CS1800

10/10 - Tues.

Admin

- HW4 due Fri 11:59, only one late day
- exam in one week! 10/17, 2 hour window 9 - 5:30
- Fri (10/13) ⇝ optional lecture (practice exam)

Agenda

1. Stars + Bars (aka, balls into bins)
2. Breaking down (subtraction, overcounting)
3. Examples

Formulas:

$$P(n,k) = \frac{n!}{(n-k)!}$$

↝ no rep

↝ when to stop

ABC BCA
CBA ACB
CAB
BAC

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

↝ no rep

↝ like a permutation but don't overcount

{A, B, C}

↳ b/c order doesn't matter

1. Stars + Bars

- order doesn't matter
- repetition is ok

} indistinguishable objects

(ex) Bagels at the Dunkin'

- 3 flavors: S, E, G
  - ↳ for a given flavor, bagels are indistinguishable "identical"
- I want 4 bagels
  - ↳ necessary repetition
- They get placed in a bag
  - ↳ order doesn't matter

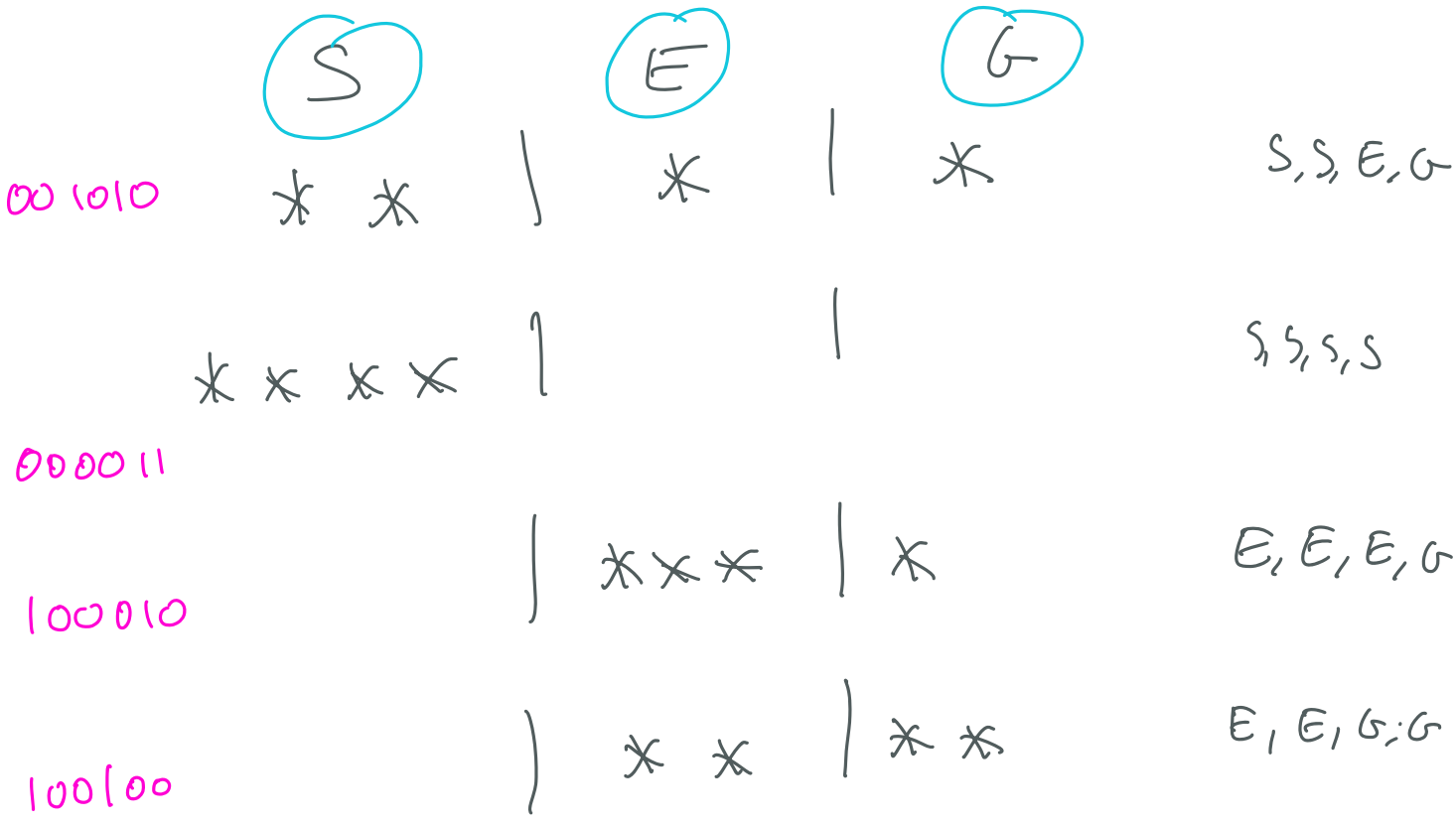
Possible outcomes:

- $\{S, S, E, G\} = \{S, E, G, S\} = \{E, S, S, G\} = \cdots$
- $\{S, S, S, S\}$
- $\{E, E, E, G\}$
- $\{E, E, G, G\}$

Approach: diagram the outcomes

* bagel

| divider between flavors



- notation makes sense ✓
- capture every possible outcome ✓

notice...

