CS1800

10/6 - Fri !!

Admin

- Hw3 due today 11:59pm
- Hw4 out now
  - ↳ incl. from 10/10
  - only one late day
- Exam #1 10/17 9-5:30pm
- Fri the 13th !! optional lecture - exam review
  practice exam

Agenda

1. Product/Sum Rule
2. Permutations
3. Combinations

II. Product Rule, Sum Rule

counting How many ways...?

task 1 – n ways to do it

task 2 – m ways to do it

Product Rule:

there are $\boxed{n \cdot m}$ ways to do task 1 and task 2

and ⟶ multiply!

Sum Rule:

there are $\boxed{n+m}$ ways to do task 1 or task 2

or ⟶ addition!

ex $L = \{GG, Only\}$ $T = \{FG, Archer\}$

Cartesian Product $L \times T = \{(GG, FG), (GG, Arch), (Only, FG), (Only, Arch)\}$

order matters ⟶ only Laney, then a Tom

$(FG, GG) \notin L \times T$

$L \times T \neq T \times L$

In general...

$$|A \times B| = |A| \cdot |B|$$

Because product rule!

task 1: select element from A (|A| ways)

task 2: select element from B (|B| ways)

Deck of cards

- 52 cards total
- 13 values per suit: A, 2, ... Q, K
- 4 suits: C, D, H, S

Universe: draw a card, put back in the deck (when relevant)

How many ways...

- to select a Q and then a K?

↳ multiply!

- task 1 x task 2

$4 \cdot 4 = 16$

Cartesian Product $\{QD, QH, QS, QC\} \times \{KD, KH, KS, KC\}$

$|Set1| \cdot |Set2|$

$= 4 \cdot 4 = 16$

How many ways ...

- to select 2 cards?
  - task 1: 52 task 2: 52

    total $52 \cdot 52 = 2704 = 52^2$

$\boxed{9S, 9S}$ repetition is ok

$\boxed{QD, JS \quad JS, QD}$ order matters

Bit strings: sequence of 0s and 1s

How many bit strings exist ...

- of length 7?

  7 tasks: select 1 or $\emptyset$

→ repetition is ok

1111111

→ order matters? yes!

$$\underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}}$$

- total : $2^7 = 128$ (n^k)

- of length 7, that start with 11

$$\underset{1}{\underline{1}} \quad \underset{1}{\underline{1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}} \quad \underset{2}{\underline{0/1}}$$

$1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

Flashback... Powerset

$S = \{a, b, c\}$

0 = not there

1 = there

a	b	c
0	0	0
0	0	1
0	1	0
0	1	1

....

task 1: a in subset? 2

task 2: b in subsets? 2

task 3: c in subset? 2

$2 \cdot 2 \cdot 2 = 2^3 = 8$

and ⇝ multiply

or ⇝ addition

How many ways...

- to pick Q then K, or K then Q
- task 1 Q then K: 16

- task 2 K then Q : 16
- totae : 16 + 16 = 32 ways

HOW MANY BITSTRINGS EXIST....

- of length 6, 7, or 8?

task 1: 2^6

task 2: 2^7

task 3: 2^8

$\boxed{\text{totae}}$ $2^6 + 2^7 + 2^8$

$= 448$

- of length 7, that start with 01 or 10?

task 1: 0 1 _ _ _ _ _ $= 2^5$

task 2: 1 0 _ _ _ _ _ $= 2^5$

total: $2^5 + 2^5 = \boxed{64}$

Counting Problem...

- repetition ok?
- does order matter?

} From the problem, or from context

now we know how to approach the problem

order matters no repetition ✓ permutation	order matters repetition ok ✓ n^k
order doesn't matter no repetition combination	order doesn't matter repetition ok stars + bars "balls into bins"

product rule

10:48

2. Permutations

$L = \{GG, Only, Rookie\}$

Watch all three shows → one of each, $\boxed{\text{no rep.}}$

$\boxed{\text{order matters}}$ → GG, only $\neq$ only, GG

Still product rule:

$$\underset{\text{Show 1}}{\underline{3}} \times \underset{\text{Show 2}}{\underline{2}} \times \underset{\text{Show 3}}{\underline{1}} = 6$$

$\boxed{\text{Also: Permutation}}$ → Formula

r-permutation

n objects, arrange r of them

$$\boxed{P(n,r) = \frac{n!}{(n-r)!}}$$

Shows example:

$n = 3$

$r = 3$

$$P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3 \cdot 2 \cdot 1 \; \boxed{= 6}$$

Formula Breakdown:

$\underline{n!}$ → select w/o repetition

$(n-r)!$ ⟶ tells us when to stop

ex 4 Queens in a hand

{QC, QD, QS, QH} ⟶ Start as set

How MANY WAYS...

- to arrange 2 of them?

$\underline{4} \cdot \underline{3} = \boxed{12}$

product rule

$n=4$

$r=2$

$$\frac{4!}{(4-2)!} = \frac{4\cdot3\cdot2\cdot1}{2\cdot1} = 4\cdot3 = \boxed{12}$$

perm formula

3. Combinations

order matters no repetition ✓ $\boxed{\text{permutation}}$	order matters repetition ok ✓ $\boxed{n^k}$
order doesn't matter no repetition $\boxed{\text{combination}}$	order doesn't matter repetition ok $\boxed{\text{stars + bars}}$ "balls into bins"

order doesn't matter... GG, only = only, GG

$$\boxed{n! = (n)(n-1)(n-2)\ldots(2)(1)}$$

Factorial

$4! = 4\cdot3\cdot2\cdot1 = 24$

$5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

...

gets v. big v. fast!

By hand...

- Pick 2 Queens
- order doesn't matter
- result goes in a set

{H, D} {C, H}

{S, C} {S, D}

{S, H} {C, D}

{D, H} xxx !! = {H, D}

Combination Formula

- n objects
- choose k of them
- order doesn't matter

$$C(n,k) = \binom{n}{k} = \boxed{\frac{n!}{k!(n-k)!}}$$

Choose 2 Queens

$n = 4$
$k = 2$

$$\frac{4!}{2!(4-2)!} = \frac{4\cdot3\cdot2\cdot1}{(2\cdot1)(2\cdot1)} = 3\cdot2 = 6$$

!!
◡

$n! \rightsquigarrow$ no repetition

$(n-k)! \rightsquigarrow$ when to stop

$k! \rightsquigarrow$ don't double count

HOW MANY BIT STRINGS EXIST....

- of length 7
- with exactly one zero?

Secretly Combination!

we are choosing the position of the zero

$\underline{0}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$

0

0

0

0

0

0

By hand

$\boxed{7}$ places to put the zero

everything else is ones

Combination:

$n = 7$

$k = 1$

$$\binom{7}{1} = \frac{7!}{1!(6!)} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7$$

How many bit strings exist of length 10 with 0, 1, or 2 zeroes?

- task 1 ~ 0 zeros 1

- task 2 ~ 1 zero 10

- task 3 ~ 2 zeros $\binom{10}{2} = \frac{10!}{2!8!} = 45$

choose positions of 2 zeros

total: $1 + 10 + 45 = \boxed{56}$

"at most 2 zeros"