

CS1800

10/6 - Fri !!

## Admin

• Hw3 due today 11:59pm

• Hw4 out now

↳ incl. from 10/10  
only one late day

• Exam #1 10/17 9-5:30pm

• Fri the 13th !!

optional lecture - exam review  
practice exam

## Agenda

1. Product/Sum Rule
2. Permutations
3. Combinations

# I. Product Rule, Sum Rule

Counting

How many ways...?

task 1 -  $n$  ways to do it

task 2 -  $m$  ways to do it

Product Rule:

there are  $n \cdot m$  ways to do task 1 and task 2

and  $\rightsquigarrow$  multiply!

Sum Rule:

there are  $n + m$  ways to do task 1 or task 2

or  $\longrightarrow$  addition!

$$\text{(ex)} \quad L = \{GG, \text{Only}\} \quad T = \{FG, \text{Archer}\}$$

$$\text{Cartesian Product} \quad L \times T = \{(GG, FG), (GG, \text{Arch}), \\ (\text{only}, FG), (\text{only}, \text{Arch})\}$$

order matters  $\longrightarrow$  only Laney, then 2 Tam  
 $(FG, GG) \notin L \times T$

$$L \times T \cong T \times L$$

In general...

$$|A \times B| = |A| \cdot |B|$$

Because product rule!

task 1: select element from A ( $|A|$  ways)

task 2: select element from B ( $|B|$  ways)

Deck of cards

• 52 cards total

• 13 values per suit: A, 2, ..., Q, K

• 4 suits: C, D, H, S

Universe: draw a card, put back in the deck (when relevant)

How many ways...

• to select a Q and then a K?

↳ multiply!

• task 1 x task 2

$$4 \cdot 4 = 16$$

Cartesian Product

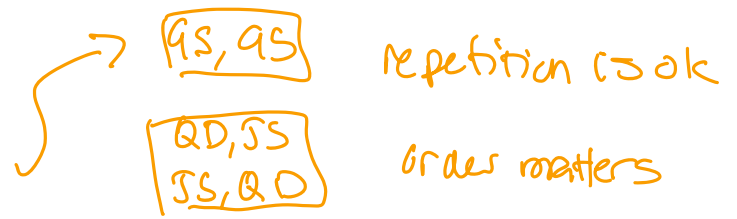
$$\{QD, QH, QS, QC\} \times \{KD, KH, KS, KC\}$$

$$|\text{Set 1}| \cdot |\text{Set 2}|$$

$$= 4 \cdot 4 = 16$$

How MANY WAYS ...

• to select 2 cards?



• task 1: 52

task 2: 52

total  $52 \cdot 52 = 2704 = 52^2$

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Bit strings: sequence of 0s and 1s

How MANY BIT STRINGS EXIST ...

• of length 7?

→ repetition is ok  
| | | | | | |

7 tasks: select 1 or  $\emptyset$

→ order matters? yes!

$$\frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2}$$

• total:  $2^7 = 128$  ( $n^k$ )

• of length 7, that start with 11

$$\frac{1}{1} \quad \frac{1}{1} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2} \quad \frac{0/1}{2}$$

$$1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$$

Flashback ... Powerset

$$S = \{a, b, c\}$$

0 = not there

1 = there

| a   | b   | c   |
|-----|-----|-----|
| 0   | 0   | 0   |
| 0   | 0   | 1   |
| 0   | 1   | 0   |
| 0   | 1   | 1   |
| ... | ... | ... |

task 1: a in subset?

2

task 2: b in subset?

2

task 3: c in subset?

2

$$2 \cdot 2 \cdot 2 = 2^3 = 8$$

and  $\rightsquigarrow$  multiply

or  $\rightsquigarrow$  addition

How MANY WAYS...

- to pick Q then K, or K then Q

- task 1 Q then K: 16

• task 2 K then Q: 16

• total:  $16 + 16 = 32$  ways

How MANY BITSTRINGS EXIST....

• of length 6, 7, or 8?

task 1:  $2^6$

task 2:  $2^7$

task 3:  $2^8$

total  $2^6 + 2^7 + 2^8$

$= 448$

• of length 7, that start with 01 or 10?

task 1:  $\underline{0} \underline{1} \_ \_ \_ \_ \_ = 2^5$

task 2:  $\underline{1} \underline{0} \_ \_ \_ \_ \_ = 2^5$

total:  $2^5 + 2^5 = \boxed{64}$

# Counting Problem...

- repetition ok?
- does order matter?

} From the problem,  
or from context

now we know how to approach the problem

|   |   |
|---|---|
| order matters<br>no repetition ✓<br><u>permutation</u>      | order matters<br>repetition ok ✓<br><u><math>n^k</math></u>                       |
| order doesn't matter<br>no repetition<br><u>combination</u> | order doesn't matter<br>repetition ok<br><u>stars + bars</u><br>"balls into bins" |

product  
rule

10:48

## 2. Permutations

$$L = \{GG, \text{Only}, \text{Roots}\}$$

Watch all three shows  $\rightarrow$  one of each, no rep.