- all outcomes are same length (6)
- 2 choices for every character (*, |)
- all outcomes have same # of stars (4)

And so... it's a bit string question!! :)

Bar = 1

Star = 0

How many bit strings of length 6 have exactly 4 zeros?

- Choose the positions for the 4 zeros

$$\binom{6}{4} = \frac{6!}{2!4!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 4 \cdot 3 \cdot 2} = 15$$

There are 15 ways to have 4 bagels from 3 flavors

A Stars + Bars question

- order doesn't matter
- repetition is ok

} def. a combination

$$\binom{?}{?}$$

$$\binom{\#\text{bars} + \#\text{stars}}{\#\text{stars}}$$

$\#\text{bars} = \#\text{dividers} = \#\text{options} - 1$

$\#\text{stars} = \text{selections}$

Bagel example: $\#\text{bars} = 2$ (3 flavors)

$\#\text{stars} = 4$ (4 selections)

S | E | G

Lanny, Krysta are playing roulette (1-36)

- Lanny wins on odd
- Krysta wins on even

How many ways for 5 spins to turn out?

Is # games won by each person

- order matters? no
- repetition ok? yes

possible outcomes:

- LLLKK
- LLLLL
- LLKKK

Lanny	Krysta
xxx	xx
xxxxx	
xx	xxx

$$\binom{?}{?} = \binom{\#\text{Ls} + \#\text{Ks}}{\#\text{Ls}} = \binom{6}{5} = \frac{6!}{5!\,1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{5 \cdot 4 \cdot 3 \cdot 2} = \boxed{6}$$

2. Breaking Down Problems (subtraction / arranging)

Word problems for counting... **what is it asking?**

- does order matter?
- is repetition ok?

Often, more than one subproblem and need to put stuff together

- AND ... multiply!
- OR ... addition!
- at least ... break into cases

"At least 5..."

- exactly 5 OR
- exactly 6 OR
- exactly 7 OR
- ...

} cases (add them!)

- at most ... break into cases
- Lots of cases? ... subtraction rule
  - compute total possibilities
  - subtract invalid cases

- "arrange" ... order matters
- "choose" ... order doesn't matter

- "identical" ... stars + bars
- outcomes are named ... order matters
  - ↳ choose A, B, C?
    vs. choose 3 things

10:54

3. Examples

(ex) How many bit strings of length 7 have at least 2 zeroes?

Case 1: exactly 2 (OR)
Case 2: exactly 3 (OR)
Case 3: exactly 4
...

But this is tedious! So... Subtraction

- total: 2^7
- invalid: at most 1 zero
  - 0 zeroes: 1
  - 1 zero: 7

  } 8 total invalid
- atleast 2 zeroes: $2^7 - 8 = 120$

(ex) 10 players

Want to make 2 teams of 5

- w/in a team, order doesn't matter
- no repetition

(A) Aces, Liberty

Pick Aces **and** pick Liberty

$$\binom{10}{5} \cdot \binom{5}{5}$$

$$= \frac{10!}{5! \cdot 5!} = \boxed{252}$$

Aces: ABCDE

Liberty: FGHIJ

$\leftrightarrows$

FGHIJ ABCDE

(B) any 2 teams

$$\frac{\binom{10}{5} \cdot \binom{5}{5}}{2!}$$

$$= \boxed{126}$$

ABCDE, FGHIJ = FGHIJ, ABCDE

3 teams

A	B	C
123	456	789
123	789	456
456	789	123
456	123	789
789	123	456
789	456	123

no names

123, 456, 789

↳ to correct for overcounting, divide by

$\boxed{3!}$

Assume

- Deck of cards: 52, 4 suits, 13 of each suit
- 200 ppl in this room
- 5 instructors for CS1800

- we are in a battle royale. How many ways to have first, second, third place?

$$P(200,3) = \frac{200!}{197!} = 200 \cdot 199 \cdot 198 = 7880400$$

- there are 800 identical dollar bills. Everyone gets at least $1. How many ways to distribute the money?

Stars + bars → give everyone $1

Solve the problem w/ leftovers

200 people, $600

Stars: 600

bars: 199

$$\binom{600+199}{600} = \text{big number } :)$$

- Flush: 5 cards, all same suit

How many flushes exist?

- choose suit $\binom{4}{1} = 4$
- choose values (order doesn't matter) (no rep) $\binom{13}{5} = \frac{13!}{5!8!} = 1287$

- choose 2nd values: $4 \cdot 1287 = 5148$

Everyone in this room shakes hands w/ everyone else exactly once. How many handshakes?

- order matters? no
- rep ok? no

$$\binom{200}{2} = \frac{200!}{2! \, 198!} = 19900$$

↳ choosing pairs