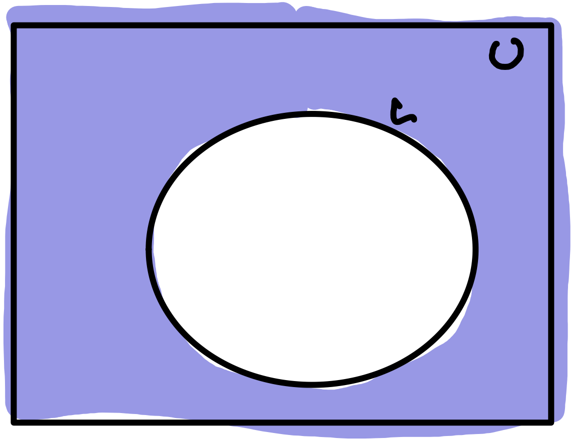
$5! - 4!$


A

Count-by-complement

How many ways are there to order 5 people such that person A is not last?

General approach: If we can count everything (U) and all items we're not interested in (L) then we can subtract the two to count the items of interest.



$$|U - L| = |U| - |L|$$

⚠ $|A - B| = |A| - |B|$ NOT TRUE IN GENERAL

Count-by-partition: motivating example

How many passwords can be made of lowercase letters which are no longer than 5 characters?

0 1 2 3 4 5

$$1 + 26 + 26^2 + 26^3 + 26^4 + 26^5$$

$$|A \cup B| = |A| + |B|$$

Partition: definition

Intuition: partition of set A divides its items into groups so each item is in exactly one group

$A = \{1, 2, 3, 4\}$

$A_1 = \{2, 3\}$

$A_2 = \{1\}$

$A_3 = \{4\}$

Definition: partition of set A is a set of sets A_1, A_2, ... such that

$i \neq j \rightarrow A_i \cap A_j = \emptyset$

EACH ITEM OF A IS IN AT MOST ONE A_i

$A_1 \cup A_2 \cup A_3 \ldots \cup A_N = A$

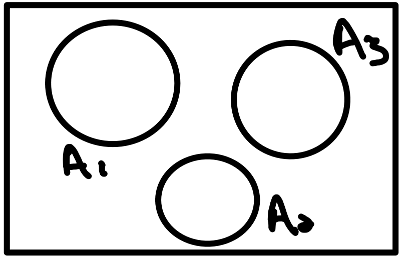
AT LEAST ONE A_i

Count-by-partition:

$$|A| = |A_1 \cup A_2 \cup A_3|$$

$$= |A_1| + |A_2| + |A_3|$$

A_i A_j ARE DISJOINT
(ALL INTERSECTIONS EMPTY)



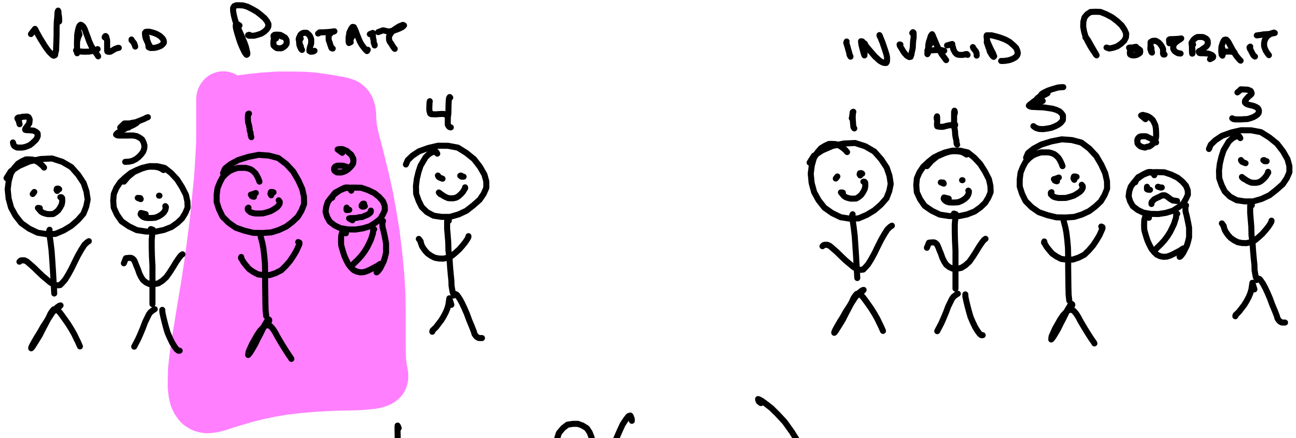
Approach:

Count items by partitioning them into subsets

(common error: ensure that every item is in exactly one subset)

Count-by-simplification:

How many ways can we order 5 family members for a portrait if person 2 is a baby and must be on person 1's immediate right?



$$4! = P(4,4)$$

Mea Culpa:

"Count-by-simplification" isn't really a particular approach like others ...

point is, be on the lookout for equivilent problems more easily counted

In Class Activity

How many passwords of length 10, made of lowercase characters, whose first 6 characters are not "qwerty"?
(hint: complement)

How many ways are there to order 3 people in a wedding photo for romeo and juliet?
Assume:

- there are 10 Montague's (Romeo's family, excluding him) who could be in the photo
- there are 7 Capulets (Juliet's family, excluding her) who could be in the photo
- Romeo and Juliet are too busy dancing to be in any picture
- Montagues and Capulets won't get in the same photo (that whole Tybalt / Mercutio thing...)

(hint: partition, simplify a bit)

How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: partition, complement)

How many passwords of length 10, made of lowercase characters, don't start with "qwerty"?
(hint: complement)

$$\cancel{\frac{26!}{16!}}$$

~~UNIVERSAL~~

~~LIMIT~~

Q W E R T Y _ _ _ _

$$26^{10} - 26^{4}$$

How many ways are there to order 3 people in a wedding photo for romeo and juliet?
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(hint: partition, simplify a bit)

ORDER 3 OF 10 MONT

$P(10,3)$

9

ORDER 3 OF 7 CAP

$P(7,3)$

$P(10,3) + P(7,3)$

How many ways are there to order 5 of 7 people in a family portrait such that person 1, if included, is not immediately to right of person 2?
(hint: partition, complement)

ALL ORDERINGS OF 5 PEOPLE FROM 7

$P(7,5)$

ORDERINGS WHERE P1 IS RIGHT OF P2

2 1 _ _ _ $P(5,3)$

_ 2 1 _ _ $P(5,3)$

_ _ 2 1 _ $P(5,3)$

_ _ _ 2 1 $P(5,3)$

$4 \cdot P(5,3)$

$P(6,4)$

$P(7,5) - 4 \cdot P(5,3)$

From discussion after sec4's class (another way of doing the same)

ORDERINGS WHERE P1 IS RIGHT OF P2 $= P(6,4) - P(5,4)$

```
matt@matt-yoga-nu:~$ python3
Python 3.10.12 (main, Jun 11 2023, 05:26:28) [GCC 11.4.0] on linu
Type "help", "copyright", "credits" or "license" for more informa
>>> from math import factorial, perm
>>> perm(6, 4) - perm(5, 4)
240
>>> 4 * perm(5, 3)
240
>>>
```

GLUE P1 TO RIGHT OF P2

PEOPLE: 21, 3, 4, 5, 6, 7

WAYS OF ORDERING 4 OF THESE $P(6,4)$

ONE IS 21 GLUED

BUT THIS ALSO INCLUDES THE $P(5,4)$ WAYS WHERE 21 IS NOT INCLUDED