Order matters  $\rightarrow$  GG, only  $\neq$  only, GG

Still product rule:

$$\underline{3} \times \underline{2} \times \underline{1} = 6$$

show 1    show 2    show 3

Also: Permutation  $\rightsquigarrow$  Formula

r-permutation

n objects, arrange r of them

$$P(n, r) = \frac{n!}{(n-r)!}$$

Shows example:

$$n = 3$$

$$r = 3$$

$$P(3, 3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3 \cdot 2 \cdot 1$$

$$= 6$$

Formula Breakdown:

n!  $\rightsquigarrow$  select w/o repetition



$(n-r)!$   $\rightarrow$  tells us when to stop

(ex) 4 Queens in a hand

$\{QC, QD, QS, QH\}$   $\rightarrow$  start as set

How MANY WAYS...

• to arrange 2 of them?

$$\underline{4} \cdot \underline{3} = \boxed{12}$$

product rule

$$n=4$$

$$r=2$$

$$\frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 4 \cdot 3 = \boxed{12}$$

perm formula

# 3. Combinations

|  |  |
|--|--|
| order matters<br>no repetition ✓<br>[permutation]      | order matters<br>repetition ok ✓<br>[ $n^k$ ]                                |
| order doesn't matter<br>no repetition<br>[combination] | order doesn't matter<br>repetition ok<br>[stars + bars]<br>"balls into bins" |

order doesn't matter... GG, only = only, GG

$$n! = (n)(n-1)(n-2) \dots (2)(1)$$

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

...

Factorial

gets v. big v. fast!

By hand...

- Pick 2 Queens
- order doesn't matter
- result goes in a set

{H, D}

{C, H}

{S, C}

{S, D}

{S, H}

{C, D}

$$\{D, H\} \times \times \times \frac{1}{n} = \{H, D\}$$

# Combination Formula

- $n$  objects
- choose  $k$  of them
- order doesn't matter

$$C(n, k) = \binom{n}{k} =$$

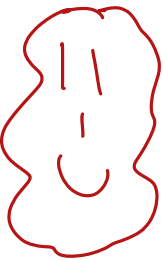
$$\frac{n!}{k!(n-k)!}$$

Choose 2 Queens

$$n = 4$$

$$k = 2$$

$$\frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(2 \cdot 1)} = 3 \cdot 2 = 6$$



$n!$   $\rightsquigarrow$  no repetitions

$(n-k)!$   $\rightsquigarrow$  when to stop

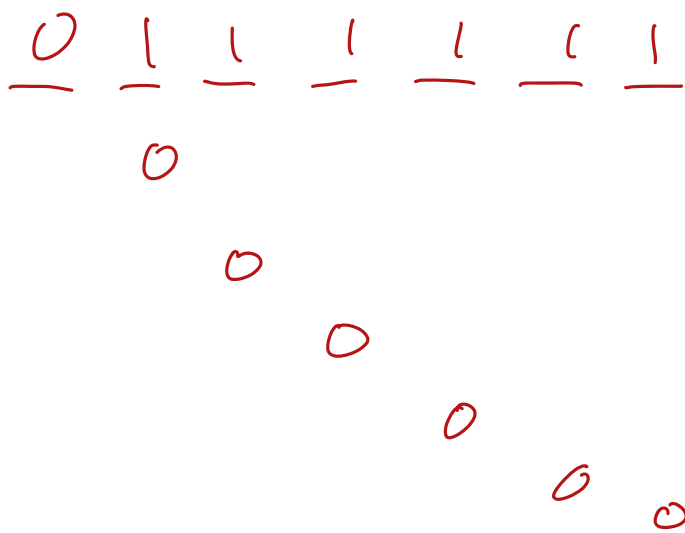
$k!$   $\rightsquigarrow$  don't double count

How many BIT STRINGS EXIST....

- of length 7
- with exactly one zero?

Secretly Combination!

We are choosing the position of  
the zero



By hand  
 $\boxed{7}$  places to put  
the zero  
everything else is ones

Combination:

$$n = 7$$

$$k = 1$$

$$\binom{7}{1} = \frac{7!}{1!(6!)} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7$$

How many bit strings exist of length 10  
with 0, 1, or 2 zeros?

• task 1 ~ 0 zeros

(1)

• task 2 ~ 1 zero

(10)

• task 3 ~ 2 zeros

$$\binom{10}{2} = \frac{10!}{2!8!} = 45$$

→  
↪ choose positions  
of 2 zeros

$$\text{total: } 1 + 10 + 45 = \boxed{56}$$

"at most 2 zeros"