CS1800

10/24 — Tue.

Admin

- Back to recitations this week!
- HW5 out on Fri, due 11/3

Agenda

1. Variance vs. EV
2. Joint / Conditional probability
3. Bayes Rule

Formulas

$$E[X] = \sum \Pr(s_i) \cdot X_i$$

Exp value

RV

$$Var[X] = \sum (x_i - \mu)^2 \cdot \Pr(s_i)$$

Cond.

$$\Pr(E|F) = \Pr(E \cap F) / \Pr(F)$$

Bayes

$$\Pr(F) = \Pr(F|E) \cdot \Pr(E) + \Pr(F|\neg E) \cdot \Pr(\neg E)$$

1. Variance

↳ B/c EV doesn't tell the whole story

↳ average

↳ EV, variance: whole picture

ex Roll a 6-sided die

(A) \$8 if 3, lose \$1 otherwise X

(B) \$1 if 2,3,4,5,6, lose \$2 otherwise Y

$$E[X] = \sum Pr(s_i) \cdot x_i$$

$E[X]$

$8 \cdot \frac{1}{6} + -1 \cdot \frac{5}{6}$

$\frac{3}{6} = \frac{1}{2}$

$\boxed{\$.50}$

$E[Y]$

$1 \cdot \frac{5}{6} + -2 \cdot \frac{1}{6}$

$\frac{3}{6} = \frac{1}{2}$

$E[X] = E[Y]$

$$Var[X] = \sum (x_i - \mu)^2 \cdot Pr(s_i)$$ → Can give us more context

$\boxed{\mu = E.V.}$

(A) $X_1 = 8, \quad Pr(s_1) = 1/6$

$X_2 = -1, \quad Pr(s_2) = 5/6$

$\mu = .5$

(B) $X_1 = 1 \quad Pr(s_1) = 5/6$

$X_2 = -2 \quad Pr(s_2) = 1/6$

$\mu = .5$

$(8 - .5)^2 \cdot (1/6) + (-1 - .5)^2 \cdot (5/6)$

$= 11.25$

$(1 - .5)^2 \cdot (5/6) + (-2 - .5)^2 (1/6)$

$= 1.25$

Higher variance $\Rightarrow$ higher risk, volatility

2. Joint / Conditional Probability

S = sample space = all possible outcomes

E = event space = $E \subseteq S$ = outcomes we care about

now we have > 1 event

- joint probability - two events occuring simultaneously
  events happen together $Pr(E \cap F)$ $Pr(E \wedge F)$

- conditional probability - one event occuring in the presence of another event. Have more information!
  $Pr(E | F)$

Probability table

- joint
- marginal
- (conditional)

(ex) roll a 6 sided die

A = roll is even

$\neg A$ = roll is not even

B = roll is prime

$\neg B$ = roll is not prime

$Pr(A \cap B)$ = joint = die is even + prime at same time

$Pr(A | B)$ = cond = even, given that it's prime

$A = \{2,4,6\}$

$B = \{2,3,5\}$

dep

	A	$\neg A$	
B	$1/6$ $A \cap B$	$2/6$ $B \cap \neg A$	$Pr(B) = 1/6 + 2/6 = 1/2$
$\neg B$	$2/6$ $\neg B \cap A$	$1/6$ $\neg B \cap \neg A$	$Pr(\neg B) = 2/6 + 1/6 = 1/2$
	$Pr(A) = 1/6 + 2/6 = 1/2$	$Pr(\neg A) = 2/6 + 1/6 = 1/2$	

→ marginal

Add all the squares... $1/6 + 2/6 + 2/6 + 1/6 = 6/6 = 1$:)

$Pr(A) = Pr(A \wedge B) + Pr(A \wedge \neg B)$

(total prob of A: happens w/ B, or w/ ¬B)

ex Draw balls out of an urn

2 urns, each has 6 red balls, 3 blue ones

- draw from urn #1, then urn #2
  (A) (B)

ind

	$A = \text{red}$	$A = \text{Blue}$	
$B = \text{red}$	$(2/3)\cdot(2/3) = \frac{4}{9}$	$(1/3)\cdot(2/3) = 2/9$	$Pr(B=\text{red}) = \frac{4}{9} + \frac{2}{9} = \frac{2}{3}$
$B = \text{blue}$	$(2/3)\cdot(1/3) = \frac{2}{9}$	$(\frac{1}{3})\cdot(\frac{1}{3}) = \frac{1}{9}$	$Pr(B=\text{blue}) = \frac{2}{9} + \frac{1}{9} = \frac{1}{3}$

10:48

$$Pr(E|F) = Pr(E \cap F) / Pr(F)$$

Pr of E given F

↳ more info!

ex Rolling a die

E = even F = prime

$Pr(E) = 1/2$ $Pr(F) = 1/2$

} what if I <u>know</u> F?

Intuition... primes are new sample space

$\{2, 3, 5\}$

$Pr(\text{even}) = 1/3$

$Pr(E|F)$

↳ $\dfrac{Pr(E \cap F)}{Pr(F)} = \dfrac{1/6}{3/6} = \dfrac{1}{6} \cdot \dfrac{6}{3}$

$\boxed{= \dfrac{1}{3}}$

ex Use Formula...

roll two dice

What is prob. one of the dice is a 4,

given that the sum is 7?

E = one die is a 4 F = sum is 7

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{2/36}{6/36} = \frac{2}{36} \cdot \frac{36}{6} = \frac{2}{6} = \frac{1}{3}$$

$E \cap F$ = all outcomes where E is true and F is true

(4, 3), (3, 4)

dependent

3. Bayes Rule

↳ expansion of conditional probability

more information ⟶ probability?

$Pr(cond) = ?$ $Pr(cond \mid test\ pos) = \underline{?}$

better answer!

What we know... (cond)

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

→ what if I don't know?

→ solve for $E \cap F$

$$\boxed{Pr(E \cap F) = Pr(F \cap E) = Pr(E|F) \cdot Pr(F) = Pr(F|E) \cdot Pr(E)}$$

What we know... joint

$Pr(F)$ ⟶ what if I don't know?

$$Pr(F) = Pr(F \cap E) + Pr(F \cap \neg E)$$

plug in

#2 $$\boxed{= Pr(F|E) \cdot Pr(E) + P(F|\neg E) \cdot Pr(\neg E)}$$

#1 $$\boxed{Pr(E|F) = \frac{P(E \cap F)}{Pr(F)} = \frac{Pr(F|E) \cdot Pr(E)}{Pr(F)}}$$

(ex) Tottenham Hotspur (soccer)

Larry's favorite player: Son

Setup:

- Son plays 20/38 matches
- Hotspurs win 75% games when Son plays
- Hotspurs win 40% games when Son doesn't play

Pr(Son was playing | we won)
?

$Pr(S) = 20/38$
$Pr(W|S) = .75$
$Pr(W|\neg S) = .40$
know

#1

$$Pr(S|W) = \frac{Pr(S \wedge W)}{Pr(W)} = \frac{Pr(W|S) \cdot Pr(S)}{Pr(W)} = \boxed{\frac{(.75)(20/38)}{??}}$$

What is prob that we won?

#2

$$Pr(W) = Pr(W \wedge S) + Pr(W \wedge \neg S)$$
$$= Pr(W|S) \cdot Pr(S) + Pr(W|\neg S) \cdot Pr(\neg S)$$
$$= (.75)(20/38) + \cancel{\&}\ (.40)(18/38)$$
$$= .58$$

$$Pr(S|W) = \frac{(.75)(20/38)}{.58} = \boxed{.68}$$

$$Pr(E|F) = \frac{Pr(F|E) \cdot Pr(E)}{Pr(F)}$$ → not known $P(E \cap F)$

→ not known! need to figure it out

$$Pr(F) = Pr(F|E) \cdot P(E) + P(F|\neg E) \cdot P(\neg E)$$